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MDCCCLXXIX.

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PART II.

LIST OF ILLUSTRATIONS.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1878, by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to JEAN BAPTISTE BOUSSINGAULT, for his long-continued and important Researches and Discoveries in Agricultural Chemistry.

A ROYAL MEDAL to Mr. JOHN ALLAN BROUN, F.R.S., for his Investigations during thirty-five years in Magnetism and Meteorology, and for his improvements in Methods of Observation.

A ROYAL MEDAL to Dr. ALBERT GÜNTHER, F.R.S., for his numerous and valuable Contributions to the Zoology and Anatomy of Fishes and Reptiles.

The RUMFORD MEDAL to Mr. ALFRED CORNU, for his various Optical Researches, and especially for his recent re-determination of the Velocity of Propagation of Light.

The DAVY MEDAL to Messrs. LOUIS PAUL CAILLETET and RAOUL PICTET, for their Researches, conducted independently, but contemporaneously, on the Condensation of the so-called Permanent Gases.

The Paper "On Repulsion resulting from Radiation, Part V.," by W. CROOKES, F.R.S., was appointed as the Bakerian Lecture.

The Paper "On the Structure of the Stylasteridæ, a Family of the Hydroid Stony Corals," by H. N. MOSELEY, F.R.S., was appointed as the Croonian Lecture.

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CONTENTS

OF VOL. 169.

PART I.

PART II.

LIST OF ILLUSTRATIONS.

X. *On the Organization of the Fossil Plants of the Coal-Measures.*—Part IX.

By Professor W. C. WILLIAMSON, *F.R.S.*

Received June 7,—Read November 15, 1877.

[PLATES 19–25.]

IN Part I. of this series of memoirs (Phil. Trans. 1871, Plate 25, fig. 16, and Plate 27, fig. 39, p. 487) I described what appeared to be a transverse section of a Calamite, in which the woody wedges showed no traces of the longitudinal canal that occupies the innermost angle of each of these primary wedges in true Calamites. Not having at that time the materials which I have since accumulated, I was unable to say much about this exceptional specimen. I now know that it is not a Calamite, but a plant having a wholly different structure, and to which I propose assigning the provisional name of Astromyelon, from the peculiar stellate form which transverse sections of its pith exhibit. The plant had branching, unarticulated stems, a feature which at once distinguishes it from the Calamites, though transverse sections of the two plants exhibit such remarkable resemblances. It is one of the more common of the forms met with in the Oldham nodules. I have rarely seen specimens of it more than 0·25* in diameter. One example alone, represented in fig. 5, has had a diameter of nearly 0·75. Fig. 1 represents the more usual aspect of transverse sections of this plant enlarged 20 diameters. It consists of a central parenchymatous medulla, *a*, surrounded by an exogenous cylinder of vessels arranged in a very regular series of primary wedges, *b*, corresponding closely in all respects with those of Calamites, except in the absence of the long canals already referred to. The central cells of the medulla are much larger than those of the circumference, some of the former having a diameter of ·011, whilst the peripheral ones are not more than ·0041. In the longitudinal section, fig. 2, *a*, and fig. 3, *a′*, these medullary cells are seen to be somewhat elongated vertically, being often ·022 in length. The cells have usually rectangular partitions, and are arranged in vertical rows, as in many living ferns; one or two of the rows in immediate contact with the vascular zone (fig. 3, *a′*) have a similar arrangement, though they are much narrower in their transverse diameter. In the great majority of cases the pith is solid. In but a few instances have I found it otherwise. Two of the latter are represented in figs. 4 and 5. Fig. 5 further

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* These measurements are, as in my preceding memoirs, made in parts of an inch.

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presents an example in which the medulla has had an unusually great diameter in proportion to that of the vascular zone.

This exogenous zone is made up of a variable number of primary vascular wedges (figs. 1, *b*, 5, *b*), each one of which is composed of numerous radiating vascular laminæ separated by medullary rays. The remarkable uniformity in their size and the regularity in the arrangement of these wedges gives to the transverse sections of the medulla the star-shaped outline already referred to. At their inner or medullary apex each of these wedges commences at a few vessels of somewhat larger size than those composing the rest of the vascular zone. These are observable in fig. 1, but they are much more conspicuous in some examples than in others. Fig. 7 represents a section, for which I am indebted to Mr. BUTTERWORTH, in which these vessels are extremely conspicuous. The medulla of this specimen is much disorganized by mineralization, but it has not been fistular. Its diameter is much less than is usually the case, contrasting strongly in this respect with fig. 5. In Mr. BINNEY'S monograph on the Calamites* he figured, at page 20, what he believed to be part of a primary wedge of a Calamite, adding the remark, that in these Calamites "the wedge-shaped bundles of pseudo-vascular tissue originate from a small circular orifice or opening, sometimes simple, as in the specimen now under consideration, but in other instances apparently divided into several parts, as shown in the annexed woodcut (fig. 3)." I have no doubt that the latter examples were specimens of Astromyelon, which, like myself, Mr. BINNEY then mistook for Calamites.

The inner extremities of the primary woody wedges, where seen in transverse sections, have generally an obtusely convex outline rather than a sharply cuneate one. Fig. 5, however, represents an example in which they are much more wedge-shaped than usual, whilst in fig. 4, partly owing to the direction of the section, but yet more to a peculiarity in the specimen, the normal contour of these woody wedges is much less obvious than ordinary; still traces of their prevailing aspect are seen at the lower side of the base of the lateral branch *a′*. Each primary wedge is composed of a numerous series of laminæ, each one of which has a mean thickness of about ·002. The vessels of each lamina are arranged with the utmost regularity in a single radiating series, reminding us, so far as the transverse sections are concerned, of what we see in the woody prosenchyma of living Conifers. On making a tangential section we see that these vascular laminæ are separated by numerous medullary rays. Very frequently single laminæ are thus separated from each other. In other instances we see as many as three such laminæ unseparated by intervening rays. The vessels (fig. 6, *b*) rarely exhibit any special structure, but here and there, as at fig. 6, *b′*, we discover proofs that they were of the usual barred type, and that where the transverse bars are not visible their absence is merely a result of mineralization. The ordinary medullary rays (fig. 6, *c*) are arranged in single vertical series. The cells are elongated vertically and much compressed laterally. Occasionally a ray consists of a single

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* 'Observations on the Structure of the Fossil Plants found in the Carboniferous Strata,' 1868.

lenticular cell, whilst in others there are several cells arranged in a vertical series, the septa between these cells being square at right angles to their longer axes. The section represented in fig. 6 having been made at but a little distance from the medulla, we see at *c* one of the primary rays separating the primary wedges. It differs from the more numerous secondary ones in consisting of more than one vertical line of cells. As we approach the pith the number of such vertical series increases, whilst at the periphery these primary medullary rays become almost undistinguishable from the ordinary or secondary ones.

Notwithstanding the large number of specimens of this plant contained in my cabinet, I never succeeded in finding a solitary fragment in which the bark is preserved. Mr. BUTTERWORTH fortunately had better success. He met with the specimen represented in fig. 7. In this example the vascular zone is surrounded by a very thin layer of parenchyma (fig. 7, *d*), rarely displaying more than three or four cells in its thickness; more generally there are but two. The structure is obviously little more than an epidermal layer. At fig. 7, *d'* we have an indication of the loose way in which this layer was attached to the vascular zone, and which probably explains the excessive infrequency with which it is found in its normal position. This specimen further indicates the existence of concentric lines, suggestive of successive interruptions to the continuous growth of the vascular zone.

I have already observed that these stems were branched, such secondary branches ascending from the primary one at an oblique angle. In fig. 2 the section has intersected the base of one such branch in a very oblique manner. In the oblique transverse section, fig. 4, I endeavoured to incline the section so as to intersect the branch longitudinally. Though almost successful I was not completely so, since I did not quite trace the pith, *a'*, of the branch into connexion with that (*a*) of the main axis. It was sufficiently so, however, to demonstrate the continuity of the one with the other. In fig. 2 these medullary cells are intersected at *a'* near the base of the branch.

I have obtained no clue whatever to the plants to which these branches belonged. It is clear that structurally the individual woody wedges exhibit no difference whatever from those of a Calamite. Neither transverse nor tangential sections of the one can be distinguished from similar sections of the other. Their large primary medullary rays also exactly resemble those of Calamites. But the absence of articulations and the consequent absence of the alternation of the woody wedges in passing from one internode to another at once distinguish the two types. Further, the absence from Astromyelon of the remarkable internodal canals of Calamites, and also the difference in the medullary tissues of the two plants, constitute further distinctions between them. In Astromyelon the medulla was not fistular. The two exceptions, seen in figs. 4 and 5, are the only ones I have met with, and these may have been the result of desiccation rather than of a characteristic tendency to the formation of a fistular pith. That the growth of the vascular zone was exogenous is obvious enough. Though consisting wholly of barred vessels, the bundles are as open as in any other exogenous plant, yet we know of no gymnospermous stems that are entirely composed of such vessels, but which are so characteristic of the true carboniferous Cryptogams, hence I am inclined to place Astromyelon in the Cryptogamic group.

The plant just described is the only one belonging to a distinct type of vegetation left unnoticed in my previous memoirs. But since the earlier parts were published numerous objects have come under my observation throwing additional light upon several of the genera which I have previously described. These I now propose to examine in detail.

Calamites.

The wide-spread interest which these plants excite, and the continued existence of two sharply-defined and very discordant views respecting their nature and affinities, render further investigations necessary. Since my first memoir, which mainly treated of these objects, was published, I have gathered numerous additional illustrations of their history. The study of these specimens has only confirmed my previously expressed conclusion, that we possess but one type of these plants, and that the separation of Calamites from *Calamodendron* has no foundation in the facts of their organization. Fig. 8 is a transverse section of a small stem or twig not much more than ·033 in diameter. The medullary cells (*b*) are here unruptured, the medullary fistular cavity having as yet no existence. Nine longitudinal internodal canals are seen (*e*), and these form the only recognizable line of demarcation between the pith, *b*, and the bark, *h*. There is but little difference observable between the cells of these two structures.

Fig. 9 is obviously a decorticated specimen, similar in all other respects to fig. 8, but with eight canals, and the section has passed through the stem rather obliquely. In fig. 10 we have a transverse section through a similar stem or branch to fig. 8. We still discover the bark, *h*, the internodal canals, *e*, again nine in number, and the medullary parenchyma ; but the bark, in this example, is a thick layer of parenchyma of coarser tissue than that composing the medulla, and the latter now displays a central fissure, *a*, which obviously indicates the commencement of the formation of the medullary fistular cavity. We have still but very slight indications of the formation of woody wedges external to each of the internodal canals. Fig. 11 is a decorticated example in the same stage of development as fig. 10, only the medullary fistular cavity, *a*, is now of large size, and the internodal canals, *e*, are eighteen in number. Fig. 12, on the other hand, is another decorticated example in which the canals, *e*, are eleven in number, and the vascular wedges, *f*, are now advanced in development; but the central fistular cavity, *a*, is in a stage of development intermediate between that of fig. 10 and fig. 11. In fig. 13, the original of which was found by Mr. Butterworth, the stem has attained to larger dimensions, being now about ·12 in diameter. The layer of medullary cells, *b*, is now extremely scanty, whilst the fistular cavity, *a*, is very large. In fact, the thin layer of medullary cells does little more than link together the large primary medullary rays, *c*, which separate the large vascular wedges, *f*. These wedges, each with its internodal canal, *e*, at its inner angle, are here thirteen in number. The cortical parenchyma is seen at *h*.

My cabinet contains other similar specimens besides those figured; the series clearly illustrates one type of structure seen in various degrees of development, and figs. 12 and 13 unmistakeably demonstrate the type to be identical with the *Calamodendron* of Brongniart and those who accept his views. We here see the gradual formation of the central fistular cavity, and of the vascular wedges, from a condition in fig 8, in which they exactly resemble what we find in living Equisetums, viz., canals with extremely few vessels at their outer border, to that of figs. 12 and 13, where exogenous additions have developed a well-formed wedge composed of numerous radiating laminæ of vessels separated by medullary rays. The structure of these wedges was fully described in my first memoir, and my recent researches require that nothing should either be added to or subtracted from those descriptions. It is obvious enough that if such examples as figs. 11 and 13 had been deposited horizontally, parallel to the stratification of a bed of shale, and subjected to vertical pressure, they would have become as flat, and almost as thin, as a piece of brown paper. In other words, they would have resembled the Calamites of Brongniart, and yet they are all true examples of the genus *Calamodendron* of that author.

We may now turn to the different conditions seen in figs. 14 and 15, which represent, two-thirds their natural size, transverse and vertical sections of a fine Calamite found near Oldham by Mr. Nield. In the transverse section, fig. 14, the fistular cavity, *a*, has a mean diameter of about ·62. It is surrounded by an extremely scanty pith, *b*, which in turn is invested by a cylinder of vascular tissue fully 2 inches in thickness. At its inner surface this zone is divided into the usual radiating wedges, *f*, which have been either fifty-eight or fifty-nine in number. Each of these wedges starts internally from the characteristic canal, and they are separated from each other by the usual radiating extensions of the pith which I have always designated the primary medullary rays. These can easily be traced, extending outwards through the vascular cylinder as much as ·25, but beyond that distance the tissues become so blended together that the rays cease to be visible. In tangential sections made near to the pith these primary rays are very regular in form and arrangement, but long before reaching the middle distance between the pith and the bark they are only represented by a very small number of vertically elongated lenticular masses of cells, such as Mr. Binney has accurately represented in Plate iii., fig. 6, of his 'Monograph on Calamites and Calamodendron.' These cellular areas are arranged at very irregular intervals in all matured stems.

The aspect of the secondary rays in radial vertical sections is shown in fig. 16*d*. The vessels, *g*, are usually arranged vertically with great regularity, especially when seen in radial sections; but in one of my tangential sections there is an irregular vertical line varying from one-sixth to one-tenth of an inch in breadth, in which the vessels, instead of pursuing their regular course, are twisted and contorted, often dichotomising and interlacing in the most extraordinary manner, the irregular interspaces being occupied by expansions of the secondary medullary rays. As I described in my first memoir, this state of things is common enough in tangential sections where a branch is about to be given off, as in figs. 26 and 28 ; but I am wholly unable to explain its meaning here. Fig. 17 represents a small portion of the section referred to, in which *d* indicates the medullary rays and *g* the contorted vessels.

On turning to fig. 15, which is a vertical section made through the centre of the specimen, we find that the section has most fortunately passed through a node, the cellular extension of the medulla forming the nodal medullary diaphragm being seen at *n*. It will be observed that there is no nodal constriction at the exterior of the vascular cylinder, *f*, like that which is seen in the pith, and which forms so conspicuous a feature in the common sandstone casts of the medullary cavity. The most valuable, because, in my experience, unique feature of this specimen is seen in its bark, *h*. This is broken up into fragments, which, though often detached, and intermingled with Stigmarian rootlets that have penetrated between these fragments, nevertheless retain substantially their normal positions. A limited layer of parenchyma exists immediately external to the vascular cylinder, but the greater portion of the cortex consists of prismatic prosenchyma, the cells of which are arranged in regular radiating lines, an arrangement best seen in the transverse sections, as represented in fig. 18, which is enlarged 26 diameters. The cells have a transverse diameter of about ·0023. Their length varies greatly, but, as is the case with the cells of the primary and secondary medullary rays of the vascular zone, we find long radial lines of them having exactly the same length, each series terminating abruptly by coming in contact with other similar series composed of cells of a different but equally uniform length.

Tangential sections made through this tissue exhibit the appearance represented in fig. 20. The cells here have almost always the oblique or acute extremities of prosenchymatous tissue. Fig. 21 represents a small portion of a tangential section, enlarged 10 diameters, in which a lenticular mass of parenchyma appears. This tissue seems to me identical in character with the similar masses of cells seen in the vascular zone, and which, as I have already shown, are the outward prolongations of the primary medullary rays. It will be curious if future observations should prove that these cellular structures are prolonged through the prismatic layer of the bark to its periphery. Yet I strongly suspect that such has been the case.

I do not suppose that this layer of prismatic prosenchyma constitutes the outermost bark. The specimen when found by Mr. Nield was worn and weathered, hence we have no means of knowing what was the entire thickness of the bark ; the probability is that there would be an outer parenchyma in which the genetic line producing the prosenchyma would be found. This, however, is a mere probability which requires verifying. One thing is clear, viz., that the bark, as we see it in figs. 8, 10, and 13 is a primitive generalized parenchyma; but as the stems became arborescent, this generalized tissue developed within its interior the thick layer of prosenchyma which resembles so closely the cork layer of living Phanerogams. It will be remembered, however, that we find the same kind of tissue, in an almost identical position, in *Lepidodendron Harcourtii* as well as in all other Lepidodendroid and Sigillarian stems.

In my first memoir of this series (Phil. Trans., 1871, Plate 26, fig. 26; Plate 27, fig. 33, and Plate 28, fig. 38), I illustrated the general arrangement of the tissues at the node of the young Calamite. Since that memoir was published I have obtained some additional examples throwing light upon those curious structures. Plate 26, fig. 25, of that memoir represents a similar section to figs. 23 and 24 of the present one. In all these sections we see each of the vascular wedges of one internode (within which internode it is separated from its neighbours by the cellular primary medullary rays) subdivided into two portions, the half of one wedge uniting with the contiguous half of its next neighbour to form a new wedge in the next internode, whether above or below. I pointed out the existence at and near each node of two very distinct verticils, of objects. One of these verticils, situated immediately *upon* the node, appeared to me to represent bundles of specialised tissues going off to supply verticils of organs, probably small twigs or leaves. The second verticil instead of being planted *upon* the node occupied the uppermost portion of each internode. I showed that, in three distinct specimens, there existed at the uppermost extremity of each primary medullary ray a peculiar cylindrical or laterally compressed canal, from which all tissues had disappeared, and which was filled with the same inorganic material as that which occupied the fistular cavity of the pith. These passages I designated the infranodal canals. I further demonstrated, in fig. 22 and p. 490–91, that the areas which these canals occupied were originally filled with the cells of the primary medullary rays through which the canals subsequently passed. These cells became first ruptured, and secondly absorbed, leaving behind a verticil of sharply-defined canals arranged in a regular radiating order and passing to the periphery of the vascular zone. Some sections of a Calamite, for which I am indebted to Mr. BUTTERWORTH, exhibit these organs under a somewhat new aspect. In fig. 23 we have the vascular wedges at *f* and the primary medullary rays at *c*, *c′*, *c″*, the former exhibiting the nodal bifurcations to which I have just referred. We here see that at the nodal extremity, *l*, of each medullary ray of the lower internode, *c′*, *c′*, there is a small mass of cells of much smaller size than those which constitute the rest of the ray. These special cells occupy the centre of a lenticular enlargement of the end of the ray. It will also be observed that the lower extremity of each of the rays, *c″*, *c″*, of the upper internode the ray communicates by a very narrow neck with a small lenticular cellular organ, *m*, the central cells of which again are much smaller than those of its periphery. There is no question whatever that the small cells, *l*, *l*, of the lower row of rays occupy the exact position of my infranodal canals, whilst those terminating some of the upper series, c'', c'', indicated by *m*, appear to be identical with those similarly indicated in fig. 25 of my previous memoir on Calamites. It was perfectly clear to me when that memoir was written, that though the cellular structures, *l*, appeared to be but modifications of the primary medullary rays through which they pass, they nevertheless had some specialised character and function; and that their existence characterised the subgenus *Calamopitus* which I proposed to establish for the reception of the plants which exhibit these peculiar organs. Mr. BUTTERWORTH's sections, especially that represented in fig. 24, confirm these conclusions. I have already shown that in all Calamites, save very small ones, soon after leaving the medulla the primary medullary rays almost entirely disappear. They have done so in the present figure, in which their normal position in the upper internode is only indicated by the dark lines *c*, *c*. This section is also tangential, but more peripheral, from the same specimen as that which furnished fig. 23; but it will be observed that as the exterior of the stem has been much enlarged by exogenous growths, all the lenticular organs have also become much larger both in length and breadth than is the case in that figure. The remarkable feature revealed by this section is, that whilst all the primary medullary rays have disappeared, their lenticularly enlarged upper extremities, fig. 23, *l*, remain as the circumscribed lenticular areas, *l*, of fig. 24. These two figures being drawn to the same scale, *i.e.*, enlarged 8 diameters, a comparison of the organs, *l* of fig. 24, with the corresponding terminations, *l*, of the primary medullary rays, c' of fig 23, will show how much these organs, whatever their functions, have increased in size as they proceeded outwards. A radial section shows that they contain no traces of vessels. Fig. 25 represents one of these lenticular organs enlarged 25 diameters. The cells of its upper and lower extremities are large, like those of the primary medullary ray of which it is a remnant, whilst those of its centre are small. Their appearance is much the same in the radial section, only in it they are slightly elongated radially, *i.e.*, in the direction in which they are proceeding to reach the bark.

These facts, I think, show that I was perfectly accurate when in my memoir Part I., I assigned great importance to these organs. In that memoir I regarded the upper and smaller series, *m*, as supplying leaves or small twigs, believing that they contained vascular bundles. I am doubtful if I was correct on this point.* I believe that the transversely divided structures are chiefly cells, as already stated. In three cases I traced the development of the lower series, *l*, into verticils of the radiating infranodal canals, illustrated by fig. 31, which represents the lower extremity of the sandstone cast of the medullary cavity of a Calamite surrounded by the disorganised carbon, *f f*, of its vascular zone. The radiating spokes, *l*, are also sandstone casts of the interiors

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* Since this was written I have studied a vertical radial section of a Calamite sent to me by Mr. BINNS, of Halifax, in which some of the vessels ascending from below are, on reaching one of these lenticular organs, conspicuously deflected outwards, making it clear that *some* vessels do accompany these cellular radii in their outward course, confirming my original conclusion.

of the infranodal canals of one internode, whilst in *l′*, we have a few of the similar organs from the next inferior internode. The positions occupied by others which originally completed this second verticil are indicated by a ring of the small rounded scars so commonly seen on the ordinary Calamites (fig. 30, *l*). There can be no doubt whatever that these spokes represent the same parts of the plant as the lenticular cell masses, *l* of figs. 23 and 24. Why these organs continued to be occupied by cellular parenchyma in some specimens whilst in others the cellular tissues were absorbed when the medullary cells underwent the same fate, the former being thus converted into radiating prolongations of the medullary fistular cavity thus produced, I cannot determine. The most probable solution is that the two conditions represent different species of plants, in some of which the cellular elements of the radii had but a transitional temporary use, whilst their function was more permanent in others.

Some further light is thrown upon these anomalous structures by the specimens represented in figs. 26 and 27. These are sections of a large Calamite found by Mr. Butterworth, the surface of the node of which exhibited a few deep circular depressions, that appeared to have been points from which branches had been detached.

A transverse section of the structure underlying one of these depressions is represented in fig. 26, and a vertical one in fig. 27, both figures being enlarged five times. Fig. 26 is a tangential section of the vascular zone of the primary stem, the vessels of which are seen at *g*, *g*, separated by their numerous accompanying secondary medullary rays. These vessels are deflected right and left to allow of the exit of a cylindrical mass of parenchyma in which numerous vessels meander in a tortuous manner, especially at the upper and lower margins of the cellular area. The diameter of this cellular mass is about ·25. Fig. 27 is a radial section through a portion of the same Calamite in which a similar branch is again intersected. At *a* we have part of the medullary fistular cavity, and at *b* a few of the true medullary cells; at *g*, *g* we have the vessels of the vascular zone, which, it will be observed, bend outwards from the medulla, both above and below the incipient branch, forming arches across the node, in the way which I have previously described as characteristic of their nodal arrangement. At *g′*, *g′* the vessels are intersected more or less transversely because of their lateral deflexion to pass round the cylindrical branch, *m*. At its inner extremity, *m′*, this cellular outgrowth from the medulla is only about ·033 in diameter, but it increases rapidly in size as it proceeds outwards, until, at a distance of ·16 from the point at which it enters the vascular zone, it becomes, like fig. 26, enlarged to ·25 in diameter. It will be noticed that whilst numerous vessels meander amongst the cells of the periphery of the central branch of fig. 26, there is no approach to a radiating arrangement of these vessels, or to the formation of a distinct vascular cylinder. It is otherwise with fig. 28, which represents a similar section to fig. 26, enlarged 30 diameters, from a decorticated Calamite in my cabinet, the diameter of which stem is about 1·25, and the thickness of its vascular cylinder averages about ·25. The primary medullary rays are seen at *c*, and the intervening primary vascular wedges at *f*. At *m* we have a verticil of small lenticular cellular areas resembling fig. 23, *m*, and in the centre of the figure is a large circular area representing a branch. In this latter structure we have a medulla composed of parenchyma, the central cells of which are larger than the peripheral ones. This medulla is encompassed by a vascular zone composed of laminæ of vessels arranged in slightly curved radiating lines, and which are, beyond all question, derived from the surrounding vascular tissues of the parent stem. The continuity between the longitudinally intersected vessels of the latter, and the transversely divided ones of the branch, is clear and unmistakeable. We further see that the latter evince an equally obvious disposition to arrange themselves in numerous small radiating vascular wedges, but which exhibit no trace of the large internodal canals invariably found at the inner extremities of the wedges of true Calamitean stems. I am not certain that these incipient wedges are identical with the very regular primary vascular wedges seen in ordinary Calamites, but they are obviously preparing to assume that condition.

The stem which furnished the last section also supplied me with some others. One of these made near the periphery of the vascular zone corresponded very closely with fig. 26. On making a second section parallel with the first one, but nearer to the medulla, I obtained the result seen in fig. 29. I think there is no room for doubting that the lenticular area, *m*, is the representative of the similar areas indicated by the same letter in fig. 23, and that it has been the virtual starting point of the young branch ; but we further find some enlarged secondary medullary rays, *m′*, *m′*, which may have contributed towards the cellular elements forming the pith of the young growth. This section seems to prove that, as in the higher exogens, a connexion is maintained between the medulla of the central stem and that of its branches through the intermediate agency of its enlarged medullary rays.

I think we can have no hesitation in concluding that in the organs just described we have the beginnings of peripheral branches before they have emerged from the parent stem, and that the specimen shown in fig. 38 of Part I. of these memoirs was, as I there suggested, a similar organ. It appears to me further obvious that the organs which I there designated infranodal canals, but which it now appears sometimes retain their primitive cellular tissues instead of their becoming absorbed, must have fulfilled some important function, since, though the large *primary* medullary rays, in which these organs originate, contract their dimensions as they proceed outwards until they become virtually merged in the inconspicuous secondary medullary rays of the woody wedges, these radial organs not only do *not* thus disappear, but actually become larger, and their boundaries become more clearly defined as they proceed outwards through the vascular zone towards the bark. This circumstance explains the great definiteness with which we discover the circular and oblong scars marking the position of these radii, even in the ordinary specimens of Calamites found imbedded in the coal-shales.

But we still remain encompassed by physiological and morphological difficulties. Thus we have seen that as the rudimentary branches pass outwards through the vascular zone they carry along with them many of the vessels of that zone, arranged as shown in fig. 28. But in none of these branches thus enclosed within the primary stem do we see any development of nodes with their peculiar vascular arrangements or of the longitudinal internodal canals. Are there two sets of branches, one with and the other devoid of these characteristic peculiarities? Such is very unlikely to be the case. Do the vascular elements of the branch of fig. 28 re-arrange themselves into the characteristic Calamitean form *after* they emerge from the parent stem? The curious "Phragmata" existing in the closest connexion with the grooved, sandstone, medullary casts of the common specimens of Calamites seem to indicate the contrary, and to show that the peculiar conditions which occasioned the grooved contours of the medullary casts of the parent stem, existed equally in the branches where the latter were in close contact with the parent medulla. Then, again, what are the examples represented in figs. 8, 9, and 10 of this memoir, in which we have states whose size indicates even a much earlier degree of development than is seen even in the branch of fig. 28? In them the formation of the longitudinal canals clearly *precedes* that of the vascular structures; a stage of growth in which they closely resemble the living *Equisetum;* whereas, in fig. 28, we have a considerable development of the vascular tissues, but, as yet, no canals. My own impression is that, minute as many of the examples represented by such figs. as 8, 9, 10, 12, and 13 are, they are young plants directly developed from spores; and that in their half-embryonic state they represent *temporarily* the permanent condition seen in the more degraded Equiseta of the present day.

Fig. 30 is a suggestive example of a Calamite in blue shale from the Upper Coal-measures of the Manchester district. It is in the cabinet of Mr. DRINKWATER, formerly of Manchester, but now resident in the United States of America. It is obviously one of those examples which some of my friends would regard as not being a *Calamodendron,* but a Calamite of the supposed Equisetaceous type of *C. Cistii* and *cannæformis.*

According to my hypothesis, *a* is the inorganic cast of the interior of the medullary fistular cavity of a primary stem, and *a′* is a similar cast, only belonging to a lateral branch. The former organic connexion of the two is shown by the carbonised area, *f,* which I have no doubt is a relic of the thick vascular layer with which the pith was originally invested. The large medullary cavity of the branch *a′* appears as if it had not reached the corresponding cavity, *a*, of the central stem, but terminated at *a″*, or fully a $\frac{1}{4}$ inch from it. If this was the case this specimen would seem to indicate that the characteristic Calamitean features of a fistular cavity, viz., of distinct nodes, longitudinal grooves and the verticils of scars marking the position of the infranodal passages, began to appear in very young branches, not at the medullary surface of the vascular cylinder of the parent stem, as the phragmata already referred to seem to suggest, but at some distance from that surface; in which case the connexion between the two medullæ, *i.e.*, of the stem and of its branch, might be maintained by means of such cellular structures as those seen in fig. 27. If this has been the case then the possibility which I have suggested would become an established fact, viz., that so long as the *young* branch was passing outwards through the vascular zone of the parent axis, it did *not* acquire those features which we have seen to be so remarkably characteristic of Calamitean organization, but that when they did completely emerge from the stem those features appeared, at first in a very moderate degree, but soon undergoing rapid development both as to definiteness and size. It is impossible to believe that the large branch of fig. 30 was connected with its parent stem only by the tip of the small mamilla, *a″*, constituting its inner extremity. There must have been a large amount of tissue of some kind uniting the two firmly together, but which tissues have disappeared during mineralization—the only trace left of the former existence of that tissue being the dark stain, *f*, which still connects the stem and branch, and which demonstrates that their union is organic and not accidental; whether the stem and branch still continue at the same distance apart from each other that they held when living may be questioned--but all the numerous facts which we now possess show that a branch possessing a medullary fistular cavity of the dimensions seen in the specimen under consideration, must have possessed a vascular cylinder of considerable thickness to sustain its weight, even when we have made all necessary allowance for specific variations in the relative dimensions of these two parts in such plants.

Since writing the above descriptions, I received from Mr. ISAAC EARNSHAW a specimen which, on being cut into, revealed important evidence bearing upon the question of the branching of Calamites. A transverse section of part of the specimen is represented, two-thirds of the natural size, in fig. 31B, and a longitudinal section is shown in fig. 31A. At 31B, *a* is the medullary cavity of the parent stem, whilst at *a′* of the same figure we have the similar cavity of a lateral branch, *m*. The vertical section, fig. 31A, shows that the branch, *a′*, is given off from the stem, *a*, nearly at right angles to the latter; we here learn many truths: 1st, that the medulla of the smaller branch is fully as large as that of the parent stem, as is the case in the specimen fig. 30; 2nd, that the woody wedges, with their large primary medullary rays, are equally developed in both; 3rd, that only the outer and newer exogenous vascular layers of the parent stem extend over and form the vascular zone of the branch; 4th, at *i*, *i*, we have the two sides of a node of the parent stem—being that from which the branch obviously originated—and at *i′*, *i′*, we have the next superior node of the same axis. On the branch I discover no trace of a similar node. These two sections clearly prove, beyond possibility of further doubt, that whatever may have been the condition of the lateral branches in their very young state, such branches as did not fall off or become abortive, gradually became invested by the successive woody layers that were added to the parent axis subsequently to the first appearance of such branches. The two grew together, and thus the relations of stem and branch became exactly the same as those observable in any ordinary exogenous tree. It thus becomes evident that the correctness of my argument in favour of my opinion, viz., that the medullary casts of both the stem and branch of fig. 30 were once enclosed within a *thick* woody zone, but which has disappeared during mineralization, is amply confirmed, and that when, in our imaginary restorations of the mature Calamite, we give to it a straight substantial stem with verticils of extremely slender twigs we make a mistake. That such was the case with very young stems is more than probable; but my specimens seem to show that many of the twigs of each verticil were arrested at an early stage of their development, whilst the few that were not so arrested did not differ materially in their external appearance from the branches of an ordinary Pinus.

I presume that these facts will, at the first glance, give seeming support to those who still believe in a group of gymnospermous Calamodendra, but I cannot accept this inference.

It is to me a surprising fact that there can be palæo-botanists who still believe in the existence of Equisetaceous Calamites apart from gymnospermous Calamodendra. Oldham, Halifax, and Autun, have now supplied immense numbers of stems of all sizes, from the delicate shoot represented in my figure 8, to the large arborescent stem shown in figs. 14 and 15. How is it that we nowhere find a solitary fragment which can be identified with the Calamites of BRONGNIART and M. GRAND-'EURY? Even the latter author is obliged to admit: "Il est au moins surprenant qu'on n'ait pas mit le main sur un Calamite avec la structure conservée."[*] There can have been nothing in the structure of an Equisetaceous plant to prevent its conservation equally with the delicate rootlets of the Stigmariæ which are so abundant in our carboniferous nodules. The supposed Calamites have been too abundant for the absence of the required specimens to be explained on the ground of rarity; yet where are they? I marvel that this fact alone does not satisfy my friendly opponents on this question, that Calamites and Calamodendra are identical plants. But this is not all; M. GRAND-'EURY speaking of Calamodendra further admits: "Le bois est formé par l'alternance des lames rayonnantes de tissus différents, cancrelé par suite de cette composition, articulé par l'arrêt et le croissement des lames aux articulations, qui, étant en outre accompagnées de cicatrices raméales, *donnent lieu, sur le surface, à une véritable forme de Calamite.*" The italics of the last noteworthy observation are mine. The statement thus emphasised is not only true, but even more thoroughly so than the above sentence indicates. In addition to the perfect identity, in the two cases, of the arrangement of the vertical lines of alternating vascular and cellular laminæ, and their change of relative position at each node, there are yet further resemblances to be noted. We have seen that at the nodes of my plants, which M. BRONGNIART regarded, and M. GRAND-'EURY still regards, as Calamodendra, there exists the two verticils of lenticular areas seen in fig. 24, and one of which verticils we have further found to be identical in position and origin with the infranodal canals represented by the elongated

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[*] 'Flore Carbonifère du Département de la Loire et du centre de la France,' par F. CYRILLE GRAND-'EURY. Mémoires présentées par divers savants à l'Académie des Sciences de l'Institut de France, 1877. Première partie, Botanique, p. 30.

spokes of fig. 31. Whatever these latter objects may have been, they are very remarkable and uncommon appendages to a vegetable stem. They coexisted with a thick vascular Calamodendroid zone, and it is their medullary extremities that are represented by the scars, *l*, of fig. 30; even when that vascular zone has disappeared, and nothing remains but the inorganic cast of the medullary cavity, these scars also remain, indicating the former existence of the canals to which they owe their origin. Both these verticils of nodal appendages are discoverable in most of the objects recognised as Calamites by BRONGNIART, GRAND-'EURY, and those who agree with the French school of palæo-phytologists on this point. Yet I am asked to believe that these extraordinary combinations of detailed structural resemblances are to be found in two classes of plants so remote from each other as are the Cryptogamic Equisetaceæ and the Gymnosperms. Surely such an instance of mimicry as this would, if true, be far more remarkable than any of Mr. WALLACE'S illustrations of that curious phenomenon. But I am convinced that it is not true, and still hold that all the supposititious distinctions upon which M. GRAND-'EURY relies in his recent volume, for separating Calamites from Calamodendra are fallacious ones, and that they are altogether outweighed by the structural identities to which I now once more call attention.

Asterophyllites.

The next specimen to be noticed is one throwing additional light upon some described in my fifth memoir as belonging to the genus *Asterophyllites*. It will be remembered that M. RENAULT described certain stems which he regarded as those of *Sphenophyllum*, and he was doubtless justified in doing so. In the memoir referred to I showed that numerous stems which I believed to be those of *Asteropyllites* had, typically, the same structure as those of *Sphenophyllum*. In his recent work on the coal-measures and coal-plants of central France, my friend M. GRAND 'EURY disputes the correctness of my determination. He says, "Il est donc, en tout cas, bien certain que nous avons à faire à la structure des tiges de *Sphénophyllum*, et non à celle des *Astérophyllites*, comme M. WILLIAMSON le prétend, d'après des exemplaires calcifiés munis de feuilles plus nombreuses, apparemment simples; mais le Bechera grandis paraît se rapporter à quelque *Sphénophyllum*, et je connais des tiges de *Sphénophyllum angustifolium* avec de nombreuses feuilles aciculaires à peine soudées légèrement deux à deux à la base."* In my memoir, Part V., pp. 48, 49, I calculated that the plants there described must have had about 24 long linear leaves in each verticil, and I further pointed out that the transverse sections of these leaves (*loc. cit.*, Plate III., figs. 14, 17) displayed a central thickening, indicative of the existence of a single midrib to each leaf. The specimen which I now figure, fig. 32, is a transverse section of the same stem as that represented in fig. 2 of Part V. of my memoirs, but that figure merely represented a section of an internode. The section now published has passed *obliquely* through a

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* 'Flore Carbonifère Botanique,' p. 50.

node, intersecting at once both some leaves and the commencement of a branch. The characteristic primary vascular, triangular, axis of the stem is seen at *c*, enclosed within a single exogenous series of vessels, *d*, as described in my previous memoir. The inner bark, *g*, has disappeared, but the outer bark, *k*, remains perfect. In the section previously figured the outline of the bark exhibited the three deep internodal furrows so common amongst these plants, but in the section now described these internodal furrows are, as is usual at the nodes, not present. On the lower margin of the figure we see in the outer bark the row of circular areolæ, *l*, *l*, which are obviously points from which a corresponding number of vascular bundles have disappeared. At l' we find one of these circular areolæ exactly opposite the base of the leaf *m*, and at l'' we find that the next similar areola is intersected more in the direction of its length and is actually being prolonged into the centre of the leaf m'. There can be no doubt whatever that these areolæ represent a corresponding number of primary vascular leaf-bundles, and that there was but *one* of these bundles to each leaf. That the objects m, m' are leaves and not branches is fortunately demonstrated by the fact that at x we have a small triangular bundle of the usual characteristic shape, which is obviously going off to supply a true branch. Since there are nine of the leaf-bundles in the left half of the section it is obvious that had that section also passed exactly through the node on the right hand side of the specimen, we should have had at least eighteen of these vascular bundles going off to a corresponding number of leaves.

In determining whether this specimen is an *Asterophyllites* or a *Sphenophyllum*, we must understand what the distinctive features of these two genera are. BRONGNIART defines *Sphenophyllum* thus:—"Feuilles verticillées, cunéiformes, tronquées, entières ou dentées, émarginées ou profondément dichotomes, quadrilobées à lobes plus ou moins profonds et grêles."* He adds that "dans quelques espèces, les lobes deviennent profonds, étroits et linéaires, et peuveut être pris pour autant de feuilles distinctes analogues à celles des *Astérophyllites* avec lesquelles il est alors facile de les confondre," and again he remarks, "il faille une grande attention pour ne pas le confondre avec certaines espèces *d'Astérophyllites*. Il se rapproche, en effet, de ces plantes par la disposition verticillaire de ses feuilles, mais il en diffère par le nombre beaucoup moindre de ces organes à chaque verticille, 6 à 8 ou 10, et par leur forme qui est triangulaire, tronquée au sommet, ou dentés et lobés quelquefois très profondément." Now, the leaves of my plants exhibit none of these features, being long, linear, and entire, and from eighteen to twenty-four in each verticil. It is impossible to look at the two leaves m and m' of fig. 27 without seeing that they show no signs of becoming cuneiform. Still less can we examine those described in my memoir, Part V., and figured in Plate 3, examples of which I made sections in every possible direction, without arriving at the same conclusion. Messrs.

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* 'Tableau des Genres de Végétaux Fossiles,' p. 52.

Coemans and Kickx, in their 'Monographie des Sphenophyllums d'Europe,'* define these plants as "foliis cuneatis, sessilibus, verticillatis, *nervo medio destitutis*, nervulis autem æqualibus, dichotomis." Here again we have an obvious distinction between a clear definition and my plants. The circle of areolæ in my fig. 27 clearly mark the positions of a verticil of median leaf bundles, whilst the leaves *m* and *m'* in that figure as clearly show that but one such leaf-bundle went to each leaf, hence we have here no dichotomization. Had these nervures branched we should have found that of 24, *m'*, doing so long before it had advanced so far into the leaf. Connecting this with the fact that none of my other sections show a trace of such multiplied nervures as are seen in those of *Sphenophyllum Stephanense*, figured by M. Renault,† I still conclude that my examples are true *Asterophyllites*. It is true that M. Grand-'Eury recognizes a group of Sphenophylla, "où les feuilles, en nombre variable des tiges aux branches, ont un seule nervure radicale et correspondant sur la tige a autant des petites cotés."‡ But the plants which he places in this group are only the several varieties of the well-known and well-marked species, *Sphenophyllum Schlotheimi* and *augustifolium*. That my plants do not belong to this group is obvious enough, yet those composing it are the only ones which, like mine, have the "nervure radicale unique." Even were it otherwise the number of the leaf bundles in my specimens are very far from being "correspondant sur la tige a autant des petites cotés." I see no reason therefore for abandoning my original conclusion that my plants are true *Asterophyllites*, and that *Asterophyllites* and *Sphenophyllum* are genera so closely allied that their separate existence finds but little justification in nature.

Before leaving the subject of *Asterophyllites*, I may express my surprise that Mr. Carruthers, in a recent article § reiterates his belief in the occurrence of *elaters* in connection with the spores of *Calamostachys Binneana*, notwithstanding my clear demonstration in my fifth memoir, that they do not exist. Mr. Carruthers implies by this reiteration his continued belief in the Calamitian character of these fruits; whereas the very remarkable difference in the structure of the cellulo-vascular axes of the two plants renders it absolutely impossible that such can have been the case. I need only quote the late Dr. Robert Brown's estimate of the value of such differences. "To the argument derived from an agreement in structure between axis of stem and strobilus I attach considerable importance, an equal agreement existing both in recent and fossil Conifera."‖

It is curious that both M. Grand-'Eury and Dr. Dawson have fallen into the accidental error of making me regard the *Calamostachys Binneana* as belonging to

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* Bulletins de l'Académie Royale de Belgique, 2^{me} série, tome xvii. p. 138.

† Annales des Sciences Nat., 5^{me} serie, Bot. tom. 18, plate 4, figs. 3 and 4.

‡ *Loc. cit.*, p. 49.

§ 'Contemporary Review,' February, 1877.

‖ "Some Account of Triplosporites, an undescribed Fossil Fruit," Trans. Linn. Soc., vol. xx. p. 471.

Calamites, whereas I have most strongly opposed that idea,* on the identical grounds recognised by Dr. BROWN in the case of Triplosporites, viz., that there must be a correspondence between the internal structure of the stem and that of the axis of the strobilus whenever the two belong to the same plant, which correspondence not only does not exist in the plants in question, but is as remote as possible from doing so.

Lepidodendron and Sigillaria.

Notwithstanding all that has been done in elucidating the relations of these two groups of plants much still remains obscure. The present position of the question appears to be this. The first stem of a *Lepidodendron* discovered, in which the internal organization is preserved, was that historic one originally described and figured by WITHAM,† and named by him *L. Harcourtii.* The same specimen was further described and figured by LINDLEY and HUTTON,‡ and again by BRONGNIART.§ This plant, which in my opinion only represents one extreme modification of the *Lepidodendroid* organization, and that the least highly organized one, was for so long a time the only known example in which the internal structure was preserved, that it came to be regarded, especially by BRONGNIART and those who implicitly accepted his conclusions, as typical of the entire Lepidodendroid group. In the volume which contained his description of his *Lepidodendron,* WITHAM described his *Anabathra pulcherrima* (*loc. cit.*, p. 74, plate 8, fig. 7–12), a plant which, as we now know, only differs from the *Lepidodendron* in having a second exogenous vascular zone, enclosing the primary one of the Lepidodendra. WITHAM does not hazard any opinion as to its botanical affinities. BRONGNIART ignored this plant in his 'Vegetaux Fossiles,' but at a later period he obtained a specimen which he made the subject of his classic memoir on *Sigillaria elegans.*‖ This latter plant possessed many of the features of WITHAM's *Anabathra*, but exhibited some differences in the structure of its inner vascular cylinder, which in *Sigillaria elegans* consisted of an *interrupted* vascular ring, whilst the *Anabathra* displayed a *continuous* one. Then followed the publication of CORDA's 'Flora der Vorwelt,' in which the author describes his genus *Diploxylon,* a form which unites the features of *Lepidodendron Harcourtii* with those of *Sigillaria elegans,* having the *continuous* inner vascular cylinder of the former associated with the exogenous zone of the latter. In his 'Tableaux des genres de Végétaux Fossiles,' p. 57, BRONGNIART very properly places CORDA's genus *Anabathra* amongst his "Sigillariées," and with equal accuracy identifies it with the *Anabathra* of WITHAM. Various other publications by Dr. DAWSON, Mr. BINNEY and myself have followed those quoted,

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* See my fifth memoir, Phil. Trans., 1874, p. 65.

† 'The Internal Structure of Fossil Vegetables, &c.,' 1833, p. 51, plate 13.

‡ 'Fossil Flora,' tom. ii. p. 45, plates 98 and 99.

§ 'Vegetaux Fossiles,' p. 38, plates 20 and 21.

‖ Arch. Mus., tom. i. p. 405, plates 25–28.

besides those of M. RENAULT and M. GRAND-'EURY to which I shall have to refer more specially. But the point at issue resides in a nutshell. One school follows BRONGNIART in believing that *Lepidodendron Harcourtii* is the true typical representative of the structure of the Cryptogamous Lepidodendra, whilst *Sigillaria elegans* and *Diploxylon* represent the Sigillariæ which are not only supposed to have no affinity with the Lepidodendra, but belong to the very different group of the flowering Gymnosperms.

In several of my previous memoirs I have arrayed a series of facts which appear to me conclusive, and which lead me to reject this separation of the Lepidodendra from the Sigillariæ, and I should have been inclined to have left those facts, and the arguments based upon them, to stand or fall by their own accuracy. But in his recent volume on the coal-measures of Central France, my friend M. GRAND-'EURY has put my arguments in so erroneous a form, that I deem it necessary to correct the errors, and to advance additional evidence in support of my views. In my third memoir, I showed in a way which no one has attempted to answer, that, in its young state, the *Anabathra* of WITHAM is a true Lycopod, of the type of *Lepidodendron Harcourtii.* But that after a time, in addition to *the inner ring of vessels characteristic of the Lepidodendron,* it began to develop *the outer exogenous ring characteristic of* BRONGNIART'S *family of Gymnospermous Sigillariæ.* The conclusion to which these facts lead us is inevitable, and the argument unanswerable, except it can be shown that I am mistaken in my facts, which no one has even attempted to do. I have very little doubt but that this plant which M. BRONGNIART'S theory converts into a Cryptogam in its youth and a Phanerogam in its mature life is really the *Lepidodendron Veltheimianum.* It must be understood that I have sought to demonstrate the organic unity of the Lepidodendra and the Sigillariæ by showing that plants which are unquestionable Lepidodendra gradually acquire the internal features supposed to be characteristic of the Sigillariæ. In the second of my memoirs,[*] I have shown that the bark of *Favularia* and of a true *Sigillaria,* have a structure which is identical with that of *Lepidodendron,* but this is only a secondary illustration of their unity, auxiliary to that based upon the development of their vascular axis. Such being the case, I was surprised to find in M. GRAND-'EURY'S recently published work the following remark, "M. WILLIAMSON dit avoir trouvé une série de spécimens établissant l'identité des Sigillaires avec les Lépidodendrons. En suivant son opuscule, on voit que l'auteur passe à cette conclusion par des faits isolés, d'après l'analogie de l'écorce et non par des exemples complets réunissant les caractères extérieurs aux caractères intérieurs" (*loc. cit.,* p. 177). I have already said sufficient to show that this paragraph involves a serious error on the part of its author, which is not removed by the remarks that follow it. On the same page as that from which I have quoted, M. GRAND-'EURY makes the important admission, "Il est au moins curieux que, à part le corps vasculaire, les autres parties des Sigillaires soient semblables aux parties correspondantes des Lepidodendrons," and he further adds, "Cette égale composition descend jusque dans

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[*] Phil. Trans., 1872, pp. 210 and 212.

le Stigmaria." About the truth of these remarks there is no doubt, and seeing that the difference in their "corps vasculaire," is, as I have shown in the case of the Burntisland plant, merely one of age and growth, I marvel that M. GRAND-'EURY does not discern whither his own admissions lead him.

I will now call attention to some further details in the organization of *Lepidodendron selaginoides*, which approaches the nearest to the type of *L. Harcourtii* of any of the Lepidodendra. This is the plant which Mr. BINNEY designates *Sigillaria vascularis.* I presume that he does so *because* it possesses a thin exogenous vascular zone. If so, he merely reasons in a circle, since, with this exception it does not possess a solitary feature connecting it with the Sigillariæ; it does not display the faintest trace of the longitudinal ridges and furrows so characteristic of the true Sigillariæ, whilst, as I have already shown,* its bark and leaves display every feature that characterizes a true *Lepidodendron,* a conclusion in which I am supported by the opinion of Mr. CARRUTHERS.† I am indebted to Mr. JAMES SPENCER, of Akroydon, near Halifax, for the specimens from which I have obtained the fine additional sections now figured.

It is not necessary to enter again into a detailed description of the structures representing the medullary axis of the plant, in which axis cells and vessels are more or less intermingled in a way that is essentially Lycopodiaceous. I have already done that in my second memoir just quoted, and later observations have suggested nothing that requires either to be added to or retracted from my previous descriptions. Fig. 33 is a portion of a transverse section of a stem in which *c* represents some of the peripheral vessels of the medullary group, which group represent the primary vascular bundle of a young axis. That these vessels increase with age up to a certain point, both in size and number, is unquestioned; but I am still as far as ever from learning where and by what agency the increase in number has been effected. At *d* we have part of the exogenous zone, composed of radiating laminæ (*e*) of vessels that increase in size from within outwards, and which laminæ are separated from each other at very frequent intervals by the medullary rays, *f.* The innermost cortical layer is seen at *g*, *g'*, *g''*, the latter being a narrow band bridging over the space so constantly left vacant, and showing that, when occupied by its original tissues, it consists of parenchyma like that seen at *g*. At *m*, *m*, are two vascular bundles passing outwards through the bark. Nothing can be more clear than that, whatever may be the mode of increase in the vessels constituting the cylinder *c*, and which represents the vascular cylinder of *Lepidodendron Harcourtii*, in the outer zone, *d*, we have a definite example of exogenous growth, but one in which, unlike the Diploxyloid forms from Burntisland and elsewhere, only undergoes a very limited development. At *m'* we find one of the foliar bundles of small vessels issuing directly from the inner or medullary series of vessels, *c*, and passing outwards through an opening in the

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* Phil. Trans., 1872, Plate XXIV., figs. 1, 5, and 6.

† 'Monthly Microscopical Journal,' Oct. 1, 1869, p. 179.

exogenous zone, *d*, in order to reach the bark, as the bundles, *m*, *m*, have already done. It will be seen that whilst the latter bundles are intersected almost transversely, *m′* is almost intersected in the plane of its vessels. Longitudinal sections will explain this difference.

Fig. 34 represents a magnificent section of a stem enlarged nearly three diameters, in which the relations of the various parts to each other are well seen. At *a* we have the medullary axis consisting of intermingled cells and vessels; the former being the most abundant in the centre and the latter exclusively forming the peripheral region, as seen in fig. 33 *c*. *d* is the exogenous layer which forms a cylinder the thickness of which is small contrasted with the diameter of the central medullary axis. At *g* we have the delicate parenchyma of the inner bark; at *h* the coarser parenchyma of the middle bark, and at *i* is the thin prosenchymatous layer seen in all these Lepidodendroid and Sigillarian stems, and which is generally preserved when all the other tissues have disappeared. The persistent bases of the numerous leaves are seen at *l*. This section demonstrates the regularity with which the foliar vascular bundles are given off; but more enlarged figures reveal their relations more clearly.

Fig. 35 represents a portion of fig. 34 enlarged 20 diameters. At *c* we perceive the large barred vessels of the medullary axis. At *d* we have the smaller barred vessels of the exogenous zone; at *g*, *g* are several portions of the delicate parenchyma constituting the inner bark, and which is so frequently more or less destroyed. At *m* we have a bundle of small barred vessels issuing from the *exterior* of the medullary axis, *c*, and passing obliquely upwards and outwards through the exogenous zone, *d*, and then ascending almost vertically through the inner bark at *m′*, an arrangement which explains why, in fig. 33, the bundles, *m*, are intersected transversely, whilst *m′* is cut through more nearly in the plane of its long axis. Both figs. 33 and 35 show that the foliar vascular bundle passes outwards accompanied by some cellular parenchyma, either derived from the medullary axis, or more probably from the primitive tissue of the bark through which the bundle passed before the exogenous layer began to be formed. Fig. 36 represents a tangential section of the thin exogenous vascular zone, of which *e* indicates the vessels, *f* the numerous small medullary rays, and *m*, *m*, two lenticular openings through which the foliar vascular bundles have been transmitted in passing from the outermost medullary vessels, where they originate, to the bark and leaves. Fig. 37 represents another portion of fig. 34, but only enlarged 10 diameters, for the purpose of illustrating the course followed by the foliar vascular bundles; at *m* one of these bundles is seen passing obliquely upwards and outwards through the exogenous zone as represented in fig. 35. After passing obliquely through that zone the bundle bends perpendicularly upwards, running for a little distance parallel to, and almost in contact with its external surface. *m′* represents a second foliar bundle ascending from the next node below. At its lower extremity, like the upper part of *m*, it ascends parallel with the exogenous cylinder; but on reaching a point a little above that at which the bundle *m* emerges from the exogenous zone it bends outwards horizontally and almost at a right angle to its former course, to reach the middle bark, where it again ascends obliquely outwards to reach the leaves. This is the course followed by each one of the numerous bundles seen in fig. 34.

On comparing the above facts with what I have already described in my third memoir,* we cannot fail to see that though the exogenous cylinder of *Lepidodendron selaginoides* is much less developed than in the conspicuously Diploxyloid Burntisland plant, the essential conditions in each are absolutely identical. Thus figs. 35 and 36 of the present memoir are virtually copies of figs. 10 and 13 of the older one, just as all the sections of the leaf bases of these two plants equally demonstrate their true Lepidodendroid character. Here, then, we have two Lepidodendroid plants which possess, *in different degrees of development*, the vascular organization which M. BRONGNIART believed, and which his disciples still believe, to be characteristic of Sigillarians plants, and which, on the evidence of that organization, they regard as Gymnospermous Phanerogams. Much too clear headed not to see the force of this kind of evidence, M. GRAND-'EURY endeavours to evade the inevitable conclusion to which it leads by saying, "Tout cela ne prouve qu'une chose; c'est que certaines impreintes de Lepidophloyos peuvent appartenir à des vegetaux Dicotylédones." Remembering the absolute identity in all the other features of the stems of Lepidodendra and Sigillariæ, and which identity M. GRAND-'EURY has acknowledged to exist, is it not more rational to admit that some of these arborescent Lycopodiaceæ have been provided with a pseudo-cambial layer from which they developed an exogenous zone, than to transfer the genus *Lomatophloios* from the Lycopodiaceæ with which all authors agree to arrange it, to the very different Phanerogamous Gymnosperms with which M. GRAND-'EURY would thus unite it. My indisputable facts stand in the way of BRONGNIART's hypothesis, which M. GRAND-'EURY adopts in its totality, and, as it appears to me, he endeavours to evade them by the adoption of an explanation which has no foundation of fact on which to rest.

The great misfortune has been that *Lepidodendron Harcourtii* was so long the only Lepidodendroid stem of which the organization was known, hence, instead of being rightly interpreted as one extreme modification of the Lepidodendroid type, of which *Sigillaria* presented the opposite extreme, it came to be regarded by M. BRONGNIART as *the sole* typical form.

I need say little here about the views of Dr. DAWSON on the affinities of Sigillariæ. They have now advanced some way in the same direction as mine. He has discovered in Canada a *Sigillaria* with a true Diploxyloid axis, and which is very different from the type of *Sigillaria* that he has described in some of his earlier memoirs. This fact, and the study of a specimen of *Lepidodendron Selaginoides* which I forwarded to him has led him to the following conclusion, which I quote, because where so much discrepancy exists in the minds of some of those most experienced in the study of coal-plants, it is

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* Phil. Trans., 1872.

important to record every step of advance in the direction of unity of view. He says, "the slice enclosed shows very well what I would call the *most perfect* kind of Lepidodendron structure, and which, but for the better development of the woody cylinder in *Sigillaria*, comes very near, if not quite up to, the lowest type of the latter." "Your Lepidodendron is *perfectly exogenous*, and therefore enters into that general type which leads up to the highest plants." "As for me, I do not doubt the truly exogenous character of the stems of *all* Sigillariæ, and *some* Lepidodendra, and that some of the forms approach very closely to each other." * All the phenomena tend to confirm my previous conclusions that Lepidodendra and Sigillariæ belong to the same type of vegetation; that they are equally Cryptogamic plants, but that the Sigillariæ represent, so far as their vegetative organs are concerned, the highest modification to which the Lycopodiaceous type has ever attained.

Lepidodendroid Reproductive Organs.

I some time ago discovered a few singular fringed macrospores, both in the Oldham and the Halifax beds. More recently both Mr. James Spencer, of Ackroyden, near Halifax, Mr. Binns, of Delph Hill, in the same district, and Mr. Earnshaw, of Oldham, have forwarded to me numerous examples of the same objects. We have not yet obtained these macrospores in union with the *Lepidostrobus* to which they have belonged, but it is most probable that one of two that I have received has been their parent strobilus. One of these is a crushed one, abounding in microspores. For this I am indebted to Mr. Binns; both that indefatigable worker and Mr. Spencer have sent me sections of another one, which I shall describe in detail. The microspores of the former of these strobili display no peculiarities; figs. 48 and 71, exhibit some of them enlarged 160 diameters. The second *Lepidostrobus*, on the contrary, is a very remarkable one.

Fig 53 represents a slightly oblique transverse section of it from Mr. Binns' cabinet. Fig. 54 is a slightly oblique tangential one, from Mr. Spencer's Cabinet, made nearly in the plane of the outer surface of the common axis of the strobilus. Fig. 55 is also an obliquely tangential section which I have obtained from a fragment forwarded to me by Mr. Spencer. Fig. 56 is part of a nearly longitudinal section from the same fragment, and fig. 57 is the central sporangium of fig. 55 enlarged 75 diameters. The medullary tissues, as well as the innermost cortical layer, have disappeared from all these sections. In fig. 53, a large Stigmarian rootlet, *x*, has nearly filled the internal cavity thus left, pushing aside the vascular axis, *a*, which has been a cylinder consisting of unusually small barred vessels, each having a diameter of about ·001.† To facilitate reference to the figures the same letters are affixed to identical organs in all

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* In litera. April 11, 1877.

† I have recently received some fine transverse sections of this cylinder, the details of which will be described in a forthcoming memoir. June 10th, 1878.

the fine sections 52 to 57. The vascular axis has been enclosed in a thick cortical layer. The innermost portion of this bark doubtless consisted of a delicate parenchyma, which has disappeared, but there remains a thick middle and outer portion, *b*, the former being composed of a less dense tissue than the latter which consists of oblong, more or less prosenchymatous cells, fig. 56*b*. From this cortical axis there spring numerous sporangiophores, *c*, arranged in the spiral order common to the Lepidodendroid stems. This is well shown in the tangential section, fig. 54, where their bases are seen at *c*, whilst at *c′* their narrower cylindrical portions are divided transversely. The centres of these latter have evidently been occupied by prolongations of the delicate parenchyma of the inner bark, but which, like that tissue, have disappeared. In some of the sporangiophores we find the vascular bundle which also connected them with the main vascular axis. At their outer extremities these sporangiophores expand into peltate disks, 54, 55, and 56 *c″*.* The section, fig. 53, appears to have been made near one extremity of the strobilus where some of these disks, seen at the right hand part of the figure, are in close contact with each other (53, *c*), and in some cases these sporangiophores are actually confluent (53, *c‴*). Fig. 55 demonstrates that each sporangiophore supported one sporangium, *d*, on its upper surface, the two having apparently been attached to each other about midway between the base and the peltate extremity of the sporangiophore; the walls of the sporangia are composed of a single layer of cells (fig. 57, *d*), which are always somewhat oblong and frequently prosenchymatous (fig. 38).

But it is in the interior of the sporangia that the chief interest of this fruit resides. Lepidostrobi containing spores have been frequently described, but this is the first instance I have met with in which the mother cells of the spores with their numerous sister cells, are preserved with such remarkable definiteness.

The sporangia, as already observed, are composed of a single layer of very strong prosenchymatous cells (fig. 38) like those composing the outer cortical tissue of the axis and its branches, though of smaller size. They differ in this respect from those of the Burntisland *Lepidostrobus*, described in my third memoir, in which the sporangium-wall consisted of at least a double row of parenchymatous cells arranged vertically to the surface of the sporangium. Within the sporangia are clustered a number of remarkable objects, some of which are represented in the series of figures from 39 to 47.

One of the common forms of these objects is shown in fig. 39, where we have a central cell, *a*, containing a protoplasmic (?) mass, *c*, and with a curious oblong appendage, *d*, *d*, at each end, which appendages we shall find to be two sections of a remarkable cell. In fig. 40, we have a similar cell, but in these we find, that in addition to the outer cell-wall, *a*, there is a second inner one, *b*, and a third, *c*, which latter has, I presume, been the outer layer of the protoplasm or primordial utricle.

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* In the latter figure two sporangia appear *vertically* between the two sporangiophores, but the lower one, *d′*, is merely a portion of the wall of a sporangium belonging to the next *lateral* series.

We still see the two terminal appendages, *d*, *d*. Fig. 41 represents a similar cell, *a*, but here the two terminal structures, *d*, *d*, are less regular, and their boundary walls are spread out into flat surfaces of cell tissue. In figs. 42 and 43 we recognise repetitions of fig. 39, only in the interior of the outer cell, *a*, we find a number of small spherical bodies, *f*, which, in fig. 42 are seen to be enclosed within the second membrane, *b*. Figs. 44, 45, and 46, represent what, at the first glance, look like very different objects, but which intermediate forms demonstrate to be identical with the rest, only seen in different positions. We now see that the cell, *a*, is a four-sided object exactly resembling, in all but size, an ordinary microspore of a *Lepidodendron.* This is best seen in 44 and 45, the projecting apex of the cell having, in fig. 46, been ground away in making the section. That this spore-shaped structure is identical with the cell, *a*, in the other figures, is shown in fig. 47, where we have the general contour of figs. 39 and 42, but combined with the projecting triradiate ridges, *e*, of figs. 44 and 45. We thus learn that the terminal appendages, *d*, of the figures just described, are transverse sections, *d'*, of the large curved cell, *d*, seen in contact with the largest curvature of the spore-shaped body in figs. 44, 45, and 46. In fig. 44 the larger or convex border of the embracing cell is seen to be slightly keeled, which explains the peculiar shape of the sections of these cells as shown in figs. 39 and 42. In fig. 46, in addition to this cell, *d*, *d'*, we find some narrower ones, *g*, *g*, so that the triquetrous, conical body, *a*, looks as if it were planted by its broad convex base upon a triangular, multicellular scale. This condition exists in a very large number of the examples discovered; I have not found a solitary instance in which the spore-shaped body, *a*, is entirely detached from its cellular surroundings. However small, some fragment of the encompassing cellular tissue is invariably seen adhering to it, as in fig. 44. In fig. 52, a spore, like fig. 44, is shown in another aspect, so that the crescentic cell, 44, *d*, is seen crossing the disk of the spore, like a flat band, 52, *d'*, and connecting the two terminal appendages *d*, *d*. The peculiar mode in which these supposed spores are combined with the cells and portions of cells to which they are attached, receives its explanation when we examine the section, fig. 57, which represents one of the sporangia of fig. 55 enlarged 50 diameters. Enclosed within this sporangium we discover nine clusters of tetraspores, each cluster being held together by its group of mother and sister cells. The peculiar triangular group of fig. 46 reappears in the clusters, *g*, *g'*, of fig. 57, only each cluster has now its full complement of four spores. The triangular contour of each group obviously indicates a cluster of cells derived from a common parent cell, which has packed itself between other similarly derived clusters and the sporangium wall, modifying its shape according to the resistance its expanding elements had to encounter. Several of these clusters exhibit the curious crescentic cell, *g'*, *g'*, seen in fig. 46, *d*, as well as in figs. 44 and 45. In like manner, many of the tetraspores exhibit in their interior the protoplasmic (?) masses seen in figs. 39, 40, and 41, *c*. A peculiar feature of these sporangia, and one in which they differ from those of all Lepidostrobi with which I am familiar, is their shape. They present nearly the same form and dimensions, in whatever direction they are intersected. In this respect they remind us of the supposed fruits of *Asterophyllites*, as represented by my *Volkmannia Dawsoni*,* rather than of true Lepidodendroid structures. But the spiral arrangements of the strobilus under consideration, and the verticillate ones of the *Volkmannia*, place them widely apart.

Several important questions arise out of the study of this fruit, but which mainly centre in one, viz., what are these tetraspores?

On first examining these specimens, I was inclined to believe that the triquetrous body, *a*, was a large microspore, and that the enclosed spherical objects seen in figs. 42, 43, and 44, *f*, were identical with those figured by Hofmeister in plate lvii., figs. 13, 14, and 15, of the Ray Society's edition of his 'Higher Cryptogamia.' But two reasons militated against this conclusion, 1st, the large size of the spore-like bodies, *a*, and 2nd, their invariable adhesion to greater or smaller portions of cellular tissue, whereas developed microspores are invariably free. To illustrate the question of size I have, in figs. 48, 49, 50, and 51, drawn some true microspores to the same scale, *i.e.*, enlarged 100 times. Fig. 48 represents three *detached* microspores from the crushed *Lepidostrobus* supplied to me by Mr. Binns, and fig. 77 three others, all from the same bed near Halifax as the specimens under consideration. Figs. 49 and 50 represent two clusters, each consisting of four microspores (only three being visible), from two Oldham examples of *Lepidostrobus*. Fig. 51 represents a similar cluster from a sporangium of the recent *Lycopodium alpinum*. It will at once be seen that the size of each of these small microspores differs so widely from that of the large bodies under consideration as to make it most improbable that the latter can also be microspores. Three other possibilities remain—1st, they are macrospores; 2nd, they are the mother-cells of microspores, the granules, *f*, seen in figs. 42, 43, and 44, being the true microspores in an early state of development; 3rd, they are *spores* of a special kind, belonging to a strobilus of the type of the recent *Lycopodium*, and which only possessed one kind of spore. The latter appears to me to be the more probable explanation of their nature, though the second supposition is not an impossible one, nor without its illustration amongst living Lycopods. Thus in plate lvii., fig. 10, Hofmeister figures the mother-cells of the microspores of *Selaginella Martensii*, which are arranged in such a manner as to give each cell one rounded side and three flat ones, terminating at a projecting angle. This is exactly the form of my examples. Whenever I obtain the specimens attached to their surrounding cells, I invariably find that each of the spore-shaped bodies, *a*, rests upon those cells by its larger concave surface, whilst its other three sides stand up in bold relief, the conical apex being uppermost and wholly free, as in figs. 44, 45, and 46. Fig. 57 demonstrates that four of these bodies were primarily in juxtaposition, as the common products of one large fertile parent cell, and that the surrounding cells, *d* and *g*, seen in fig. 46, were barren cells belonging to the same cluster as each of those fertile parent cells.

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* Part V., Plate 5, fig. 28.

Viewing them in this light it is easy to understand the cohesion of all these clusters As grandmother cells of spores, they had not reached the stage of development at which they would be free from their attachment to the other cells of the cluster in one of which they had been specially developed. If this is a correct explanation of these objects, the small spheres, *f*, seen in figs. 43, 44, and 45, become groups of free cells, each one of which would in turn have developed within its cavity, by subdivision of its protoplasm, a cluster of 4 microspores, like those seen in figs. 49, 50, and 51. On comparing the size of these latter objects with the small spheres, *f*, of figs. 43, 44, and 45, it will be seen that a very moderate measure of further development would bring them up to the dimensions of the group of four perfect microspores. I presume that when they reached this stage of growth all the barren cells, *d* and *g*, would have disappeared. I have already observed that in all the specimens of *Lepidostrobus* which I have examined in which the spores have attained to maturity, each male sporangium is crowded with microspores, but it shows no traces of the mother cells whence those spores were derived. In like manner the mother cells of macrospores are as invariably wanting. Hence the existence of numerous barren cells in this *Lepidostrobus* in such a perfect state of preservation indicates that we probably have before us a strobilus in an early stage of its development, whatever may be the true nature of its tetrasporal contents. If these tetraspores are merely the grandmother cells of microspores, the small cells seen in figs. 42, 43, and 44, being their true mother cells, then we have the materials capable of supplying the enormous number of spores seen alike in the microspores of the bisexual Selaginellæ and the ordinary spores of the Lycopodia. If, on the other hand, they are true, and fully grown, spores, belonging to plants of the unisexual type, their magnitude distinguishes them conspicuously from all known similar spores, whether living or fossil.

I have already stated that numerous macrospores of a very remarkable character have been found near Halifax by Mr. Binns, Mr. Spencer, and Mr. Earnshaw, associated with the spores already described. I am far from certain that all these belong to the same plant, but I think there is reason to believe that figs. 58 to 66 do so; of these figs. 58, 59, and 60, 65 and 66 are enlarged 43 times; 61 and 62, 100 times; and 63 is enlarged 214 times; fig. 64 is enlarged 50 diameters.* Fig. 60 is the specimen which I first discovered; subsequently Mr. Binns forwarded to me the examples drawn in the other figures just referred to. Shortly after these drawings were made Mr. Spencer sent me a series of five sections, as well as many more at a later date, which were of material service to me in studying these objects.

Many of the examples consist of a double spore-wall. The outer layer is thick and variously furnished with peripheral radiating appendages. It exhibits no sign of

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* It would have been more convenient had all these figures been drawn to the same scale, but this plan would either have left the smaller ones too indefinite, or would have caused the larger ones to occupy too much space.

structure beyond a granular texture which may merely be the result of mineralization. The inner wall, *a*, figs. 65, 66, 66B, and 66C, though sharply defined, is very thin. So long as I only saw isolated specimens, those represented in figs. 65 and 66, seriously perplexed me, being apparently sporangia full of minute spores. The discovery by Mr. BINNS of the specimen fig. 64, left no further room for doubting that these curious objects are macrospores, of which three are inclosed within a cellular sporangium-wall of the usual type. This sporangium-wall is not, like those described in the previous pages (fig. 57D), composed of prosenchyma, but of parenchymatous cells, whose slightly elongated axes are vertical to the surface of the spore. The latter is also the case with the crushed *Lepidostrobus*, microspores from which are represented in figs. 48 and 77. This difference shows that whilst we have two types of strobilus in the Halifax beds, there is no difference, so far as the sporangium-walls are concerned, between those containing the macrospores and those with true microspores. Hence there exists the possibility, at least, that the latter two may belong to the same plant.

The fringes of radiating appendages which clothe the exteriors of these spores vary much both in their shape and arrangement. They are often simple, as is seen in fig. 58. In fig. 63 most of them are so, whilst in fig. 71 there are no branched ones, yet the former of these two figures is so intermediate between the latter and the common form represented by figs. 58 and 60, as to leave little room for doubting that these several examples merely represent different states of the same spore. I noticed in several of the specimens the peculiar outline represented by *d*, *e*, in figs. 65 and 66, and which looked as if the section had intersected a small projecting peduncle surrounded by a thickened ring. This peculiar form is seen in the two left-hand spores of fig. 64, whilst fig. 64* represents the same appendages as they appear in another spore in Mr. SPENCER's cabinet, enlarged 40 diameters. In fig. 64 these appendages of the several spores appear to converge towards a common centre. That these can have been peduncles is, of course, out of the question. It appears to me more probable that they may have been appendages similar to the dome-shaped projections developed at the apex of the macrospore of *Pilularia globulifera*, each of which encloses a funnel-shaped entrance into the spore; in these recent objects we also find a thickened ring surrounding the base of this appendage, very similar to that seen at *e*, *e*, in figs. 64, 65, and 66. In 64* we see clear indications that a canal, *f*, passed along this appendage in the case of the fossil spores.

Figs. 65 and 66 are further interesting because of the spherical bodies contained within their interior. These are manifestly small cells developed within the spore. In many of those of fig. 66 we see the protoplasmic and other cell contents preserved in the interior of each cell. I think we can scarcely regard these objects otherwise than as the endospermic cells of the macrospore, destined to be developed into a prothallus. If this is a correct explanation of them their presence in a fossil coal-plant becomes an interesting fact.*

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* Since writing the above Mr. ISAAC EARNSHAW, of Oldham, has kindly prepared for my inspection a

I have already referred to the second or thin inner layer of the macrosporal-wall. This is seen at *a*, in figs. 65, 66, and 66C, but in spores more recently sent to me, both by Mr. SPENCER and Mr. BINNS, I find it yet more conspicuously exhibited in large spores like fig. 58. Fig. 70 represents two hairs from another spore in which there appears to be a thin outer layer detached from the exterior surface of the spore, but I cannot determine whether this was a feature of the living spore or whether it is merely a result of mineralization. Some other specimens render this latter suggestion possible.

Associated with these macrospores both Mr. BINNS and Mr. SPENCER have found some other very remarkable objects, some of which may be young states of the macrospores just described, or they may belong to distinct species of plants. Fig. 72 represents the exterior of what I presume has been a small macrospore, in which the free extremity of each peripheral appendage is trifid; somewhat similar conditions are seen in the yet smaller examples represented in figs. 73 and 74. It is impossible to overlook the striking resemblance of these little objects to the fossil Xanthidia of the chalk flints, and to the zygospores of some of the Desmideæ. In figs. 75 and 76 we have two objects which appear to be of a different nature. Fig. 76 is obviously a cavity about ·007 in diameter. The figure represents an optical section, whilst the faint areolation represents the ends of the cells bounding the cavity but seen a little out of focus. Figs. 75, 75A, 75B, and 75C, are apparently examples of the same organism, only in them less of the surrounding parenchyma is preserved. The diameter of fig. 75 is about ·0012; of 75A and 75C about ·0111, whilst fig. 75B is about ·0166. Fig. 75A, 75B, 75C, and 76 are empty; but fig. 75 is filled with small parenchymatous cells which have a mean diameter of about ·0012. Fig. 75C also contains a mass of parenchyma; some of the cells in the latter specimen are further enlarged in fig. 75D, and two of them are there seen to contain three or four small rounded spheres, *a*, which may possibly be reproductive spores of some kind. These objects appear to be conceptacles of some kind that have been formed in the midst of parenchymatous tissue, but from which they have shown a remarkable tendency to become detached with a somewhat definite yet irregular contour.*

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large number of sections of the Halifax material, and amongst them I have found the two very fine macrospores represented in figs. 66A, and 66B, and in one of Mr. SPENCER's slides I find the spore, fig. 66C. In 66A, which is enlarged 50 times, the endospermic cells are of uniform size, being about ·003 in diameter. In fig. 66B, these cells are unequal in size, the largest being about the same magnitude as in fig. 66A, whilst the smaller ones are ·0005, a few odd ones being even still more minute. In fig. 66C, we find the cavity of the macrospore filled with irregular parenchyma, apparently resulting from the successive subdivision of a multitude of free endospermic cells, which are thus forming themselves into a prothallus. In the centre of the specimen a small fragment of a barred vessel, which clearly formed no part of the endosperm, suggests the possibility that these cells may belong to some algoid plant that has found its way into an empty cavity, the fragment of a vessel having been accidentally introduced along with the parasites; but two other specimens correspond so closely with that figured that I see little reason for doubting that this parenchyma belongs to the macrospore.

* Fig. 76B not only exhibits this defined contour, but further shows a tendency in the peripheral cells to arrange themselves in fan-shaped or radiating columns at the three points *a*, *a*, *a*. Other specimens

Fig. 76A represents a somewhat similar conceptacle, but in this instance the outer wall of the organism consists of a single layer of cells, *a*, the flattened inner ends of which form the boundary of the conceptacular cavity. The dark ring in the figure merely represents part of this surface of the inner cavity which has not been crossed by the section at its greatest diameter, and which is consequently seen a little out of focus. This example differs from the figs. 75, in the fact that in the former the *outer* extremities of the cells bear no appearance of having been forcibly detached from some other surrounding parenchyma, and only consists of a single row of elongated cells disposed vertically upon the cavity which they invest. This specimen, which is from one of Mr. BINN'S Halifax sections, has a maximum diameter of ·0166. In its interior is what appears to be a collapsed, structureless membrane, *c*, that originally lined the cavity.

I have not yet obtained the slightest clue to the plants to which these curious objects belong; but some of them inevitably remind the botanist of the spore cavities seen in sections of the fertile frond of *Ophioglossum vulgare.*

Fig. 77 represents two detached microspores and one united cluster of four, but of which only three can be seen in any one position. These, which are from a fragment of a Halifax strobilus, are introduced to serve as standards of comparison in reference to the other structures just described. They are enlarged, like the figs. 63–73, 74, and 76, 214 diameters.

Both Mr. BINNS, Mr. SPENCER, and Mr. EARNSHAW have supplied me with examples of the very remarkable objects represented in figs. 67, 68 and 69. Fig. 67 represents a section enlarged 50 diameters, its actual breadth being ·025. Its outer wall, exclusive of its projecting hairs, is about ·0023 in thickness. Fig. 68 represents a segment of the circle enlarged 214 times. The wall appears to consist of a row of cells, fig. 68, *a*, *a*, shaped like dumb-bells, arranged perpendicularly to the surface of the organism. At the outer and inner surfaces of the structure the enlarged extremities of these cells are in unbroken contact, looking like two layers of ordinary tabular parenchyma; but the middle constricted portions of the cells appear to stand isolated, and to be surrounded by a continuous cavity, fig. 68, *b*, like a chamber whose roof and floor are connected by a series of pillars with bases and capitols. Supposing it possible that the lozenge-shaped spaces, fig. 68, *b*, might be occupied by a central series of cells, I examined the specimens carefully in search of a double cell-wall; but having failed to find anything of the kind, I consider that these are what I have already suggested, viz., a continuous network of inter-cellular spaces traversed vertically by the constricted parts of the cells, and shut in by the continuous outer and inner surfaces of the organism. The specimen just described is from Mr. SPENCER'S cabinet. Fig. 69 represents part of a second specimen sent to me by Mr. BINNS. It is obviously

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obtained more recently exhibit the same arrangement. I propose grouping these and some allied objects in a provisional genus *Sporocarpon*, and the present species may be designated *Sporocarpon cellulosum.* June 10th, 1878.

the outer surface of part of a crushed specimen in which the perfect contact of all the outer expanded extremities of the cells composing the wall of the organism is clearly shown.

The hairs, *c*, which project from the outer surface of these specimens are merely the external dilated extremities of some of these cells prolonged outwardly. They are very turgid at what, viewed externally, appears to constitute the base of each hair, but at their peripheral portions they are drawn out into long cylindrical, uncellular structures, $c' \; c''$, which appear in some instances to be bifid or trifid at their extremities. Each hair therefore appears to consist of a single cell, the lower end of which is at first turgid, then dumbbell-shaped, the latter portion being prolonged through the wall of the organism, so as to reach its inner surface.

These objects have obviously been spherical bodies. In fig. 67, *d*, we find an organic spherical tissue occupying the interior of the organism. The latter has a shrivelled membraneous look about it, but otherwise displays no traces of structure.

Fig. 69A, is another specimen of the same conceptacle from one of Mr. EARNSHAW's slides. In it, the central membrane seen in fig. 67, *d*, is filled with small spherical cells, the protoplasmic contents of some of which, as at *a*, have become contracted and detached from the cell-wall. In other respects this example exhibits the same structure as is seen in figs. 67 and 69. These objects may be distinguished provisionally as *Sporocarpon elegans.*

I am acquainted with no recent objects that exactly correspond with these conceptacles. There is some little resemblance between them and the outermost layer of the sporangiocarp of *Pilularia globulifera.* In this latter structure we have a layer of large cells, many of which are prolonged externally into conspicuous hairs, each one of which has a turgid base like those of my fossil; but the subjacent hour-glass form of each cell is entirely wanting in the recent type.

Mr. BINNS, Mr. SPENCER, and Mr. EARNSHAW, have furnished me with slides from the Halifax beds, containing examples of the remarkable type of conceptacle represented in figs. 76A,* 78, and 78A. Fig. 76, *a*, for the original of which I am indebted to Mr. BINNS, represents a sphere composed of a single layer of oblong cells, *a*, the inner ends of which are flattened and in close contact so as to bound a very regular and smooth internal, spherical cavity. These cells continue in close contact throughout the greater part of their length, but their peripheral extremities are free, and more or less rounded. The dark circle, *b*, in the figure, represents a portion of this smooth inner bounding wall of the cavity which happens not to have been cut through, at either surface of the section, in the plane of its greatest diameter. At *c* we have a collapsed inner and apparently structureless membrane apparently corresponding to *d* in fig. 67. At the first glance this conceptacle appears similar to that represented by figs. 75, 76. But this difference exists between the two: in the former the bounding wall of the conceptacular cavity consists of a single layer

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* I have already referred to this figure in p. 182.

of cells, with perfectly free outer extremities. In the two latter, that boundary wall consists of at least a multi-cellular layer, whilst such examples as 75A, and 76, indicate that the entire structure has been embedded in and formed part of a parenchymatous structure. Such has certainly not been the case with the organism now under consideration. I have as yet discovered in the specimens of the latter, no trace of a peduncle, or point of attachment to any other structure. Fig. 76A, is enlarged 324 diameters. I propose to distinguish this object as *Sporocarpon compactum.*

Fig. 78 represents a second form of the same type of conceptacle which I found in a slide sent to me by Mr. EARNSHAW. It differs from the specimen last described only in the greater number and narrower transverse diameter of its peripheral cells, *a*, a feature which results in producing a more definitely regular peripheral outline. The cells are also longer than in fig. 76A, causing the thickness of the bounding wall to be greater in proportion to the diameter of the enclosed cavity. The maximum diameter of the entire organism is about ·009, being enlarged in the plate 214 times. The specimen, fig. 76A, has about the same dimensions. Fig. 78A, represents a single cell of fig. 78; its peripheral extremity, *a*, is broader than its narrower opposite or inner one; its exposed face exhibits two flattened or slightly concave surfaces, resultants of the pressure of two contiguous cells. Each cell of this specimen has a length of about ·0023 to ·0029 and a maximum breadth of ·0005 to ·0006. I have formed no opinion as to the relations of these objects. Fig. 78 may be designated *Sporocarpon tubulatum.*

Before leaving these curious reproductive structures, I would further call attention to a single transverse section of a remarkable little fruit, fig. 103, which I only discovered accidentally in a larger section of one of the Oldham nodules made for the sake of a very different object. Though I carefully examined contiguous portions of the nodule, I could find no further traces of the specimen, a fact indicating that it was either a very short fruit, or only a fragment of a more elongated one. Its mean diameter is about ·05. It consists of a central angular axis surrounded by a circle of what appears to be three, or possibly four curious structures *b*, *b′*, which are evidently symmetrical in their contour and arrangement; the two marked *b*, *b*, appear to retain their original form; it is not so clear whether those marked *b′* retain their normal form, or whether they were originally united, forming a figure with three centripetal and three centrifugal prolongations, like the uppermost one, *b′*. These three or four structures are enclosed within a more peripheral zone, *c*, whilst yet more externally we have numerous bracts, *d*, each one of which bears on its inner surface a rounded or ovoid sporangium, *e*. In the figure these sporangia do not appear so distinct from the bract to which they are attached as they do in the specimen, since in the latter, whilst the bracts are opaque, black, and carbonised, the sporangia are translucent and of a rich, dark amber colour, like the hue of the spores found in coal.* The most definite of these sporangium-bearing bracts are those seen at *d′*, *d′*. The bract appears to have had a V-shaped section, the sporangium being lodged in the inner, concave upper surface of the bract. Each

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* These are now tinted in the figure.

sporangium has a diameter of about ·005. That this has been a strobilus of some kind is unquestionable, and it is equally obvious that it is very distinct from any fruit the internal oganization of which has hitherto been described. It is from the Oldham deposits. It has a Lypocodiaceous look about it, but until we learn more respecting it I would apply to it the provisional name of *Volkmannia* (?) *parvula*.

Ferns.

Two new structures, apparently belonging to this family, have been met with since I wrote my sixth memoir. One of these is a transverse section (fig. 79) of a petiole found in one of the Oldham nodules, by Mr. ISAAC EARNSHAW, to whom I have been indebted for the loan of several instructive specimens from that locality. It obviously belongs to the same type as the *Chorionopteris gleiche* of CORDA.* In CORDA's specimen, the walls of the vessels of the vascular bundles are very regularly reticulated. Though Mr. EARNSHAW's plant is very opaque, I detect distinct evidences that its vessels were of the same reticulate character. Within the doubly incurved vascular bundle is a mass of very small cells, whilst the cortical layer is composed of larger ones. Since I can detect no difference between this Oldham specimen and CORDA'S, I venture to attach his specific name to it and designate it, in accordance with my previously adopted plan, *Rachiopteris gleiche.* Its mean diameter is about ·11.

Amongst the new objects with which the Halifax beds have rewarded the researches of Messrs. BINNS and SPENCER, are some stems or roots which for the present I have located in the provisional genus *Rachiopteris,* though I am far from certain that the objects are true ferns. Fig. 80 represents the transverse section of an axis which appears to be of the maximum size of these objects so far as the specimens hitherto discovered enable us to judge. Its diameter is about ·066. In its centre is a cylindrical bundle, about ·025 in diameter, composed of barred vessels. The central ones are very small; the peripheral ones, on the other hand, very much larger, having a mean diameter of ·004. This vascular bundle is surrounded by a thick ring of very minute cells, *a;* the outer margin of this zone, which appears to form a sort of bundle sheath, is rather sharply defined. It is enclosed in a cortical layer of larger cells, *b*, from ·008 to ·016 in thickness, the cells of which exhibit a little tendency to arrange themselves in irregular concentric circles, after the fashion of those composing the root-stems of *Psilotum triquetrum.* Figs. 81, 82 and 83 are three successively smaller axes. In each of these the central vascular bundle consists of but few vessels of various sizes and not disposed in any uniform manner; the central vessel of fig. 81 and the three central ones of fig. 82 being not only the largest in the bundle but actually larger than those occupying its centre in fig. 80. Fig. 83 is further enlarged to 140 diameters in fig. 85. The latter figure exhibits the vascular bundle composed of very small vessels. The line of demarcation between the exterior of the

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* Flora der Vorwelt, taf. liv., fig. 8.

cellular bundle-sheath and the outer cortical layer is indicated by a rather irregular ring of cells, *a*, which have had some peculiar cell-walls, giving them a darker hue than the rest of the section. Fig. 84 represents a segment of fig. 82, also enlarged to 140 diameters. In it the tendency of the inner cells of the cortical layer *b*, to arrange themselves in concentric circles is very marked; also the disposition of the innermost ones to become compressed and narrowed in the radial direction. The bundle-sheath in this specimen has disappeared. Fig. 86 is a portion of a longitudinal section enlarged 60 diameters; the vascular bundle is seen to consist of barred vessels. The bundle-sheath, *a*, is composed of very narrow, vertically elongated, square-ended cells, whilst in the cortex, *b*, the cells are of larger size, of coarser texture, and exhibit a strong tendency to become prosenchymatous. Fig. 87 is a transverse section of a stem like fig. 80, but giving off a long, straight, lateral branch. In this section the large and small vessels of the central bundle are more irregularly intermingled than is the case in fig. 80. On the whole the largest vessels are found at the periphery of the bundle. Judging from the size of its vessels and from its average diameter I presume that a transverse section of this branch would be intermediate between figs. 82 and 83. The section fig. 87, exhibits an extension of the bundle-sheath, *a*, of the central stem prolonged outwards to form the bundle-sheath, *a'*, of the branch, and the cortical layer, *b*, is similarly extended, *b'*. The connexion of the vessels of the branch with those of the central axis is not seen, owing to the slight angle at which the former have been given off from the latter, and which has prevented their direction from corresponding exactly with the plane of the section. Fig. 88 is a transverse section of another stem which exhibits the general features of those just described, but which differs from them in the small size of its central vascular bundle and bundle-sheath, contrasted with the diameter of its cortical outer layer. It is giving off a lateral branch, *a*, which is obviously ascending almost parallel to the main axis. The difference between the size of the vascular axis of the branch and that of the main stem is much less than in fig. 87. Still since this section occurs in the same slide as those represented in figs. 80–87, and since the same slide contains other sections which seem somewhat intermediate between the two extreme modifications, I am disposed to regard them all as belonging to the same plant. The differences between them are not greater than exist in similar sections of the aerial and subterranean stems of *Psilotum triquetrum*, to which the entire series of sections of this plant displays a considerable resemblance.

Three possibilities suggest themselves as to the affinities of this plant. It may be a fern stem, though I know no recent type of fern which it resembles; it may be the root of some type of fern, an idea suggested by the tendency to a concentric arrangement of the cortical cells; or it may belong to some dwarf type of Lycopodiaceous plants. For the present these points must remain undetermined. Meanwhile I would distinguish the plant by the temporary name of *Rachiopteris cylindrica*. I have not seen this form in the Oldham nodules. All the specimens figured are in a slide from the cabinet of Mr. Binns.

Cordaites.

Two fragments alone, both from the Halifax deposits, whence they were sent to me by Mr. BINNS, seem to resemble objects figured by M. GRAND-'EURY[*] which belong to Cordaites. One of these closely resembles the section of a leaf represented in Plate 18, fig. 1 of the work cited. The other is a fragment of epidermis (fig. 89) with numerous large and closely grouped stomata. The epidermal cells have disappeared, but the stomata are clear and distinct. From the large size of the area enclosed between each pair of guard-cells I presume that there has been a second inner pair to each stoma, but I cannot trace any division line in that area separating such cells. It is possible that this specimen may have belonged to some other plant than Cordaites, but it corresponds most closely with M. GRAND-'EURY'S fig. 1″, in his Plate 18. Fig. 89 is enlarged 360 diameters; the vertical length of each stoma is about ·0009, and its diameter about ·0006.

I am indebted to Professor YOUNG and Mr. J. YOUNG, of the Glasgow University, for a fragment of very curious wood from the Volcanic Ash of Arran; fig. 90 represents a portion of a transverse section of this specimen enlarged 16 diameters. Fig. 91 is part of the same section further enlarged to 50 diameters, and fig. 92 is a tangential section, enlarged like fig. 90, 16 diameters. The structure of this specimen will be best understood by first examining the tangential section. It exhibits a series of vascular laminæ, *a*, composed of rather large reticulated vessels, and which form a very irregular network, *a*, *a*, enclosing a multitude of areas, *b*, *b*, which are equally irregular in size and form and which are filled with intersected cells. On turning to the transverse section, fig. 90, we find that these vessels are arranged in rather regularly disposed laminæ which have radiated outwards from the central axis of the stem. This is seen clearly wherever these laminæ have been intersected at right angles to the direction followed by the vessels, as at *a*, *a*; but in other places, as at *a′*, *a′*, the vessels are cut through nearly in the plane of their course as they bend round to rejoin other neighbouring laminæ, and thus enclose the cellular areas, *b*, *b*.[†] The same section shows that the cellular tracts, *b*, are composed of a modified form of prosenchymatous cells, grouped in masses which have radiated from the centre of the stem to the bark, the long axis of each of the component cells following the radial direction of these masses as a whole. These cell masses are, in fact, huge medullary rays of a most extraordinary form. Fig. 91, *b*, illustrates the character of these oblong cells, as well as the minutely reticulated structure of the walls of the vessels, *a*. Some of the cells have square ends; more frequently they have oblique overlapping ones. They have the appearance of possessing thickened walls, which are translucent, whilst their cavities are occupied by carbonaceous matter, but whether this effect is due to original lignification of the cell-walls, or

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[*] Flore Carbonifère du Departement de la Loire, &c.

[†] The upper and lower margins of this figure severally represent the medullary and cortical borders of the section.

to some mineralization of them, I am unable to determine. It is needless to point out how much fig. 91 resembles the ordinary aspect of a radial section of a wood with the mural cells of its medullary rays crossing the vessels at right angles to the vertical course of the latter.

The only stems which I have examined that bear any resemblance to the one now described are those of *Lyginodendron Oldhamium*, illustrated in my fourth memoir, and in which the medullary rays are of unusual size; but in that plant the vertical length of each ray, as seen in tangential sections, many times exceeds its breadth. This is not the case with the plant under consideration. Though opposed as a rule to giving names to detached fragments of wood, this one appears to have such remarkably distinctive features, that I venture to designate it *Lyginodendron anomalum*. It may be remembered that the late Mr. GOURLIE described casts of the bark of a *Lyginodendron* from the Scotch carboniferous deposits. It will be an interesting fact if the specimens now described prove to belong to the woody zone of Mr. GOURLIE's plant. Should they do so, the genus *Lyginodendron* will be characterized by the possession of a most distinctive internal organization.

The Oldham calcareous nodules very frequently contain fragments of wood of various kinds, which, on examination, often prove to be masses of cortical cellular tissue. A very large fragment of this kind was found by Mr. NIELD, to whom I am indebted for the specimen from which the sections that I am about to describe were made. The entire fragment is about 3½ inches in diameter from its medullary to its peripheral surfaces. At its inner portion it consists of parenchyma, the cells of which are not arranged in any regular order. Their transverse diameter (fig. 96) is about ·01, whilst they are a little elongated vertically, many of them having a diameter, in that direction, of ·02 (fig. 97). In these vertical sections many of the cells exhibit clear evidences of their meristem origin in the secondary, but perfected, divisions seen within many of the older cells (fig. 97, *a*, *a*). Proceeding from within outwards, we soon find, in transverse sections, that these cells tend to arrange themselves in radial lines (fig. 93A) whilst radial vertical sections (fig. 94) show that as they do this, the cells increase in vertical length, and their longer sides approach more and more to perfect parallelism with each other. But in addition to this parallelism, the cells display a disposition to group themselves in well-defined clusters, each cluster being longer in the radial than in the vertical direction. This arrangement, as is seen in fig. 94, *a*, continues to prevail until we reach the peripheral margin, *b*, of the specimen. No signs of this peculiar grouping appear in transverse sections *of this part of the specimen* where, at about an inch from the inner or medullary surface of this bark, we obtain the condition represented in fig. 98, which is part of a transverse section, and in fig. 99, which is a tangential one of the same part. The whole tissue has now assumed a prosenchymatous character, the cells being arranged in very regular and uniform radiating lines proceeding from within outwards. These cells now have a transverse diameter of about ·0041 to ·0027, whilst many of them are ·025 in length; fig. 93, *a*, exhibits their collective aspect in transverse sections, and fig. 94, *a*, in vertical radial ones, whilst fig. 100 represents a portion of the radial section enlarged 15 diameters. The latter figure illustrates the way in which these cells are arranged, in long radial lines, most of the component cells of each of which are of exactly equal length, such groups constituting the peculiar limited clusters already referred to. The cells now present conspicuous examples of what has been termed "prismatic parenchyma." As we approach the peripheral surface of the bark, we find the new and very characteristic conditions illustrated by the figures 93, 94, 95, and 101. The outermost layer is seen to be an irregular parenchyma (fig. 93, *b*, 94, *b*), but even in this portion we again discover a tendency on the part of the cells to throw themselves into groups, the component cells of which have evidently some common bond of union separating them from similar neighbouring groups ; but intermediate between this outer portion, *b*, and the prismatic cells, *a*, we find a curious arrangement. In fig. 93, some of the long contiguous parallel lines of prismatic cells gradually diverge in opposite directions, and allow of the intercalation of long, wedge-shaped rows of cells, *c*, *c*, whose longer division walls are again parallel to each other, and to the periphery of the bark as well as transverse to the radial direction of each individual wedge. On tracing these wedges inwards, *i.e.*, towards the medullary surface of the bark, *we find them to be continuous with one or two of the lines of prismatic cells*, fig. 93, *a*. Hence it is clear that the one tissue stands in some definite relationship to the other. At their peripheral extremities these lines of parallel cells merge in those of the superficial parenchyma, *b*. On turning to the vertical radial section, fig. 94, we find that though in the transverse section, each of these wedges usually consists of but a single linear series of cells, in the radial vertical one it consists of several such series (fig. 94, *c*) superimposed vertically upon each other, and interposed between the separated layers of prismatic cells, fig. 94, *a′*, *a′*. This section further reveals to us the fact that these cells are grouped in lenticular clusters, which form a part of the continuous series of such clusters seen in the prismatic tissue of the bark at fig. 94, *a*. The vertical partitions of the cells of each group, 94, *c*, are parallel to each other at the more internal parts of the bark, the cells having about the same dimensions as they possess in the transverse section, fig. 93, *c*, but as we approach the peripheral parenchyma, *b*, the cells of each cluster become shorter, broader, and less regular, assuming more and more the form of ordinary parenchyma.

Fig. 95, which is a tangential section of the tissues under consideration, made along a line between the two stars of fig. 93, reveals a remarkable arrangement. We now find the prosenchymatous or prismatic cells forming a regular network, *a′*, *a′*, enclosing numerous lenticular areas filled with parenchyma, fig. 95, *c*, *c*, and as the section has penetrated more deeply into the bark at *a″*, than it has on the left hand portion of the figure, we see that these parenchymatous areas become gradually smaller (fig. 95, *c′*, *c′*), and finally almost disappear, the cells merging in the mass of prismatic tissue, *a″*. It thus appears that the cells seen in fig. 93, 94, and 95, *c*, are really tabular cells whose broad parallel sides are parallel to the surface of the bark, whilst their shorter axes are radial. In a word, these cells stand upon their thin bevelled edges, with their two flat parallel surfaces severally directed towards the medulla and the periphery of the bark. It is also clear that they are grouped into lenticular, vertically extended, wedge-shaped masses, whose thin edges project inwards between the rows of prismatic cells.

But fig. 101, which is a portion of fig. 95, enlarged 16 diameters, reveals some further peculiarities. The prismatic cells, *a, a,* now exhibit their somewhat prosenchymatous aspect, whilst, in the parenchyma, the darker outlines, *c*, obviously indicate the boundaries of the tabular cells seen in figs. 93 and 94, *c;* but we now discover that these cells are in the condition of extremely active meristem, most of them containing secondary subdivisions, their thin secondary cell walls, *c′*, intersecting the cavities of the primary cells, *c*, in every direction. It is thus clear that the network of prismatic or prosenchymatous cells, *a, a′, a″*, represents permanent, or formed tissue, whilst the cellular masses, *c*, which that network encloses, have been in an active state of cell multiplication; the same activity also appearing, though less conspicuously, in the peripheral parenchyma, *b*, of the bark.

I can arrive at no other conclusion than that we have here a sub-epidermal plane of parenchymatous tissue in a state of genetic activity, and that it is from this growing tissue that the additions have been made to *the exterior* of the prismatic tissue, *a*, which is thus developed into a thick modification of corky structure, only unlike the phellem-layers of living exogens, growing by additions to its outer side instead of being produced by a more internal "phellogen," or cork-cambium. Whilst functionally this prismatic tissue appears to have been like the phellem layer of living exogens, it is impossible to overlook the morphological resemblance between the tangential sections (fig. 95) of this tissue, and corresponding sections of the phellem layer of the bark of many exogens, in which the bast-layers form a similar network to that formed by the prismatic cells, *a*, of the above figure.

In my memoir, Part II., I have already described (p. 220) and figured (Plate 31, figs. 54–57) a small fragment of bark similar to that now described, but the specimen I then possessed left me in doubt as to which was the medullary and which the peripheral side of the example figured. This point now is no longer doubtful.

The next question is—to what plant does this bark belong? I think the answer must be, to *Sigillaria.* The prismatic or prosenchymatous layer of the bark is as I have shown in previous memoirs, common to *Sigillaria* and *Lepidodendron,* in a smaller degree to *Stigmaria,* and, as I have shown in the earlier part of this memoir, to Calamites. It is obviously a protective analogue of the corky layer of living exogens. But the characteristic feature of the example now described is found in the peculiar arrangement represented in fig. 95. In my previous memoir (*loc. cit.*) I have pointed out that a similar condition exists in association with a Diploxyloid stem now in my cabinet. I find it again in a modified shape in Syringodendroid *Sigillariæ,* and it has been described by M. Renault, in a most marked form, in his memoir on *Sigillaria spinulosa.*[*] In the case of the latter plant, the fragments of the bark, with its peculiar network of prosenchymatous structures, attracted the notice of the late M. BRONGNIART, who gave to it the provisional name of *Dictyoxylon,* a name which I had previously given to what is now the *Lyginodendron Oldhamium.* M. RENAULT, however, successfully demonstrated (*loc. cit.*, Plate IV., fig. 24) that BRONGNIART'S *Dictyoxylon* was merely part of the bark of his *Sigillaria spinulosa.* The elaborate and well-illustrated description of the structure of the bark of this plant by M. RENAULT shows that in all its essential features it agrees thoroughly with that which I now describe. All these facts tend to justify my conclusion that the latter is *Sigillarian,* and it would further appear probable that the peculiar network formed by the peripheral portion of the prismatic layer (which layer M. RENAULT designates the corky layer, or "couche subereuse") are characteristic of most of the *Sigillariæ.* As shown in some of my previous memoirs, the bark of *Lepidodendron* exhibits, even in its young state, the same subdivision (Phil. Trans., 1877, Plate 41, figs. 6 and 7) into the three layers found in *Sigillaria,* viz., an inner parenchyma, a median prosenchymatous layer, and a second external parenchyma. But I have not yet found in any *Lepidodendron* the peculiar reticulated arrangements of the prosenchyma which I have just described. But in this latter respect the Lepidodendra agree with the *Sigillaria elegans* of BRONGNIART.[†] The physiological conclusions which these studies justify are, I think, obvious; viz., that there are two vertical zones in the bark of these plants in which active meristem growth has gone on. In other words, there were two planes of genetic activity. One of these was at the junction of the vascular zone and the innermost bark, representing the true cambium of ordinary exogens, adding periodically, though perhaps irregularly, to the thickness of the vascular zone, and possibly also, though on this point I am more doubtful, making additions to the inner surface of the inner cortical parenchyma. The other genetic plane was a peripheral one, multiplying the cells of the superficial parenchyma, and also making additions to the exterior of the prosenchymatous analogue of the corky layer of living exogens.

The prevalence of this peculiar regular form of prismatic cellular tissue amongst these carboniferous cryptogams is a curious fact. We have seen that it prevails alike in the bark of the matured *Calamite,* and that it forms the medullary rays of the same plant. (See my memoirs, Part I., Tab. 23, fig. 8; Tab. 24, fig. 11.) We find it in *Favularia* (Memoirs, Part II., Plate 27, fig. 29), in *Lepidodendron Harcourtii* (*idem,* Plate 26, fig. 13, *i*), in *Syringodendron* (*idem,* Plate 29, fig. 42, *i*), in *Stigmaria* (*idem,* Plate 30, fig. 41, *k;* Plate 31, figs. 55-56), in the Diploxyloid *Lepidodendron* of Burntisland (Memoirs, Part III., Plate 43, fig. 17), and in *Asterophyllites* (Memoirs, Part V., Plate 1, fig. 7). I have not met with its long continuous lines of cells in any of the ferns. The fact that it constitutes a very peculiar form of medullary ray in the

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[*] 'Memoires présentés par divers savants à l'Académie des Sciences de l'Institut National de France,' tom. 22, No. 9.

[†] 'Archives du Museum,' 1839.

Calamite is curious. Without losing a certain mural arrangement, the cells, in that plant, are, as pointed out in my first memoir, like bricks standing upon their ends. This vertical elongation of the mural cells of a medullary ray is not without its living representative: I find it in sections of the wood of the ebony.

The only objects which at present remain to be described, are some remarkable ones from the Halifax nodules, found by Mr. BINNS; two of these are represented in fig. 102, imbedded in a mass of spores and other fragments of carboniferous plants. These objects consist of a central cell, surrounded by a ring of 8 or 9 somewhat obovate scales, each one of which seems to be a single cell. In two cases the central area is further subdivided (101, *a*) into a small six-sided centre, surrounded by 6 regular subdivisions; but whether these surfaces represent 7 distinct cells, or whether they are merely the superficial impressions of other cells with which they were formerly in close contact, I am unable to determine. I incline to the latter conclusion, since they appear to me to be only impressed on one surface of the area, corresponding with the centre of fig. 102, *b*. The peripheral appendages are evidently thin, because their margins often overlap each other very distinctly. That these are free objects and not mere sections of some elongated structure is obvious from their number and from the frequent way in which two of them partially overlap each other. They may possibly be some new form of sporocarp. Until we discover further information respecting them it may be convenient to recognise them by the name of *Oidospora anomala*. Their diameter is about ·0023.

I have once more to acknowledge the aid I have received from allies, old and new: Mr. BUTTERWORTH and Mr. NIELD; JOHN AITKEN, Esq., of Bacup; Mr. ISAAC EARNSHAW, Messrs. SPENCER and BINNS, of Halifax; Professor YOUNG and JOHN YOUNG, Esq., of the Glasgow University; E. WUNSCH, Esq., of Glasgow, and Professor BALFOUR, of Edinburgh, have all kindly given me valuable aid by supplying me with important specimens; whilst I am again indebted to the Messrs. PATTISON, of Manchester, for the use of their very effective marble-cutting machinery.

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PLATE 19.

Astromyelon.

Fig. 1. Transverse section of a stem of Astromyelon enlarged 14 diameters. *a.* Medulla. *b.* Vascular wedges.

Fig. 2. Longitudinal section of a branching stem enlarged 6 diameters. *a.* Medulla. *a′*, *a″*. Medulla of Branch.

Fig. 3. A small portion of fig. 2, enlarged 44 diameters, showing the junction of the medullary cells, *a*, with the innermost vessels, *b*.

Fig. 4. Obliquely transverse section of a specimen like fig. 2. *a*. Medullary cells of stem intersected transversely. *a'*. Medullary cells of branch intersected almost longitudinally.

Fig. 5. Transverse section of a stem with an unusually large medulla, *a*, and numerous primary vascular wedges, *b*, resembling those of some Calamites, but devoid of any longitudinal internodal canals. *x*, *x*. Intruded Stigmarian rootlets. Magnified 6 diameters.

Fig. 6. Tangential section of a part of a primary vascular zone exhibiting the vascular laminæ, (*b*) separated by primary (*c'*) and secondary (*c*) medullary rays.

Fig. 7. Transverse section of a stem in which the bark (*d*) is preserved, and also exhibiting the bundle of enlarged vessels occupying the medullary angle of each primary vascular wedge.

Calamites.

Fig. 8. Transverse section of a very young Calamite enlarged 26 diameters. *b*. Medulla. *e*. Longitudinal internodal canals. *h*. Cortex.

Fig. 9. Similar section to fig. 8, but deprived of its cortex. Enlarged 26 diameters.

Fig. 10. Similar section to fig. 8, but somewhat more advanced in development. *a*. A fissure in the medulla indicating the commencing formation of a *fistular* medulla. Enlarged 26 diameters.

Fig. 11. Similar section to fig. 10, but decorticated, and with the fistular medullary canals (*e*) ; further enlarged. *b*. Medulla.

PLATE 20.

Fig. 12. Transverse section of a young decorticated stem in which also the fistular medullary cavity, *a*, is yet more enlarged than in fig. 10. Enlarged 26 diameters. *c*. Primary medullary rays. *e*. Internodal canals. *f*. Vascular wedges.

Fig. 13. Transverse section of a larger stem in which the vascular wedges (*f*) are in the same stage of development as in fig. 12, but in which the fistular medullary cavity (*a*) is larger. *h*. Cortex. Magnified 26 diameters.

Fig. 14. Transverse section of a large Calamite. *a*. Fistular medullary cavity. *b*. Thin layer of medullary cells. *f*. Primary vascular wedges merging into a continuous vascular structure towards their peripheral extremities. *h*. Thick, shattered layer of bark chiefly composed of prismatic prosenchyma. Two-thirds natural size.

Fig. 15. Vertical section of the specimen, fig. 14, passing through a node at *n*. Reference letters as in fig. 14. Two-thirds natural size.

Fig. 16. Radial vertical section of the vascular cylinder of fig. 14. *d.* Secondary medullary rays. *g.* Vessels. Enlarged 14 diameters.

Fig. 17. Small portion of another radial section, like fig. 16. *d.* Secondary medullary rays. *g.* Peculiarly contorted vessels. Enlarged 20 diameters.

Fig. 18. Transverse section of a small portion of the outer bark, composed of cells of prismatic prosenchyma arranged in lines radiating from within outwards. Enlarged 26 diameters.

Fig. 19. Radial section of fig. 18. Enlarged 14 diameters.

Fig. 20. Tangential section of fig. 18. Enlarged 32 diameters.

Fig. 21. Portion of a tangential section of the same bark, with a lenticular mass of parenchyma passing through it radially. Enlarged 10 diameters.

Fig. 23. Tangential section of the node of a vascular zone of another Calamite intersected near its medullary surface. *c, c', c'', c'''*. Primary medullary rays separating the primary vascular wedges, *f.* *l.* Clusters of small cells located at the upper nodal extremity of each primary medullary ray. *m, m''*. Small lenticular areas usually isolated upon the node, as at *m*, but in one instance, *m''*, connected with the primary medullary ray, *c'''*. Enlarged 8 diameters.

Fig. 24. Similar section to fig. 23, and from the same segment of the same Calamite, but made nearer to the cortical surface of the vascular cylinder. The cellular masses, *l*, at the upper extremities of the primary medullary rays, fig. 23, *c'*, now appear as isolated lenticular areas, *l*, enclosed within the coalesced vascular wedges *f.* The primary medullary rays with which they are connected in fig. 23 have disappeared, and are only represented by dark vertical lines, similar to those shown at *c.* *m.* Verticil of lenticular cellular (?) areas, apparently identical with those at fig. 23, *m.* Enlarged 8 diameters.

Fig. 25. One of the lenticular areas, *l*, of fig. 24, further enlarged.

PLATE 21.

Fig. 26. Tangential section from the node of a large Calamite cutting, at *m*, transversely through the medulla of a developing branch. Enlarged nearly 6 diameters.

Fig. 27. Radial longitudinal section of another part of the node of the Calamite, fig. 26, passing longitudinally through the medulla of a lateral branch. *a.* Fistular medullary cavity of the primary stem. *b.* Medullary cells of the parent stem. *g.* Vessels of the same. *m, m'*. Parenchyma of the developing branch, originating in the pith of the parent stem at *m'*, and passing outwards. Enlarged nearly 6 diameters.

Fig. 28. Tangential section across the node of a decorticated Calamite, about 1·25 in diameter, and with a vascular cylinder about ·25 in thickness. *c.* Primary medullary rays. *f.* Primary vascular wedges. *m.* Small lenticular cellular (?) areas, apparently identical with fig. 23, *m.* In the middle of the section is a transverse section of a small emerging branch, having a central cellular medulla, surrounded by a cylinder of radiating lamellæ of vascular tissue. Enlarged 20 diameters.

PLATE 20.

Fig. 29. Part of a tangential section of a node of the same Calamite as fig. 28. The cellular area, *m*, is the medullary extremity of a branch intersected transversely, and a similar section of the peripheral part of which is almost identical with fig. 26. *m′*, *m′*. Enlarged *secondary* medullary rays. Enlarged 33 diameters.

PLATE 21.

Fig. 30. A Calamite in shale with the base of a large lateral branch. Natural size.

Fig. 31. Lower extremity of the sandstone cast of the fistular medullary cavity of a Calamite with two verticils, *l*, *l′*, of the casts of the infranodal medullary canals passing through the decomposed carbonaceous matter of the vascular zone, *f*, *f*. Slightly enlarged.

Fig. 31A. Vertical section through fig. 31B intersecting the two fistular medullary canals. *a.* Medullary canal of primary stem. *a′.* Medullary canal of branch. *f.* Vascular zone of primary stem. *f′.* Vascular zone of branch. *f″.* Vascular area intermediate between the two fistular medullary canals, *a* and *a′*, fig. 31. Two-thirds natural size.

Fig. 31B. Transverse section of a decorticated Calamite giving off obliquely a lateral branch. *a.* Medullary canal of primary stem. *a′.* Medullary canal of the branch *m.* *f.* Vascular zone of primary stem. *f′.* Vascular zone of branch with its primary vascular wedges separated by the primary cellular medullary rays. Two-thirds natural size.

Asterophyllites.

Fig. 32. Obliquely transverse section of an Asterophyllites passing through a node. *c.* Primary vascular axis. *d.* Exogenous vascular zone, consisting of a single layer of vessels. *g.* Space left vacant by the disappearance of the inner bark. *k.* Outer bark. *l.* Vacant canals left by the destroyed vascular bundles of a corresponding number of verticillate leaves. *m*, *m.′* Leaves. *x.* Triangular vascular bundle going off to a lateral branch.

PLATE 22.

Lepidodendron.

Fig. 33. Transverse section of part of a stem of *Lepidodendron selaginoides.* *c.* Medullary vascular zone. *d.* Exogenous vascular zone. *g.* Inner bark. *n.* Inner parenchymatous portion of outer bark. *m.* Foliar vascular bundles. *m′.* Foliar vascular bundle, issuing from the exterior of the medullary vascular zone *c*, and passing outwards through the exogenous vascular zone *d.* Enlarged 14 diameters.

Fig. 34. Longitudinal section of a branch of *Lepidodendron selaginoides.* *a.* Medullary axis composed of intermingled cells and vessels, the latter alone composing the periphery of the structure. *d.* Exogenous vascular layer. *g.* Inner parenchymatous bark. *h.* Middle parenchymatous bark. *i.* Prosenchymatous layer of the bark. *l.* Persistent bases of leaves. *m.* Vascular leaf-bundles. Enlarged nearly 3 diameters.

Fig. 35. Portion of fig. 34 further enlarged. *c.* Vessels of the medullary axis. *d.* Exogenous vascular zone. *g.* Inner bark. *m.* Foliar vascular bundle passing outwards through the exogenous zone. Enlarged 14 diameters.

PLATE 21.

Fig. 36. Tangential section of a portion of the exogenous vascular zone of fig. 34. *e.* Vessels of the exogenous zone. *f.* Medullary rays. *m, m.* Lenticular openings through which the foliar vascular bundles passed outwards through the exogenous zone. Enlarged 16 diameters.

Fig. 37. Portion of the section, fig. 34. Enlarged 10 diameters. *c.* Vessels of the medullary axis. *d.* Exogenous vascular layer. *g, g.* Inner parenchymatous bark. *m, m′.* Two foliar vascular bundles.

PLATE 22.

Fig. 38. Prosenchymatous cells from the sporangium wall of a *Lepidostrobus* from Halifax. Enlarged 106 diameters.

Figs. 39 to 47. Various states of the mother or sister cells and the spores of the Halifax *Lepidostrobus.* Enlarged 106 diameters.

Fig. 48. Three microspores from a crushed *Lepidostrobus* from Halifax. Enlarged 106 diameters.

Figs. 49–50. Clusters of microspores from two Lepidostrobi from near Oldham. Enlarged 106 diameters.

Fig. 51. Similar cluster of four spores from the recent *Lycopodium alpinum.* Enlarged 106 diameters.

Fig. 52. Spore like figs. 44 and 47, but exhibiting the cell, *d*, of fig. 46 crossing the disk of the spore at *d′*.

Fig. 53. Transverse section of the *Lepidostrobus*, from which the spores, figs. 39 to 47 were derived. *a*. Vascular axis displaced. *b*. Prosenchymatous cells of the outer cortical layer. *c*, *c′*, *c″*. Sporangiophores. *d*. Sporangia with spores in groups of 4, and mother or sister cells. *e*. Sporangium walls. Enlarged 12 diameters.

Fig. 54. Slightly oblique tangential section of the same *Lepidostrobus* as fig. 53, made nearly in the plane of the outer surface of the vascular axis of the strobilus. Enlarged 5 diameters.

Fig. 55. Oblique tangential section of the same made a little external to fig. 54. Enlarged 4 diameters.

Fig. 56. Longitudinal section of a portion of the same. Enlarged 14 diameters.

Fig. 57. A single sporangium of the same *Lepidostrobus*, with numerous clusters of tetraspores and sister (?) cells. Enlarged 50 diameters.

PLATE 23.

Figs. 58, 59 and 60. Macrospores from Halifax. Enlarged 43 diameters.

Figs. 61 and 62. Similar macrospores. Enlarged 100 diameters.

Fig. 63. Similar macrospore, but with numerous unbranched peripheral appendages. Enlarged 214 diameters.

Fig. 64. Three macrospores enclosed within their sporangium. Enlarged 50 diameters.

Fig. 64*. Peduncular appendage like those (*d* and *e*) of the two lower macrospores of fig. 64, and at *d*, *e*, in figs. 65 and 66. Enlarged 40 diameters.

Figs. 65-66. Two macrospores containing endospermic cells. Enlarged 43 diameters.

Fig. 66 A. Macrospore containing endospermic cells of uniform size. Enlarged 33 diameters.

Fig. 66 B. Similar macrospore to fig. 66 A, but with the endospermic cells of various sizes. Enlarged 33 diameters.

Fig. 66 C. Macrospore filled with parenchymatous cells.

Fig. 67. Transverse section of a remarkable conceptacle. Enlarged 50 diameters.

Fig. 68. Segment of the peripheral wall of fig. 67. Enlarged 214 diameters.

Fig. 69. A crushed specimen of the same conceptacle.

Fig. 69 A. Another specimen of the same conceptacle, in which the central structure, 67, *d*, is seen to be occupied by a mass of parenchymatous cells. Enlarged 66 diameters.

Fig. 70. Two peripheral appendages from a macrospore like figs. 58–66, *apparently* consisting of two layers of tissue.

Fig. 71. A macrospore with unbranched peripheral appendages. Enlarged 100 diameters.

Fig. 72. A very small macrospore (?) with trifid peripheral appendages. Enlarged 214 diameters.

Fig. 73. A minute macrospore (?). Enlarged 214 diameters.

Fig. 74. A minute macrospore (?) resembling fig. 73, but with shorter subdivided peripheral appendages. Enlarged 214 diameters.

Fig. 75. A peculiar conceptacle filled with large cells. Enlarged 140 diameters.

Fig. 75A. A similar example to fig. 75, enlarged 100 diameters, but without the contained cells, and exhibiting the cellular areolation of the wall bounding the central cavity.

Fig. 75B. Transverse section of another example of the same. Enlarged 100 diameters.

PLATE 24.

Fig. 75C. Another example, but containing a mass of parenchyma. Enlarged 100 diameters.

Fig. 75D. Four cells from the part, *a*, of the parenchyma of fig. 75C. Enlarged 400 diameters.

Fig. 76. A similar cavity to fig. 75A. Enlarged 214 diameters.

Fig. 76A. A transverse section of a cellular conceptacle. Enlarged 214 diameters.

Fig. 77. Microspores from Halifax, a little larger than those represented in fig. 43. Enlarged 216 diameters.

Fig. 78. Transverse section of a conceptacle similar to fig. 76A, but with narrower peripheral cells. Enlarged 214 diameters.

Fig. 78A. One of the peripheral cells of fig. 78. Enlarged 866 diameters.

Rachiopteris gleiche (?) ; *Chorionopteris* of CORDA.

Fig. 79. Part of the transverse section of a Petiole.

Rachiopteris cylindrica.

Fig. 80. Transverse section. Enlarged 33 diameters.

Figs. 81, 82, and 83. Transverse sections of three successively smaller stems. Enlarged 33 diameters.

Fig. 84. Portion of the cortical layer, *b*, of fig. 82. Enlarged 143 diameters.

Fig. 85. The same section as fig. 83. Further enlarged to 143 diameters.

Fig. 86. Longitudinal section of a stem like fig. 80. Enlarged 40 diameters.

Fig. 87. Section of a stem giving off a lateral branch. Enlarged 33 diameters.

Fig. 88. Transverse section of a stem found in the same slide as figs. 80 and 87.

Cordaites (?).

Fig. 89. Portion of an epidermis, probably of *Cordaites*. Enlarged 360 diameters

PLATE 25.

Lyginodendron (?) *anomalum.*

Fig. 90. Transverse section of the vascular zone of a stem from Arran. Enlarged 14 diameters.

Fig. 91. Part of fig. 90. Further enlarged to 50 diameters.

Fig. 92. Tangential section of the same stem. Enlarged 14 diameters.

Sigillarian or Lepidodendroid Bark.

Fig. 93. Transverse section of the outermost bark. Enlarged 8 diameters.

Fig. 94. Radial vertical section of fig. 93. Enlarged 8 diameters.

Fig. 95. Tangential vertical section of fig. 93, made in a line between the two stars. Enlarged 8 diameters.

Fig. 96. Transverse section of a fragment from the innermost portion of the same specimen. Enlarged 32 diameters.

Fig. 97. Tangential section of fig. 96. Enlarged 32 diameters.

Fig. 98. Transverse section of a portion of fig. 93, made in the line of *a*, *a*. Enlarged 32 diameters.

Fig. 99. Tangential section of fig. 98. Enlarged 32 diameters.

Fig. 100. Radial vertical section of fig. 98. Enlarged 32 diameters.

Fig. 101. Small portion of fig. 95. Enlarged 16 diameters.

Oidospora anomala.

Fig. 102. Unknown bodies. Enlarged 360 diameters.

Volkmannia parvula.

Fig. 103. Transverse section. Enlarged 54 diameters.

XI. *New Determination of the Mechanical Equivalent of Heat.*

By JAMES PRESCOTT JOULE, *D.C.L., LL.D., F.R.SS. L. and E., &c., President of the Literary and Philosophical Society of Manchester.*

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[PLATE 26.]

THE Committee of the British Association on Standards of Electrical Resistance having judged it desirable that a fresh determination of the mechanical equivalent of heat should be made, by observing the thermal effects due to the transmission of electrical currents through resistances measured by the unit they had issued, I undertook experiments with that view, resulting in a larger figure (782·5)* than that which I had obtained from the friction of fluids (772·6).†

The only way to account for this discrepancy was to admit the existence of error, either in my thermal experiments or in the unit of resistance. A committee, consisting of Sir WM. THOMSON, Professor P. G. TAIT, Professor CLERK MAXWELL, Professor B. STEWART, and myself, were appointed at the meeting of the British Association in 1870; and with the funds thus placed at my disposal I was charged with the present investigation, for the purpose of giving greater accuracy to the results of the direct method.

The plan I adopted was, in regard to the measurement of work, similar (as I afterwards found) to that used by HIRN, who has laboured so earnestly and successfully on this subject. He has described it as follows:—" L'appareil qui m'a servi pour cette étude consiste: 1°, en un cylindre en laiton de $0^{m}\cdot3$ de diamètre, de 1^{m} de longeur, poli à sa périphérie externe, monté sur un axe solide en rapport avec un moteur d'un mouvement très régulier, et pouvant recevoir une vitesse variant à volonté de 60 à 600^{t} par minute; 2°, en un cylindre fixe, poli à son intérieur, concentrique au premier, éloigné partout de $0^{m}\cdot03$ de celui-ci. Les disques ou plateaux formant les extrémités de la cylindre étaient munis, à leur partie centrale, de boîtes à étoupes par où sortait l'axe du cylindre interne. Tout l'intervalle compris entre les deux cylindres pouvait être rempli ainsi d'un liquide quelconque que les boîtes à étoupes empêchaient de s'écouler par les centres.

" Lorsque le cylindre intérieur tournait, le frottement que sa surface externe exerçait sur le liquide, et que le liquide, mis ainsi en mouvement lui-même, exerçait à

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[*] Brit. Assoc. Report, Dundee, 1867, p. 522.

[†] Phil. Trans., 1850, p. 82.

son tour sur la surface interne du cylindre externe, tendait à faire tourner celui-ci. Deux leviers parfaitement parallèles, adaptés aux deux extrémités, et portant des plateaux de balance, permettaient d'empêcher la rotation à l'aide de poids qui indiquaient ainsi la valeur du frottement. La tare des leviers, la valeur du frottement des boîtes à étoupes, etc., étaient déterminées aisément en faisant tourner très lentement le cylindre interne dans les deux sens alternativement. Deux tuyaux verticaux, soudés aux deux disques de fermeture, et aussi près que possible des boîtes à étoupes, permettaient d'établir dans l'appareil un courant continu et parfaitement régulier d'un liquide voulu. La température de ce liquide était prise à l'entrée et à la sortie. Autant que possible, la température à l'entrée était tenue à autant de degrés au-dessous de celle de l'appartement que celle du liquide sortant était supérieure. Du reste, la loi de refroidissement de l'appareil était soigneusement déterminée de manière à ce qu'il fût facile de faire les corrections nécessaires.

"Cet appareil, qui dans son ensemble constitue une véritable balance à frottement des liquides, pouvait très aisément servir à faire connaître, d'une part, le travail dépensé pour tel ou tel liquide, pour telle ou telle vitesse, et d'autre part, à l'aide des corrections convenables, à faire connaître le nombre de calories produit par ce frottement dans un liquide dont la capacité calorifique était connue.

"Les résultats obtenus ont été en général d'une régularité satisfaisante. Six expériences consécutives faites sur l'eau, et avec différentes vitesses, avec des quantités diverses de liquide introduites par seconde entre les deux tambours, m'ont donné 432^{p}[*] pour le travail produisant une calorie, et par suite pour la valeur de l'équivalent."[†]

The method I adopted was to revolve a paddle in a suspended vessel of water, to find the heat thereby produced, measuring the work by the force required to hold the vessel from turning, and the distance run as referred to the point at which the force was applied. Fig. 1 represents the apparatus drawn one-eighteenth the actual size. A massive wooden framework, *a a*, resting on the asphalted floor of a cellar, is still further strengthened by means of timber abutting against the walls on every side. The perpendicular shaft *b* is supported by a conical collar[‡] turned on it at *c*. It is revolved, along with the fly-wheel *f*, weighing about 1 cwt., by means of the doubling hand wheels, *d e*. A counter[§] is placed at *g*, for the purpose of reading off the number of revolutions. The calorimeter *h* has an accurately turned groove, from which silk threads pass over the light, accurately-turned pulleys *j j*, to the scales *k k*. The

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[*] This equals 787·4 in the measures I have adopted, viz., British feet, and degrees Fahr.

[†] 'Théorie Mécanique de la Chaleur,' 1865, p. 55. Maxwell has independently, in 1875, devised an apparatus of a similar description. He employs channelled cones, the revolution being on a vertical axis.

[‡] Its surface, though only half a square inch, was found amply sufficient when castor-oil was employed as the lubricator. Other oils failed on trial.

[§] In most of the experiments a second counter of my own construction was used to check the indications of the other. They were found in every instance to agree exactly.

hydraulic supporter, *w v*, was not employed in the first two series of experiments, and will be described further on. Three sides of the frame are boxed in permanently; the fourth, or front, has shutters with windows which can be removed at pleasure. A delicate thermometer, suspended within the frame, is observed through a telescope, as is also the thermometer employed in reading the temperature of the calorimeter.

Fig. 2 represents the section of the calorimeter, with its paddle, all of stout sheet brass; and fig. 3 gives a plan of the same. The dotted lines in the latter show the position of the fans in the upper part. The axle of the paddle works easily in the collar *m*, and is screwed into the boxwood piece *n*. There is another boxwood piece, *o*, fig. 1, placed to prevent any considerable quantity of heat arising from the friction of the shaft being conducted downwards. This friction was, however, so small that the precaution was afterwards found to be needless.

It will be seen in figs. 2 and 3 that there are four stationary vanes in the calorimeter, and two sets of rotating vanes, each of five arms, the upper set being fixed on the axis 9° behind the lower set. Hence no two vanes pass the fixed ones at the same moment, and inasmuch as the momentary alteration of resistance at crossing takes place 40 times in each revolution the resistance may be considered as practically uniform.

The circumference of the groove of the calorimeter was found by measuring its diameter in various places, and also by measuring it directly with a fine wire, allowing for the thickness of the latter. The results, obtained with a rule verified by the Warden of Standards, are—

Diameter in inches.	Circumference.
10·5850 . .	$\times\pi=$33·2538
10·5855 . .	$\times\pi=$33·2553
10·5855 . .	$\times\pi=$33·2553
Measured by wire $\frac{1}{120}$ in. diameter	$=$33·2538
" " $\frac{1}{55}$ "	$=$33·2563
Average	33·2549 inches$=$2·77124 feet.

The diameter of the silk cord, which was the finest that could be used with safety, was exactly $\frac{1}{100}$th inch. Hence the distance to be considered as run against the weights of the scales was, for each revolution, 2·77386 feet.

When a silk thread with a weight of 11,000 grains at each extremity was thrown over the small pulleys, 30 grains added to one of the weights was sufficient to keep both in motion. This friction, which includes the rigidity of the silk cord, taken with the distance traversed by the weights in their slight upward and downward motions during an experiment, gives the loss of work on the calorimeter from this cause. It did not amount to more than $\frac{1}{200000}$ subtractive from the equivalent, and could therefore be neglected.

The thermometer used to indicate the temperature of the calorimeter was the same which I employed in my former experiments. Those designated A[*] and D were calibrated with great care. I have recently compared them together at 50 different temperatures between 32° and 80° Fahr., the result being that if the less sensitive was assumed to be correct, the other, or A, nowhere appeared more than 0°·023 in error; but taking the averages for each consecutive 10° this error amounted to no more than 0°·008. I was anxious to compare these instruments with an air-pressure thermometer, and with that view have constructed an apparatus in which the height of the mercurial column is measured by a plummet hung over the axis of a graduated wheel, a method which I find capable of extreme accuracy, and which I purpose to apply to the construction of a new barometer. But owing to the use of caoutchouc in the connexion between the receiver and the rest of the apparatus, I fear that the zero point was subject to a slight displacement. The figures at which, after much labour, I have hitherto arrived, could not therefore be accepted as any improvement on REGNAULT's determinations of the expansion of air by heat.

The freezing-point of the standard D had risen from 13·3 divisions of its scale in 1844 to 15·14 in 1877. I think it probable that the boiling-point of this thermometer, if kept constantly at this temperature, would in the course of time fall as much. The five careful determinations of this boiling-point referred to 30 bar. and 60° are respectively 706, 706·4, 706, 705·9, and 706·15—mean 706·09. Subtracting 1·84, 704·25 will be the probable ultimate reading, from which if we take 15·14 we shall have 689·11 as the range between the fixed points cleared from the effects of imperfect elasticity of the glass. Mr. E. HODGKINSON has pointed out[†] that the "set" of imperfectly elastic bodies is proportional to the square of the force applied. Therefore the effect of imperfect elasticity in the glass of the thermometers will be insensible for the small ranges used in the experiments, and the factor 3·3822 for reducing the indications of D to those of A may be confidently relied on.

We have therefore $\frac{180}{689\cdot 11\times 3\cdot 3822}=0^\circ\cdot 07723$ as the most probable value of one division of A. In my former papers the number was taken as 0°·077214, which is so near that I shall continue to use it, trusting by long-continued observations of the fixed points to give it ultimately greater accuracy, and also by experiments above indicated to state it in terms of the absolute interval between these points.

The elevation of the mercurial column in A caused by the atmospheric pressure is five divisions, but inasmuch as in the limited time of an experiment the barometer never altered 0·1 inch, error from this cause was neglected. The depression occasioned by capillarity was 0·33 of a division.

A delicate calibrated thermometer E, each division of which indicated 0°·11195, was first used for taking the temperature of the air; but in consequence of a slight hitch

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[*] Phil. Trans., 1850, p. 64.

[†] Brit. Assoc. Report, 1843, p. 23.

in the motion of the mercury, an instrument called G was afterwards employed, each of whose divisions was equal to 0°·1911.

In registering the temperature of the air surrounding the calorimeter it was necessary to make allowance for the time which a thermometer takes in altering its temperature. I found that in a regularly rising or falling temperature E was $3^{m}\cdot8$ behind time, and G $3^{m}\cdot127$. This lagging of the thermometers was always carefully allowed for.

The capacity for heat of the calorimeter, calculated from the specific heat of brass given by Regnault, was equal to that of 5002 grains of water. But Regnault has shown how considerably the specific heat of metals of the same chemical composition is altered by changes in their hardness, and moreover there were the stoppers and other adjuncts to be taken into account. I therefore constructed the special apparatus represented by fig. 4, where B, B is a wooden box containing the calorimeter *h*; the projecting rim of the latter being supported by bits of string fastened at the top of three wooden legs, one of which is shown in fig. 5. In the lid of the box are three holes which the tubulures of the calorimeter just enter without touching. The paddle of the calorimeter can be agitated by means of the boxwood piece *n*. C is a copper vessel covered with a non-conducting substance: its lid is perforated to admit a stirrer, a thermometer, and a rod furnished with a caoutchouc stopper.

In experimenting with this apparatus, the calorimeter was first weighed after the water which it might have contained was shaken out. It was then placed on its three supports, and left for three or more hours in an apartment of uniform temperature, until its thermometer ceased to show alteration. The vessel C, containing an adjusted quantity of hot distilled water, and placed at some distance, had its gradually descending temperature noted from minute to minute. At a given moment it was rapidly transferred to the position shown in the figure; and then on pulling the plug out, *h* was filled in a few seconds. C was then quickly removed, and the caoutchouc stopper belonging to the tubulure through which the water had entered having been replaced, the temperature of the water was noted again from minute to minute while *n* was constantly moved. These observations afforded the means of eliminating the effects of radiation. Finally the calorimeter, as filled with the water, was again weighed.

In the first half of the following Table, A was employed in determining the temperature of the water introduced into the calorimeter, and D was the thermometer plunged into the calorimeter. In the latter half their positions were reversed. The temperatures are all given in divisions of A. *w* includes the estimated value of the air displaced, reckoned at 8 grains of water.

EXPERIMENTS on Capacity for Heat of Calorimeter.

No.	Water already in calorimeter. w.	First temperature of calorimeter. T.	Grains of water poured in. W.	Temperature of water poured in. T′.	Corrected resulting temperature. T″.	Thermal capacity of calorimeter. $\frac{W(T'-T'')}{(T''-T.)}-w$.
1	323·2	326·0	78887·6	457·63	449·47	4890·4
2	195·4	322·36	78996·6	464·62	456·5	4586·6
3	225·9	331·11	78984·6	475·84	467·4	4665·4
4	238·0	336·94	79042·6	505·36	495·35	4756·7
5	315·0	358·0	78916·8	504·04	495·4	4647·5
6	217·9	354·1	79029·6	512·36	502·13	5243·6
7	182·1	377·95	79127·7	514·1	506·18	4705·1
8	173·8	382·7	79044·7	534·42	525·29	4887·4
9	198·8	379·25	79059·2	613·83	599·82	4822·8
10	153·3	362·5	78959·2	614·04	598·56	5024·6
11	153·3	363·0	78920·2	673·03	654·57	4841·3
12	182·3	353·0	78914·0	641·11	623·77	4871·3
13	151·2	353·45	78789·2	658·06	640·11	4782·4
14	182·4	343·68	78574·0	668 01	649·02	4704·3
15	146 9	319·9	78817·7	654·83	634·85	4853·2
16	142·2	308·0	78933·2	640·97	621·13	4859·0
17	141·7	291·75	79021·7	668·07	646·07	4764·8
18	128·3	305·3	78946·7	647·78	627·8	4762·7
19	137·1	319·6	78895·2	681·64	660·31	4802·1
20	138·9	330·1	78821·7	654·51	635·38	4800·4
21	151·4	228·99	78774·7	624·35	600·3	4950·9
22	137·3	201·16	78996·2	638·52	612·66	4827·1
23	144·3	189·48	78965·7	637·71	611·18	4823·6
24	163·7	160·45	78884·7	636·91	607·95	4941·3
25	125·6	172·45	78801·2	643·44	615·12	4915·7
26	130·5	196·24	79026·7	613·78	589·46	4757·2
27	141·4	234·24	79094·2	665·48	640·53	4715·7
28	119·3	284·22	79031·2	669·39	647·19	4714·4
29	142·2	236·34	78913·2	669·26	643·56	4838·1
30	132·9	207·82	78976·7	662·14	635·38	4810·0
31	125·8	200·2	79019·7	674·56	646·63	4817·9
32	126·8	217·38	78915·7	667·95	641·0	4893·7
33	120·9	225·1	78824·7	672·3	646·14	4776·6
34	114·3	210·75	78986·2	673·03	646·03	4785·1
35	131·2	197·93	78806·7	648·93	622·51	4772·6
36	113·4	235·0	79024·2	684·54	658·32	4781·3
37	121·3	248·95	78882·2	675·41	650·4	4793·0
38	138·8	258·63	78865·7	682·38	657·24	4835·2
39	127·3	213·94	78696·0	682·26	654·53	4825·7
40	139·8	218·6	78811·7	681·67	654·57	4759·2
Average . .		278·91	. .	624·71	604·25	4815·15

The average temperatures T′ and T″ are 78°·38 and 76°·8. Hence in order to express the foregoing result in terms of the capacity of a grain of water at 60°, we have, from the experiments of REGNAULT, $4815{\cdot}15 \times 1{\cdot}00132 = 4821{\cdot}5$. Two further corrections were needed, one amounting, as was ascertained by means of experiments devised for the purpose, to 17·6, on account of the time allowed before the final reading of T″, limited to 8^{m}, not being sufficient to enable the caoutchouc stoppers and boxwood appendages to receive what would be their ultimate thermal distribution ; the other, amounting to 3·3, arose from the thermal effect of the fall of water from one vessel to the other. Hence the final result for the capacity of the calorimeter, appendages, and thermometer, is 4842·4.

I thought it desirable to test this result by obtaining the sum of the capacities of the materials which composed the calorimeter. I had in my possession cuttings from the same sheets of brass that were used in the manufacture of the vessel and its paddle. These were formed into a compact bundle.

A copper vessel, A (fig. 6), filled with water, had a narrower vessel, C, immersed in it, to the bottom of which the material experimented on was let down by a fine wire. A Bunsen burner, *b*, kept the water at a constant temperature for not less than three hours, a continual agitation being given by revolving the stirrer *s*, formed on the principle of a screw propeller. The temperature having been noted, the material was rapidly lifted by the thin wire, and transferred to a small copper vessel, V, filled with distilled water, and furnished with a thermometer and stirrer. After 5^{m}, which time was required for the equal distribution of temperature, the immersed thermometer was read off, and its observation was repeated each succeeding minute for some time, in order to obtain the cooling effect of the atmosphere.[*] The following is a table of the results. The weight *w* of the bundle of brass was 2951·6 grains.

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[*] The method first employed was the opposite one of plunging the material at the atmospheric temperature into a small vessel filled with hot water, and observing the temperature of mixture. The following specific heats were obtained by that method with brass and copper:—

	Brass.	Copper.
	·09200	·09516
	·08734	·09183
	·08945	·09295
	·09232	·08794
	·08734	. .
Averages .	·08969	·09197

The wide discrepancy between the several results is owing to the great effect of the atmosphere on the small vessel, necessitating an absolute uniformity of stirring in order to give true temperatures.

No.	Thermal capacity of small vessel of water. W.	Temperature to which the brass was heated. T.	Temperature in small vessel before immersion of brass. T′.	Corrected temperature after immersion of brass. T″.	Time occupied in transferring the brass to the small vessel, in seconds.	Specific heat uncorrected for transfer. $\frac{(T''-T')\ W.}{(T-T'')\ w.}$
1	4733·2	901·25	174·78	213·24	5	0·08964
2	4762·8	900·58	175·5	213·8	4	0·08999
3	4747·0	947·27	213·26	252·29	3	0·09032
4	4727·7	952·13	158·0	200·56	4	0·09070
5	4750·3	1030·46	125·5	173·52	4	0·09018
6	4724·6	984·47	210·36	251·65	3	0·09019
7	4764·0	1069·6	197·95	244·3	4	0·09065
Average .		969·4 or 104°·92	179·34 or 44°·08	221·34 or 47°·32	3·86	0·09024
1	4756·0	894·49	104·35	145·72	30	0·08903
2	4794·2	985·82	154·5	197·44	30	0·08847
3	4717·4	994·44	165·33	208·97	30	0·08880
Average .		958·25 or 104°·06	141·39 or 41°·16	184·04 or 44°·44	30	0·08876

From the above we may estimate the correction arising from the time of transfer in the first seven experiments at ·00023, which, added to ·09024, gives ·09047 for the specific heat of brass at 76° compared with water at 46°. REGNAULT, in two trials, arrived at ·0939, but this appears to be in reference to water taken as 1·008. When reduced to water taken as unity it becomes ·09315, which still differs considerably from my result. The method of cooling used by REGNAULT in this instance does not appear to me to be capable of as great accuracy as the method of mixtures used by the same physicist for other substances.

The interest I felt in this part of my subject induced me to try some experiments of a similar nature with copper sheet. It was tied in a bundle like the brass. Its weight *w* was 2777·9 grains.

No.	W.	T.	T′.	T″.	Time of transfer.	$\frac{(T''-T')\ W.}{(T-T')\ w.}$
1	4734·5	855·26	199·3	232·62	6	0·09121
2	4772·7	900·58	172·4	209·57	4	0·09242
3	4738·5	946·26	218·85	255·49	5	0·09048
4	4732·2	948·07	166·18	206·18	5	0·09185
5	4786·3	1030·46	106·0	152·48	4	0·09121
6	4749·0	985·48	197·02	237·2	4	0·09180
7	4849·6	1069·0	103·35	208·46	4	0·09152
Average .		962·16 or 104°·36	166·16 or 43°·07	214·57 or 46°·80	4·57	0·09150
1	4749·9	891·11	106·6	145·2	30	0·08848
2	4815·1	985·14	161·46	201·14	30	0·08773
3	4768·0	997·47	170·35	210·99	30	0·08869
Average .		957·91 or 104°·03	146·14 or 41°·52	185·78 or 44°·58	30	0·08830

In the first seven the average time of transfer is $4^s{\cdot}57$ and the proximate specific heat 0·091497. In the last three we have 30^s and 0·088302. From these the specific heat of the sheet copper at 75° is determined at 0·092094.

The boxwood piece *n*, fig. 2, had a brass nut in its centre by which it was screwed on the axle of the brass stirrer. Being a bad conductor, and having nearly the whole of its surface in contact with the air, only a small portion of its capacity for heat could be counted in reckoning the whole capacity of the calorimeter. I determined this portion by ascertaining the heat communicated to a can of water when the boxwood piece was immersed in it after having been screwed on the calorimeter filled with hot water, for different periods of time. Calling the difference between the temperatures of the air and the calorimeter T, the gain of temperature in the small can t, the capacity of this can of water c, and C the modified or virtual capacity of the boxwood piece, we have $C=\frac{tc}{T}$. The following results were obtained showing the gradual approach of this virtual capacity to a certain limit:—

Time that the boxwood was screwed on the calorimeter.	Virtual capacity.
3^m	45·6
6^m	57·5
8^m	63·9
12^m	67·3
60^m	76·0

The virtual capacity of the caoutchouc stoppers was determined in the same manner—

Time.	Capacity.
3^m	15·35
8^m	21·8
30^m	27·45

The several capacities making up that of the calorimeter are therefore summed as follows*:—

Brass, 51979 grains × ·09047 =	4702·54
Caoutchouc stoppers	27·45
Boxwood piece	76·00
Thermometer	44·78
Total	4850·77

I had, therefore, great confidence in employing the value 4842·4, obtained, as already described, from experiments with the calorimeter itself.

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* The specific heat of boxwood, which I obtained by immersion in mercury, was 0·417; that of the caoutchouc, 0·29.

In making an experiment for the equivalent the weight of the calorimeter filled with distilled water was first carefully ascertained. It was then screwed on to the axis, and the fine silk cords attached to the scales, *k k*, fig. 1, were adjusted. Thermometer A was then introduced into one of the tubulures, and after sufficient agitation of the water by means of the paddle itself, its indication was observed through a telescope. The thermometer was then removed and a caoutchouc stopper placed in the tubulure. The axle was then brought rapidly up to the velocity which produced friction sufficient to raise the weights about a foot from the ground. My son, Mr. B. A. JOULE, who turned the wheel, could, by observing the position of the scales in a mirror, keep them very steadily at a constant height during the whole time of revolution. The wheel having been rapidly brought to a standstill, the temperature of the calorimeter was again ascertained.

In the experiments in Table I. the number of revolutions of the axis when the weights were off the ground was added to half the number occupied in the acts of starting from rest and returning to rest.

Previously to, and subsequently to, every such experiment others were made under similar conditions as to the observation of temperatures, &c., in order to ascertain the effect of the atmosphere on the temperature of the calorimeter. The indications of the thermometer for temperature of air are always reduced to the graduation of thermometer A.

Experience had already shown me that the thermal effect of the air on the calorimeter was not exactly proportional to the difference of their temperatures. This might arise from variations in the radiating powers of brass and glass from day to day. By making experiments for the air-effect immediately before and after one for the equivalent, I sought to neutralize any error arising from this circumstance. The last column but one of the first part of the following Tables gives the amount of correction required to be applied to the temperature of the air so as to make the effect proportional to the difference of temperatures. The figures in the last two columns are then used for calculating the corrected rise of temperature in the last column but one of Part 2 of the Tables.

Table I., Part 1.—Experiments to ascertain the Effect of Radiation, &c. Time occupied by each of the first fifteen, 50^{m}; by the last two, 41^{m} and 41^{m} 30^{s}.

No.	Mean temperature of calorimeter.	Mean temperature of air.	Difference.	Rise of temperature of calorimeter.	Correction to air temperature.	Thermal effect of unit difference of temperature.
1*a*	392·410	395·790	3·380+	0·20+	1·525−	0·1086
1*b*	434·310	404·540	29·770−	3·40−		
2*a*	390·086	398·330	8·244+	0·62+	2·330−	0·1048
2*b*	430·562	404·630	25·932−	2·96−		
3*a*	391·056	405·800	14·744+	1·52+	0·232−	0·1047
3*b*	435·884	423·227	12·657−	1·35−		
4*a*	395·083	397·514	2·431+	0·26+	0·084−	0·1108
4*b*	437·583	410·050	27·533−	3·06−		
5*a*	401·315	409·848	8·533+	0·61+	2·714−	0·1048
5*b*	441·385	414·240	27·145−	3·13−		
6*a*	325·315	330·736	5·421+	0·49+	0·864−	0·1075
6*b*	368·900	342·236	26·664−	2·96−		
7*a*	325·880	327·820	1·940+	0·16+	0·383−	0·1028
7*b*	368·940	341·300	27·640−	2·88−		
8*a*	338·980	346·286	7·306+	0·74+	0·071−	0·1028
8*b*	383·250	360·957	22·293−	2·30−		
9*a*	344·225	336·196	8·029−	0·75−	1·460+	0·1145
9*b*	383·790	342·847	40·943−	4·52−		
10*a*	327·350	344·610	17·260+	2·00+	0·564+	0·1122
10*b*	373·705	361·645	12·060−	1·29−		
11*a*	333·920	359·248	25·328+	3·12+	2·237+	0·1132
11*b*	381·685	373·528	8·157−	0·67−		
12*a*	326·597	345·308	18·711+	2·26+	1·598+	0·1115
12*b*	372·650	355·450	17·200−	1·74−		
13*a*	311·930	318·630	6·700+	0·66+	0·600−	0·1082
13*b*	355·000	328·244	26·756−	2·96−		
14*a*	301·305	322·640	21·335+	2·37+	1·551+	0·1036
14*b*	347·650	333·546	14·104−	1·30−		
15*a*	327·560	347·235	19·675+	1·92+	0·742−	0·1014
15*b*	372·875	355·375	17·500−	1·85−		
16*a*	296·165	315·265	19·100+	1·71+	1·534−	0·0971
16*b*	344·810	321·837	22·973−	2·38−		
17*a*	280·810	295·307	14·497+	1·15+	1·561−	0·0888
17*b*	328·730	305·500	23·230−	2·20−		

Table I., Part 2.—Experiments with Friction of Water and Brass. Weight, W, lifted in the first fifteen, 14619·5 grains; in the last two, 18122·9 grains. Average proportion of metallic to total friction, $\frac{1}{7\cdot 7}$. Time occupied by each of the first fifteen, 50^{m}; by the last two, 41^{m} and $41^{m}\ 30^{s}$. Value, or V, of one division of the thermometer, 0°·077214. Circumference of groove of calorimeter, P, 2·77386 feet.

No.	Number of revolutions. R.	Capacity of the calorimeter. C.	Mean temperature of the calorimeter.	Mean temperature of the atmosphere.	Difference.	Rise of temperature of calorimeter.	Ditto, corrected for radiation, &c. T.	Mechanical equivalent or $\frac{RWP}{CTV}$
1	5545·0	84359·5	414·480	400·190	14·290 –	42·761	44·478	776·15
2	5378·0	84413·4	412·570	399·130	13·440 –	41·830	43·482	769·52
3	5522·6	84369·5	415·594	416·030	0·436 +	44·606	44·585	771·06
4	5685·2	84309·1	418·020	402·390	15·630 –	43·905	45·646	775·89
5	5321·3	84439·5	423·522	409·792	13·730 –	41·347	43·071	768·43
6	5756·5	84429·4	349·325	335·160	14·165 –	44·890	46·506	769·98
7	5753·5	84424·1	349·502	332·934	16·568 –	44·509	46·251	773·87
8	5740·7	84429·1	363·325	351·585	11·740 –	44·923	46·137	774·01
9	5714·0	84148·6	366·028	338·175	27·853 –	42·682	45·703	777·55
10	5725·7	84415·0	352·730	351·466	1·264 –	46·088	46·167	771·60
11	5695·0	84383·1	360·146	366·080	5·934 +	46·631	45·706	775·51
12	5702·2	84370·1	351·998	349·282	2·716 –	45·878	46·003	771·59
13	5681·2	84355·0	335·844	322·280	13·564 –	44·295	45·827	771·80
14	5702·3	84353·0	326·922	325·294	1·628 –	45·883	45·891	773·64
15	5670·8	84291·0	352·669	348·867	3·802 –	45·368	45·829	770·97
16	4965·5	84282·1	324·334	316·810	7·524 –	48·750	49·630	772·85
17	4953·3	84398·0	307·643	298·298	9·345 –	48·535	49·503	771·87
	Average		366·16 or 58°·46	..	..	..	..	772·72

The mean temperature of the atmosphere was derived from observations taken from minute to minute, but there were only two readings of the temperature of the calorimeter, viz., at the commencement and termination of an experiment, from which to determine its average temperature. Suppose *a b*, fig. 7, to represent the line of air temperatures during an experiment lasting 41^{m}: the temperatures of the calorimeter will be represented by a line similar to *c d e f*. The wheel was set in motion 2^{m} after the first reading was taken. The temperature then rose until, at 35^{m}, the wheel was stopped. The temperature then declined slightly, until, at 41^{m}, the last reading was taken. The line is slightly curved; a few seconds are occupied in starting and stopping the wheel, and the thermometer reads a little backwards. Taking all these circumstances into account, I found that the average temperature for the whole time was very accurately represented by $\frac{37\frac{c+f}{2}+4f}{41}$. The mean temperature of the calorimeter for other times of experiment was estimated in a similar manner.

To obtain the corrected rise of temperature in the last column but one, the correction to the air temperature indicated in the first part of the Table was supplied. For instance, in the first experiment the temperature of the air was virtually 14·29+1·525=15·815 lower than that of the calorimeter. Hence 15·815 × 0·1086=1·717, which, added to 42·761, gives the value for T, 44·478.

TABLE II., Part 1.—Experiments to ascertain the Effect of Radiation, &c. Time occupied by each experiment, 41ᵐ. Weight raised for an instant, 18229·0 grains.

No.	Revolutions. r.	Mean temperature of calorimeter.	Mean temperature of air.	Difference.	Rise of temperature of calorimeter.	Correction to air temperature.	Thermal effect of unit difference of temperature.
1*a*	25·5	259·220	268·008	8·788+	0·945+	1·410+	0·0926
1*b*	23·5	305·925	277·775	28·150−	2·478−		
2*a*	26·0	261·740	273·313	11·573+	1·572+	3·938+	0·1013
2*b*	22·92	308·925	286·733	22·192−	1·850−		
3*a*	23·60	267·270	291·597	24·327+	3·590+	5·741+	0·1194
3*b*	24·12	318·620	307·351	11·269−	0·660−		
4*a*	22·75	298·510	308·706	10·196+	1·00+	1·502+	0·0855
4*b*	23·17	344·910	316·151	28·759−	2·33−		
5*a*	22·0	311·560	311·188	0·372−	0·12+	1·685+	0·0914
5*b*	21·08	355·710	317·040	36·670−	3·38−		
6*a*	21·7	285·095	300·273	15·178+	1·59+	3·054+	0·0873
6*b*	21·04	331·865	308·541	23·324−	1·77−		
7*a*	21·3	292·760	304·393	11·633+	1·28+	2·709+	0·0892
7*b*	19·8	340·435	312·739	27·696−	2·23−		
8*a*	21·0	277·730	295·705	17·975+	2·46+	5·890+	0·1031
8*b*	21·2	328·775	311·147	17·628−	1·21−		
9*a*	19·83	301·550	321·510	19·960+	1·90+	0·955+	0·0908
9*b*	18·79	351·365	324·762	26·603−	2·33−		
10*a*	18·33	300·370	325·183	24·813+	2·34+	2·198+	0·0866
10*b*	17·85	352·595	335·507	17·088−	1·29−		
11*a*	18·2	277·800	304·655	26·855+	2·80+	2·587+	0·0951
11*b*	17·24	329·570	317·940	11·630−	0·86−		
12*a*	16·0	288·400	321·267	32·867+	3·20+	0·935+	0·0947
12*b*	18·74	336·980	331·820	5·160−	0·40−		
13*a*	18·0	313·820	331·190	17·370+	3·73+	6·279+	0·1577
13*b*	17·66	364·075	358·747	5·328−	0·15+		
14*a*	17·32	308·585	329·613	21·028+	2·33+	3·334+	0·0956
14*b*	18·0	356·840	340·123	16·717−	1·28−		
15*a*	32·0	309·885	331·872	21·987+	2·00+	1·485+	0·0852
15*b*	18·6	359·100	340·480	18·620−	1·46−		

TABLE II., Part 2.—Experiments with Friction of Water and Brass. Weight, W, lifted, 18229·0 grains. Average proportion of metallic to total friction, $\frac{1}{8\cdot3}$. Time occupied by each experiment, 41^{m}. V = 0°·077214 ; P = 2·77386.

No.	Number of revolutions. R + *r*.	Capacity of the calorimeter. C.	Mean temperature of the calorimeter.	Mean temperature of the atmosphere.	Difference.	Rise of temperature of the calorimeter.	Ditto, corrected for radiation, &c. T.	Mechanical equivalent, or $\frac{RWP}{CTV}$
1	4898·5	84349·7	286·242	271·743	14·499 −	47·686	48·898	773·94
2	4826·5	84242·7	288·94	277·634	11·306 −	47·477	48·223	774·09
3	4929·75	84191·7	296·43	298·242	1·812 +	50·046	49·144	776·48
4	4839·5	84324·0	324·9	311·823	13·077 −	47·288	48·278	774·80
5	4829·5	84270·0	336·865	312·924	23·941 −	46·119	48·153	775·93
6	4734·3	84256·0	311·387	303·190	8·197 −	46·893	47·342	773·74
7	4897·5	84234·5	320·1	308·110	11·990 −	48·168	48·996	773·84
8	5061·7	84295·7	306·684	303·337	3·347 −	50·640	50·378	777·30
9	5091·8	84294·0	330·182	321·677	8·505 −	50·237	50·922	773·88
10	5165·9	84250·0	330·3	328·046	2·254 −	51·800	51·805	772·38
11	5045·2	84302·0	307·173	309·280	2·107 +	50·820	50·374	775·28
12	4613·66	84292·0	315·95	324·655	8·705 +	47·250	46·337	770·62
13	4733·9	84304·0	342·25	343·864	1·614 +	48·326	47·081	778·11
14	4782·17	84284·0	335·99	333·819	2·171 −	47·771	47·660	776·73
15	4834·0	84244·0	338·125	333·930	4·195 −	48·225	48·456	771·42
	Average		318·101 or 54°·76	..	..	..	..	774·57

Instead of reckoning one-half of the revolutions which took place in the acts of starting and stopping the wheel, as was done in the case of Table I., I have eliminated them in the last and subsequent Tables by starting the wheel till the scales were raised for an instant and then immediately stopped it at some period in each experiment for determining radiation. The revolutions called *r* in the first part of the Table being subtracted from the revolutions called R + *r* in the second part, give the numbers used in calculating the equivalent. This latter plan obviated some slight error to which the former method was possibly liable.

The irregularities in the values of R arise from the variations from time to time in the friction of the bearing which supports the calorimeter on the axis. In the subsequent experiments I adopted a method which removed nearly the whole of the metallic friction. In fig. 1, *v* and *w* represent two concentric vessels. The inner one has a lid surmounted by three uprights, such as that represented by fig. 5. When water is poured into the space between the vessels, the uprights are raised so as to press against the bottom rim of the calorimeter, thus relieving its weight on the axis. The arrangement was eminently successful in producing an almost absolute uniformity of motion.

TABLE III., Part 1.—Experiments to ascertain the Effect of Radiation, &c. Time occupied by each experiment, 41ᵐ. Weight raised for an instant, 16477·4 grains.

No.	Revolutions r.	Mean temperature of calorimeter.	Mean temperature of air.	Difference.	Rise of temperature of calorimeter.	Correction to air temperature.	Thermal effect of unit difference of temperature.
1*a*	19·0	385·526	427·236	41·710+	4·05+	1·237+	0·1060
1*b*	19·5	432·690	424·879	7·811−	0·62−		
2*a*	18·58	366·510	376·830	10·320+	1·02+	0·683+	0·0927
2*b*	19·42	409·710	386·590	23·120−	2·08−		
3*a*	18·75	338·865	364·127	25·262+	2·325+	0·862+	0·0890
3*b*	20·33	386·535	368·066	18·469−	1·567−		
4*a*	19·5	339·675	368·504	28·829+	2·51+	0·429−	0·0884
4*b*	20·75	385·410	372·488	12·922−	1·18−		
5*a*	19·17	357·290	378·033	20·743+	1·78+	0·027−	0·0859
5*b*	19·31	402·025	386·340	15·685−	1·35−		
6*a*	20·67	357·885	389·996	32·111+	2·97+	0·843+	0·0901
6*b*	20·93	404·650	397·150	7·500−	0·60−		
7*a*	19·33	355·780	383·923	28·143+	2·56+	0·600+	0·0891
7*b*	19·75	401·960	389·234	12·726−	1·08−		
8*a*	19·4	878·970	388·517	9·547+	0·90+	0·612+	0·0886
8*b*	21·2	421·730	392·446	29·284−	2·54−		
9*a*	19·46	342·575	361·843	19·268+	1·83+	0·503+	0·0926
9*b*	20·0	387·000	369·210	17·790−	1·60−		
10*a*	18·83	347·205	369·358	22·153+	1·75+	0·240−	0·0799
10*b*	20·43	391·930	373·889	18·041−	1·46−		
11*a*	19·0	349·530	378·721	29·191+	2·78+	1·341+	0·0911
11*b*	19·0	395·970	386·502	9·468−	0·74−		
12*a*	19·0	344·970	383·753	38·783+	3·54+	1·383+	0·0881
12*b*	19·0	393·040	390·749	2·291−	0·08−		
13*a*	18·0	348·125	372·988	24·863+	2·17+	0·627+	0·0851
13*b*	17·75	393·550	380·002	13·548−	1·10−		
14*a*	18·12	346·520	378·315	31·795+	3·00+	1·185+	0·0910
14*b*	17·0	393·310	386·409	6·901−	0·52−		
15*a*	17·73	356·395	390·069	33·674+	3·49+	0·688+	0·1016
15*b*	19·23	404·170	400·922	3·248−	0·26−		
16*a*	18·0	345·085	380·701	35·616+	3·17+	0·560+	0·0876
16*b*	16·72	392·365	386·442	5·923−	0·47−		
17*a*	19·0	366·715	388·706	21·991+	1·93+	0·086+	0·0874
17*b*	18·0	411·775	395·102	16·673−	1·45−		
18*a*	19·57	374·230	400·624	26·394+	2·46+	1·102+	0·0895
18*b*	18·97	420·020	408·859	11·161−	0·90−		
19*a*	18·72	396·755	412·830	16·075+	1·49+	0·501+	0·0899
19*b*	19·0	440·415	415·996	24·419−	2·15−		
20*a*	19·33	367·770	403·836	36·066+	3·46+	1·611+	0·0918
20*b*	17·77	415·070	411·934	3·136−	0·14−		
21*a*	17·0	386·805	415·792	28·987+	2·93+	1·158+	0·0972
21*b*	18·3	433·470	426·345	7·125−	0·58−		

Table III., Part 2.—Experiments with almost solely Friction of Water. Weight, W, lifted, 16477·4 grains. Average proportion of metallic to total friction, $\frac{1}{106}$. Time occupied by each experiment, 41^{m}. V = 0°·077214; P = 2·77386.

No.	Number of revolutions. R + r.	Capacity of the calorimeter. C.	Mean temperature of the calorimeter.	Mean temperature of the air.	Difference.	Rise of temperature of the calorimeter.	Ditto, corrected for radiation, &c. T.	Mechanical equivalent, or $\frac{\text{R W P}}{\text{C T V}}$
1	4904·95	84160·7	412·530	422 744	10·214 +	45·509	44·295	775·78
2	4940·58	84124·2	390·866	381 225	9·641 –	43·909	44·740	774·04
3	4925·28	84118·1	367·765	364·627	3·138 –	44·564	44·767	771 14
4	4922·92	84071·2	365·695	369·159	3 464 +	45·125	44·857	769·56
5	4938 0	84012·2	382 816	380·063	2·753 –	44 747	44 986	770·39
6	4936·3	83930·2	384·452	391·613	7·161 +	45·580	44·859	772·82
7	4958·0	83940·7	382·015	386·005	3 990 +	45·530	45·121	771·82
8	4925·3	83882 2	403·368	390·360	13·008 –	43·643	44·741	773·64
9	4925·9	83907·2	367·897	364·284	3·613 –	44·564	44 852	771·68
10	4927·6	83884·2	372·565	370·435	2·130 –	44·706	44·857	772·09
11	4934 3	83911·2	375 915	381·468	5·553 +	45·438	44·810	773·81
12	4935·0	83889·2	372·285	386·270	13·985 +	46·150	44·795	774·38
13	4929·0	83880·2	373 936	375·248	1·312 +	44·930	44·765	774·22
14	4923·22	83888·7	373·092	380 689	7 597 +	45 487	44·688	774·61
15	4923·0	83872·7	383·670	395·008	11 338 +	45·850	44·629	775·60
16	4923·0	83897 2	372·033	382·095	10·062 +	45·707	44·776	773 00
17	4928·0	83889·7	392·412	390·767	1·645 –	44 652	44·788	773·47
18	4926·0	83904 7	400·189	402·495	2·306 +	45·105	44·800	772·69
19	4925·75	83891·2	421·652	413·149	8·503 –	44·148	44·867	771·68
20	4926·33	83944·2	394·541	406·037	11·496 +	45 763	44·559	776 68
21	4924·0	83938·7	413 330	419·217	5·887 +	45 459	44 774	772·76
	Average		385·86 or 59°·98	..	..	..	..	773·136

An error of four or five seconds in the time at which the wheel was started and stopped will account for the divergence of the revolutions in Nos. 1 and 7 from the average. For the rest, it will be seen with what great constancy the resistance of the paddle was kept up.

The weights were also so steady that the total distance run by them in their risings and fallings only amounted to about 30 feet in each experiment. This, taken with the friction of the pulleys = 30 grains, gives a quantity to be subtracted from the equivalent too small to require estimation.

TABLE IV., Part 1.—Experiments to ascertain the Effect of Radiation, &c. Time occupied by each experiment, 41^{m}. Weight raised for an instant, 7730·56 grains.

No.	Revolutions. r.	Mean temperature of calorimeter.	Mean temperature of air.	Difference.	Rise of temperature of calorimeter.	Correction to air temperature.	Thermal effect of unit difference of temperature.
1*a* 1*b*	9·5 8·62	362·355 374·950	358·090 363·536	4·265 – 11·414 –	0·45 – 1·10 –	0·684 –	0·0909
2*a* 2*b*	10·0 9·9	361·090 375·040	363·847 368·448	2·757 + 6·592 –	0·18 + 0·60 –	0·599 –	0·0834
3*a* 3*b*	10·0 10·24	355·495 368·430	352·673 357·570	2·822 – 10·860 –	0·31 – 1·02 –	0·688 –	0·0883
4*a* 4*b*	10·57 10·25	346·515 361·985	359·807 365·790	13·292 + 3·805 +	1·01 + 0·15 +	2·150 –	0·0906
5*a* 5*b*	9·66 10·74	340·260 354·970	346·208 356·487	5·948 + 1·517 +	0·52 + 0·08 +	0·711 –	0·0993
6*a* 6*b*	10·82 10·66	357·855 373·890	368·538 382·515	10·683 + 8·625 +	1·01 + 0·78 +	1·646 –	0·1118

TABLE IV., Part 2.—Experiments with almost solely Friction of Water. Weight, W, lifted, 7730·56 grains. Average proportion of metallic to total friction, $\frac{1}{43}$. Time occupied by each experiment, 41^{m}. $V = 0^{\circ}{\cdot}077214$; $P = 2{\cdot}77386$.

No.	Number of revolutions. $R + r$.	Capacity of the calorimeter. C.	Mean temperature of the calorimeter.	Mean temperature of the air.	Difference.	Rise of temperature of the calorimeter.	Ditto, corrected for radiation, &c. T.	Mechanical equivalent or $\frac{R\,W\,P}{C\,T\,V}$
1	3336·66	83963·7	369·487	361·043	8·444 –	13·495	14·325	768·32
2	3344·2	83944·7	369·065	366·560	2·505 –	14·078	14·337	769·39
3	3343·42	83959·7	362·809	354·425	8·384 –	13·646	14·447	763·18
4	3341·23	83949·7	355·191	352·160	6·969 +	14·927	14·490	760·44
5	3330·75	83975·0	348·411	350·362	1·951 +	14·416	14·293	768·30
6	3335·33	83965·0	366·732	375·533	8·801 +	15·040	14·240	772·19
Average			361·949 or 58°·14	..	..	..	..	766·97

It will be obvious that, in the experiments of the above Table, where the heat evolved was able to raise the temperature of the calorimeter little more than 1°, great accuracy could not be expected without taking the average of a very large number of observations. In fact, the degree of accuracy will increase nearly with the square of the rise of temperature per unit of time,* and the square root of the number of observations.

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* *I.e.*, supposing the "Differences," for calculating the air correction, increase with the values of T.

Table V., Part 1.—Experiments to ascertain the Effect of Radiation, &c. Time occupied by each experiment, 41^{m}. Weight raised for an instant, 21729·56 grains.

No.	Revolutions. r.	Mean temperature of calorimeter.	Mean temperature of air.	Difference.	Rise of temperature of calorimeter.	Correction to air temperature.	Thermal effect of unit difference of temperature.
1*a*	22·0	386·760	419·637	32·877+	3·52+	3·099+	0·09784
1*b*	20·64	454·455	427·134	27·321−	2·37−		
2*a*	21·16	390·200	433·283	43·083+	4·40+	2·444+	0·09665
2*b*	22·0	460·345	444·140	16·205−	1·33−		
3*a*	20·0	380·885	410·976	30·091+	2·77+	1·369+	0·08805
3*b*	21·22	448·250	418·488	29·762−	2·50−		
4*a*	22·0	386·310	420·428	34·118+	3·62+	1·965+	0·10032
4*b*	23·16	455·170	436·858	18·312−	1·64−		
5*a*	21·42	397·770	428·568	30·798+	3·46+	3·283+	0·10152
5*b*	22·0	465·440	441·275	24·165−	2·12−		
6*a*	19·0	396·300	438·413	42·113+	3·84+	2·100+	0·08685
6*b*	22·0	465·630	447·180	18·450−	1·42−		
7*a*	21·0	377·300	405·069	27·769+	2·60+	1·509+	0·08880
7*b*	22·0	444·240	414·354	29·886−	2·52−		

Table V., Part 2.—Experiments with almost solely Friction of Water. Weight, W, lifted, 21729·56 grains. Average proportion of metallic to total friction, $\frac{1}{108}$. Time occupied by each experiment, 41^{m}. V=0°·077214; P=2·77386.

No.	Number of revolutions. R + r.	Capacity of the calorimeter. C.	Mean temperature of the calorimeter.	Mean temperature of the air.	Difference.	Rise of temperature of the calorimeter.	Ditto, corrected for radiation, &c. T.	Mechanical equivalent, or $\frac{\text{R W P}}{\text{C T V}}$
1	5653·0	83924·2	425·354	419·585	5·769−	67·336	67·597	774·93
2	5653·16	83960·2	430·165	436·986	6·821+	68·582	67·687	773·54
3	5652·55	83970·5	419·268	413·363	5·905−	67·214	67·613	774·35
4	5648·66	84001·7	425·460	426·763	1·303+	67·877	67·549	774·00
5	5645·4	83986·2	436·432	433·830	2·602−	67·522	67·453	774·90
6	5646·5	83973·7	435·661	439·140	3·479+	68·215	67·730	772·17
7	5659·0	83970·7	415·580	408·720	6·860−	67·232	67·707	774·03
	Average		426·846 or 63°·14	..	..	..	..	773·99

The average number of revolutions per minute in the last two tables were 101·4 and 171·5. The fluid resistances, 7630·2 and 21548·5, were therefore almost exactly proportional to the squares of the velocities.

The foregoing results are collected in the following Table :—

Table.	Number of experiments.	Proportion of metallic to total friction.	Mean rise of temperature per A.	Temperature of the calorimeter.	Mechanical equivalent of unit of heat.
1	17	$\frac{1}{7\cdot7}$	45·907	58°·46	772·72
2	15	$\frac{1}{8\cdot3}$	48·803	54°·76	774·57
3	21	$\frac{1}{106}$	44·777	59°·98	773·136
4	6	$\frac{1}{43}$	14·355	58°·14	766·97
5	7	$\frac{1}{120}$	67·620	63°·14	773·99

The average of the first two gives 773·65 as the equivalent at a temperature of the calorimeter 56°·61 ; but inasmuch as the metallic friction is as much as $\frac{1}{8}$ of the whole, I prefer to use the last three, and to give each its due weight I will multiply the squares of the rise by the square root of the number of determinations :—

$$\text{For the 3rd series } (44\cdot777)^2 \times \sqrt{21} = 9188.$$
$$\text{For the 4th series } (14\cdot355)^2 \times \sqrt{6} = 504\cdot76.$$
$$\text{For the 5th series } (67\cdot62)^2 \times \sqrt{7} = 12097\cdot7.$$

Then—

$$\frac{733\cdot136 \times 9188 + 766\cdot97 \times 504\cdot76 + 773\cdot99 \times 12097\cdot7}{9188 + 504\cdot76 + 12097\cdot7} = 773\cdot467$$

is the equivalent at 61°·69 ; or, using Regnault's law of the increase of the specific heat of water with its temperature, 773·369 at 60°.

The latitude of the part of Higher Broughton, Manchester, where the experiments were made, is 53° $28\frac{1}{2}'$ N. ; its elevation about 120 feet above the sea level. The equivalent at the sea level and the latitude at Greenwich will therefore be 773·492 foot lbs., defining the unit of heat to be that which a lb. of water, weighed by brass weights when the barometer stands at 30 inches, receives in passing from 60° to 61° Fah. With water weighed in vacuo the equivalent is finally reduced to 772·55.

XII. *On the Structure and Development of the Skull in the Common Snake* (Tropidonotus natrix).

By William Kitchen Parker, *F.R.S.*

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[Plates 27–33.]

The most important work on the development of this type appeared as long ago as 1839. It was by the late Professor H. Rathke, 'Entwickelungsgeschichte der Natter (*Coluber natrix*).' Konigsb., 4to.

From time to time during the last twelve years I have worked at the Snake's skull, my guide being Professor Huxley's translation of so much of Rathke's work as bears especially upon my line of research; this I have had at hand, both in the 'Croonian Lecture' and in the 'Elements of Comparative Anatomy.' My earliest studies of the embryonic conditions of the Snake's skull were made on half-grown embryos of the Python (*Python Sebæ*), and of the Anaconda (*Eunectes murinus*); these were the gift of my friends Dr. Edwards Crisp, F.Z.S., and Dr. P. L. Sclater, F.R.S. Meantime I have been acquiring younger and still younger specimens of the Common Snake, but only lately have I succeeded in obtaining any embryos sufficiently immature for my *first* and *second* stages. During the last winter (February, 1877) these earlier stages happily came to hand. At the instance of Mr. P. H. Carpenter, M. Braun, of Wurzburg, sent me a considerable number of reptilian embryos in very early stages, from Professor Semper's laboratory. These included, besides the longed-for specimens of the Common Snake, others of the Blind Worm (*Anguis fragilis*), the Nimble Lizard (*Lacerta agilis*), and the Gecko (*Platydactylus*). These other three types I intend to work out in due time. The Snake has the simplest skull of all; and yet, simple as it is, it is a key to that of any member of that great bundle of Vertebrate life which, including the reptiles and birds, is termed "Sauropsida." (See Huxley, "On the Classification of Birds," Proc. Zool. Soc., April 11th, 1867, pp. 415–472.)

Having wrought very much at the morphology of the Birds' skull, the simple condition of the skull in such a type as the Snake—which lies down at the very base of the huge group of which the Bird forms the very top and crest—has been to me the most instructive; many things in the utterly specialized avian skull would, but for this key, have been quite beyond my power of interpretation.

In this stage of my work I am able to compare the early conditions of the Ophidian embryo with what may be seen in several other important Vertebrate types; in none is this comparison more instructive than in that of the Selachians with the Snakes. (See Trans. Zool. Soc., Vol. 10, plates 34–42, pp. 189–234.) Moreover, Mr. F. M. BALFOUR's researches on the embryology of the Sharks and Skates* begin to shed a most welcome light upon the development of those most instructive fishes from the blastoderm; his work dovetails with mine in the most convenient manner. But for the development of the "Sauropsidan" embryo the reader is referred to a joint work by Professor FOSTER and Mr. F. M. BALFOUR—their 'Elements of Embryology,' Part I., 1874. So completely at one are the processes by which a Snake and a Bird—the root and the branch of the great group of the Sauropsida—are developed, that these accomplished authors might have taken the *Snake* and not the *Bird* as their subject; scarcely a single figure would need to be altered, if a counterpart work, taking the Snake as the subject, were to be prepared and published. My earliest snake-embryo (Plate 27, figs. 1, 2) corresponds very accurately with that of the Fowl at the end of the fourth day of incubation (FOSTER and BALFOUR, p. 142, fig. 46); the *allantois* in my specimens had exactly the relative size shown in their figure of the chick at that date.

I shall, once for all, refer the reader to RATHKE's descriptions of the early stages of the skull (both in the original and in Professor HUXLEY's translation), and use my own terms and modes of speech in endeavouring to make the matter plain.

First Stage. Embryo Snakes, $\frac{3}{4}$ of an inch in absolute length.

Notwithstanding the length of these embryos, the *testa* of a mustard seed would have sufficed to enclose one of them; the figure (Plate 27, fig. 1) shows only the root of the tail, which is tucked in amongst the folds of the body; the neck is partly unwound, to display the huge head better.

The embryonic membranes are now perfect, and the embryo, on the whole, is a finished form; yet the huge loop-shaped heart is only covered by the pericardium, as it lies in the re-entering angle of the throat (fig. 1, *h.*, *pcd.*).

The head is bent upon itself at an angle which is less than a right angle (Plate 27, fig. 1; Plate 29, fig. 1); this great degree of flexion is retained even in the next stage (Plate 27, fig. 3, and Plate 29, fig. 2).

Of the three brain-vesicles, the hindmost (C^3) is the longest, and is least elevated; the fore brain (C^1) is becoming lobulate by reason of the budding out of the rudiments of the hemispheres; and the mid brain (C^2) is most regular in shape.

There are large spaces in each division, filled with a glairy fluid (Plate 29, fig. 1), and the more solid tract is of great length in the hind brain.

The hollow space within the loop of the brain ("middle trabecula" of RATHKE) is

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* Journ. of Anat. and Phys. (a series of most important papers), and Phil. Trans., 1876, Part I., Plates 16–18, pp. 175–195; and his work on the 'Elasmobranchs,' 8vo., MACMILLAN and Co., 1878.

very large and projects forward, horizontal in position, in front of the parachordal region (Plate 29, fig. 1, *m.tr.*, *py.*, *nc.*).

The infundibular (ascending=posterior) part of the fore brain (*inf.*) has not yet united with the small pituitary involution (*py.*), which is *closed* above and *open* below.

The cranial notochord is slightly curved downwards at its apex; it has (relatively) retreated somewhat from the pituitary space (Plate 29, fig. 1, *nc.*, *py.*).

The hypoblastic lining of the alimentary canal has met and united with the epiblast of the face and outer part of the rudimentary throat (Plate 27, figs. 1 and 2; and Plate 29, fig. 1); the oral cavity is, therefore, open.

Behind the mouth those two layers of the blastoderm meet in the *clefts* between the descending visceral folds (see also FOSTER and BALFOUR, p. 118, fig. 37); these folds are the rudiments of the visceral (hæmal) arches and their investments.

These visceral folds are very rudimentary at present; behind the mouth there are only *four* marked off from the rest of the tissue at present; a fifth appears in the next stage.

All but the first two of the post-oral folds are evanescent, and develop no cartilage; in front of the mouth two pairs more of these visceral fringe-like reduplications are developed (figs. 1, 2, *mx.p.*, *n.f.p.*).

The structure, the transformations, and the relations, of these descending or hæmal growths of the head are of the utmost importance to morphology; they form the palate, the jaws, the hyoid arch, and, in branchiate types, the gill arches.

They are related to the cranium, the sense-capsules, and the cranial nerves; their interspaces are the mouth, the naso-lachrymal passages, and the post-oral clefts, (tympano-eustachian passages, and gill openings).

So that, above all things, it is necessary to find out how these parts arise at first; from what layers of the blastoderm they are formed; and with what region, right and left, of the body, they may be compared.

The developing core of these folds, the solidifying floor and walls of the brain-case, and the capsules of the principal organs of the special senses—these three groups of skeletal structures together make the Vertebrate skull.

So that it is easy to see that this part of a Vertebrated creature differs as much from the rest of the skeleton as the brain and cranial nerves differ from the spinal marrow and spinal nerves. Moreover, the capsules and labyrinths of the special sense-organs add largely to the sum of differences.

Both the problem itself, and the methods to be used in attempting to solve it, are quite similar to the growth of a flower from axial and foliar rudiments, and the patient work necessary for tracing out its steps and stages.*

The Primordial or Membranous Cranium.—At present, the investment of the brain

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* The vegetative processes that have mysteriously brought the cell-layers of the blastoderm into the fundamental "Sauropsidan" pourtrayed in fig. 1, are surely well worthy to be sought out; that, however, is the work of the general embryologist, and is, in order, antecedent to my special inquiry.

is merely a layer of cells belonging to the "mesoblast," and clothed with the epiblastic cells of the epidermis; dura-mater, skull-wall, skin—all these are, as yet, one stratum.

This stratum, however, is thickening, and becoming differentiated into secondary laminæ in the floor of the head. One remarkable fact is that this floor is imperfect at one part. Where the fore and hind brain meet below and behind the mid brain, there we see a large circular fontanelle, with a clear margin. Through this gap the brain can be clearly seen.

In the gap, however, there is a remarkable structure, the rudiment of the pituitary body, which is either formed by an involution of the buccal cavity (epiblast), or cut off from the fore-gut (hypoblast). The latter is the view taken by WILHELM MÜLLER; the former by GOETTE. (See FOSTER and BALFOUR, pp. 91–93.)

Where this remarkable structure passes into the cranial cavity, there the epiblast and hypoblast unite; there the axial notochord ends, and, as on a pivot, the skull first bends itself into a hook and then gradually straightens itself again.

On each side of the cranial notochord (Plate 27, fig. 2, and Plate 29, fig. 1, *nc.*, *iv.*) the floor of the hind brain is thickening to form the investing mass or parachordal tracts. The innermost cells of the stratum ultimately become cartilage.

These bands, however, which correspond to a vertebral tract, although not segmented, pass forwards on each side of the basal opening or pituitary space; these *prochordal* bands are the trabeculæ cranii (*tr.*).

These are the first, and at present the only, rudiments of skeletal structure in this membrano-cranium; the fore part of the floor, the roof and side walls, are at present merely a single cellular lamina (Plate 29, fig. 1).

But each division of the brain has on each side of it the rudiment of what will soon become a highly complex sense-capsule.

The lowermost swelling of the brain, the rudiment of the hemispheres, has on each side, a large, shallow, inverted cup, whose lower margin is sinuous and lobate (Plate 27, figs. 1, 2, *ol.*); these rudiments of the *nasal sacs* grow by their base to the skull wall, and look like cup-shaped Fungi (*Stereum*).

Down on each side, in the hollow between the fore and mid brain, the young eyeball is seen (fig. 1, *e.*); it is an imperfect oval, the reduplication which makes it being unclosed below; the notch widens into a triangle above, and shows the crystalline lens as a small seed-like body.

The forepart of the eyeball comes very near the olfactory cup, and is far from the third sense-capsule (or ear); its hind part rests against a visceral fold in front of the commencing mouth (*m.*).

The rudiment of the ear is like a depressed gourd, whose short neck is somewhat constricted from the bulb; this sac lies on each side of the hind brain, behind its middle, and is more than its own longest diameter from the eye.

This oval sac is encrusted with crowded mesoblastic cells below and at the sides, but a large oval space above is only covered with a thin gelatinous layer. (For a description of the development of this capsule in the Chick, see Foster and Balfour, pp. 111–117.)

The visceral folds and clefts can now be considered ; they are equally related to the sense-capsules and to the brain pouches or membrano-cranium.

These descending visceral or hæmal parts are represented in the body by a pair of laminæ on each side, the somatopleure and the splanchnopleure (Foster and Balfour, pp. 62–67) ; but this division only exists *above* and *in front* when the visceral folds are first formed ; there is a cavity in each fold, but it soon closes.

The pericardium (Plate 27, fig. 1, and Plate 29, fig. 1, *h.*, *pcd.*) is the foremost permanent part of the cavity formed by division of the ventral lamina into two folds.

Thus the hæmal parts of the mouth and throat are formed in a *generalized tract ;* and, moreover, this does not grow into a *wall,* but (at first, at any rate) into a mere *balustrade*—columns and interspaces.

Then as there are no structures in the body answering to the sense-capsules, and as the visceral folds are most intimately related to those special cephalic structures, we get *three* things in which the *pleural* elements of the head and throat differ from the costal elements of the body.

A *fourth* difference might be mentioned, namely, that each rib arises from a distinct axial segment ; whereas the visceral growths of the head, even when they can be most clearly followed up to the axial parts, are downgrowths from a continuous structure.

These folds with their interspaces (Plate 27, figs. 1 and 2 ; and see also in the Chick, 'Foster and Balfour,' p. 118, fig. 37; p. 142, fig. 46 ; p. 180, fig. 56 ; and p. 181, fig. 57) form a regular series, some of which are in front of the mouth and some of them behind it.

Of course those in front of the mouth are the most aberrant in form, the parts from which they grow being so much more specialized than the rootstock of the post-oral rods.

At present there are only two post-auditory folds ; these answer to the gill-arches of the Ichthyopsida ; a third appears afterwards (Plate 27, fig. 3). These are the smallest of the series in the Snake.

These folds are somewhat bulbous, so that the clefts are larger and more open above than further down ; their present development very partially walls-in the face and throat, and the pericardium lies between and below the three hinder pairs.

The second post-oral or *hyoid* processes are larger than the branchial, and each has a large root above ; this broad, fan-shaped "origin" embraces the whole lower face of the ear-sac.

In front of this "root" we see the largest of the clefts, and, moreover, that the flexure of the head brings the bulbous distal ends of this fold, and the one in front of it, together.

Thus this cleft becomes high in position as well as wide ; it is called the "spiracle" in Sharks and Ganoids, and is well seen in the Selachians ; in adult Batrachians, and in the higher Vertebrata, it forms the tympano-eustachian passage.

Between the eye and the ear we see the fore part of the hyoid "root;" the root or origin of two more folds; and *three* visceral clefts. This state of things asks some consideration. The two large visceral folds in front of the hyoid rudiment arise from a thick mass of mesoblast lying in the valley between the mid and hind brain, below.

The cleft between these two is the *oral* cleft (*m.*), the thick bulbous fold behind is the mandibular rudiment (*mn.*), and the fold in front, which is high above and rounded below, is the maxillary rudiment or maxillopalatine fold (*mx.p.*).

The oral cleft is, like those behind the ear, larger above than lower down, and then gently widening below.

These clefts all open freely into the buccal and pharyngeal cavities, and these cavities, which meet inside the mandibular rudiments, are lined, the former with epiblast and the latter with hypoblast.

In my last paper (Phil. Trans., 1877, Part II., Plate 21, fig. 4) I showed in the unhatched Axolotl's embryo the *future oral cavity* filled with mesoblastic cells, which cells separated the buccal from the pharyngeal cavity.

Here, however (Plate 29, fig. 1), we are past that stage, and these two spaces pass the one into the other.

The oral cavity is now almost uniform with the whole space between the head and throat, for the maxillary, hyoid, and branchial folds, are all imperfect below, only the mandibular have united (fig. 2). What, then, is the relation of the *oral* to the other clefts in further stages? If we compare this with the next stage, we shall get some light upon the subject (Plate 27, figs. 1 and 3; Plate 29, figs. 1 and 2).

In front of the pericardium the mandibular rudiments have united, and thus have formed the *lower lip* (fig. 2, *mn.*); whilst the maxillary rudiments, which seem to be narrow as seen from the side, are, indeed, thick masses, directed inwards. (See lower view, fig. 2, *mx.p.*)

In the next stage (fig. 3) the folds in front of and behind the mouth have become widely divergent, so that the apex of the front fold is set on the apex of the hind fold at more than a right angle; hence the gaping oral cavity (figs. 3 and 4, *m.*).

The angles of the mouth are the top part of the right and left *oral clefts;* the opening of the mouth is caused by the suppression of the ventral floor of the head; the opening into the cavity of the mouth (buccal cavity) is caused by absorption of the mesoblastic mass before spoken of. Hence the mouth of the Vertebrata, like that of the higher Invertebrata (all but the lowest), is "deuterostomatous."

In front of the maxillary rudiment we have the *foremost cleft;* it is the naso-lachrymal (*l.cl.*), and runs between the postero-inferior margin of the eye (*at present;* afterwards more completely below the eye) and the maxillary rudiment.

On account of the immense development of the head in front, and the forward position of the nasal sacs, this cleft is a good way behind the foremost visceral fold.

In the Batrachia ("Frog's Skull," Plate 3, figs. 2 and 10) this cleft cannot be seen; this arises from the fact that the maxillo-palatine fold is suppressed until the time of transformation.

But in the Salmon ("Salmon's Skull," Plates 1–3) and in the Pig ("Pig's Skull," Plate 28, figs. 1–3) this part is clearly seen; in the Fowl ("Fowl's Skull," Plate 81, fig. 1) it is represented by the line between the eye (*e.*) and the sub-ocular bar (*pa.*).

I am not certain whether this cleft is quite open within; if not, its morphological importance is the same.

The ventral region is finished in front by an elegant double fold of skin, separated by a rounded emargination at the mid line, which grows from the head across the valley between the fore brain and the rudiment of the hemispheres.

These, together, form the "naso-frontal process (*n.f.p.*);" they are attached each to the corresponding olfactory dome, at its postero-internal part. These two folds grow more together, and receive, afterwards, a double growth of cartilage from the trabeculæ—the conjoined trabecular cornua, rudiments of the foremost visceral arch. (See Plate 28, figs. 6 and 9, *c.tr.*)

The relation of the cranial nerves to these ventral (visceral) folds, and their intervening clefts, is of the utmost importance in the morphological interpretation of the parts of the face and throat.

Leaving out of question for the present those cranial nerves which cannot yet be classified, such as the olfactory, optic, and motores oculorum, the rest have very definite relations to the folds and clefts of the head.* The trigeminal, facial, glosso-pharyngeal, and vagus nerves (5th, 7th, 9th, and 10th) break up into two or three principal branches that run on each side of the clefts.

Both the facial and trigennial have three main branches; the glosso-pharyngeal two; and the vagus as many as there are branchial arches to be supplied by it behind the glosso-pharyngeal, this latter nerve supplying the back of the hyoid arch and the front of the first branchial.

The facial nerve (7th), besides sending its main part to the front of the hyoid arch, and giving off the *chorda tympani* branch to the back of the mandible, sends forward a third branch, the vidian, which is distributed to the maxillopalatine regions.

This nerve runs outside and below the first branch of the great trigeminal or 5th, whose second and third branches supply the parts round the oral cleft.

But the first branch of the 5th, the ophthalmic or orbito-nasal, runs over the optic nerve into the nasal region, and, whilst the second and third branches in a normal manner fork over the oral cleft, the first branch runs in front of the lachrymo-nasal cleft to supply the rudimentary terminal arch, whose skeleton is formed by the cornua trabeculæ.†

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* The "portio mollis," 8th or auditory nerve, may be a dorsal branch of the facial or seventh nerve; I leave it out of special consideration here.

† The later researches of Mr. Balfour (Phil. Trans., 1876, Part II., Plates 16–18, pp. 175–195), showing that these nerves are formed in the "Elasmobranchs" from epiblast, give us some hope of a true classification of the cranial nerves; for it now looks as though the olfactory and optic nerves were not so widely diverse from the others as was supposed. Dr. A. Milnes Marshall has set this at rest with regard to the 1*st nerve;* it does arise like the rest (see Quart. Journ. Micr. Soc., vol. 18, new ser., plates 1, 2, pp. 1–31).

Thus the highly-specialized parts round the oral clefts get an additional nerve-supply from the 7th; and the 5th gives off its first branch to the front of the face, which has no other chance of a supply of this kind, the neural outgrowths of the fore brain being specialized to the utmost for the 1st and 2nd sense-capsules, and three of the motor nerves going to the orbital muscles.

Second Stage. Embryo Snakes, about 1 *inch in length.*

This stage (see Plate 27, figs. 3–6, and Plate 29, fig. 2) gives us some important characters in the fast modifying head, which is now one-third larger than in the last stage.

The bend of the head is fully as great as it was (figs. 1 and 3); the mid brain (C^2) projects still more; and the hemispheres are now fairly divided (fig. 4, C^{1a}).

The brain is much more solid and complex (Plate 29, fig. 2); the space between the mid and hind brain (middle trabeculæ, *m.tr.*) is much less; the pituitary body is much larger, but is quite distinct from the infundibulum (*py.*, *inf.*).

The notochord (Plate 29, fig. 2, *nc.*) is straight and has receded from the open, oval, large pituitary space (Plate 27, fig. 6, *py.*); the roof of the head, over the long hind brain (figs. 3 and 5, C^3) is very thin, and the cerebral cavity is still large (Plate 29, fig. 2, C^3).

The large splanchnic pouch (pericardium) has retreated; its end was opposite the first post-oral cleft (fig. 2); it is now opposite the middle of the third post-oral fold (fig. 4). It is evident that the visceral folds are above and in front of the permanent division of the body wall into "somatopleure" and "splanchnopleure."

The oral cavity (*m.*) is now a large square space, with extended angles that open out between the maxillary and mandibular rudiments (fig. 4, *m.xp.*, *mn.*).

A comparison of the side views shows how this has taken place; those rudiments, the second pre-oral and first post-oral folds, instead of being nearly parallel, are placed at right angles to each other, the dorsal end of the maxillary fold abutting against the front of the dorsal end of the mandibular fold.

The distal ends of these folds are much enlarged; this is best seen in a lower view (fig. 4). They both project inwards, but the foremost pair are far apart; the mandibular folds have an extensive *symphysis*, and form a very solid chin.

The pre-oral cleft (*l.cl.*) is shallow and imperfect; the first post-oral cleft is a large, long, sinuous, and perfect slit; it passes downwards and backwards, a definite part of the root of the next fold lying between it and the ear-capsule.

The next or hyoid fold is now, relatively, much smaller, and like the other three—for there are *three* branchial folds now—it is curved forwards; the three other clefts are much smaller than the first post-oral; they are arcuate, and bend forwards.

The small hyoid fold does acquire some cartilage above, but the other three folds remain membranous, and they are now at their fullest development.

In the first stage (fig. 1) the ear-sac was embraced by the foot of the hyoid fold, and thus the rudiments of the 7th and 9th nerves were emerging at a considerable distance from each other.

Now that distance has greatly increased, for the main branch of the facial nerve (7th) creeps to the front of the hyoid fold, close behind the hyomandibular cleft; thus the space between these serial nerves is greater than the long diameter of the enlarging, egg-shaped ear-capsule.

This capsule (figs. 3 and 5, *au.*) is now well covered above with skin; it is a long distance from the eye.

The eye makes its orbit by nestling in the hollow outside the re-entering angle of the crozier-shaped head; it has no superorbital eave as in the Salmon and the Shark for the chondrocranium, which in those types is very massive, is reduced to its smallest limits in the Snake.

The position of the eyeball in front of the maxillary fold will soon be changed; the straightening of the head will throw it *over* that part.

At first blush, this stage suggests to the observer that the lachrymal cleft and the maxillary fold have their distal ends behind, against the mandibular rudiment. A reference to the first stage (fig. 1) corrects this view; their distal ends are close to the nasal pouches.

These pouches, or inverted cups, are now large and well differentiated, and are growing nearer to each other; between them the leafy folds of the fronto-nasal process (first visceral folds) have approximated and united, so that their interspace is a mere emargination of a single flap (figs. 4 and 6, *n.f.p.*).

If these cyathiform olfactory rudiments (figs. 3, 4, and 6, *ol.*) be compared with their counterparts in the adult Selachian (*Skate* or *Shark*), their likeness will be at once seen. These inverted domes are far apart, the fore brain, growing into hemispheres, swelling down between them.

The crescentic folds that adorn the bottom of these cups, lying concentrically, and enclosing towards the inner side lobate masses of cells, do not become transformed into the elegant, pectinate *membranous turbinals* of the Selachian.

These folds, traced from without inwards, become the septo-maxillary, the two upper labials, the nasal gland, and the vomer; afterwards, these parts crowd towards the septum of the nose, and lie below the olfactory region.

A consistent granular tract, ready to become cartilage, is now formed in the base of the membrano-cranium, which is thus breaking into three main layers (besides the epidermis), viz., the cutis, the layer of young cartilage, and the dura-mater.

When the ventral parts of the post-oral folds are cut away, the hind brain severed behind its middle, and the head examined from below, we get what is shown in fig. 6.

Besides the best view of the nasal cups and first visceral folds, the floor of the skull is better seen.

In the preparation figured the notochord was severed at its apex, and the granular investing mass (parachordal tracts) was cut through the middle.

Behind, these tracts are in contact with the notochord, but in their middle part they are narrow, and form the "posterior basi-cranial fontanelle" of RATHKE.

Then in the part from which the notochord has retired these bands approximate up to the oviform pituitary space (*py.*).

From thence, forwards, they bear the name of "trabeculæ cranii;" they are narrowed between the maxillary rudiments, and become wider as they meet in front of the great opening.

In front of the pituitary space the trabeculæ are close together again, but a dividing line of gelatinous tissue can be seen as they pass to the frontal wall; they *will* be continued as a coalesced process of cartilage into the combined fronto-nasal rudiments; there, of course, they become retral.

The horizontal sections of the maxillary, mandibular, and hyoid, rudiments are very instructive. The first of these (*mx.p.*) is very massive, for in it the maxillary, palatine, and transverse bones will be developed; but it does not acquire any cartilage in its interior.

But the two first post-oral folds have already acquired a pith, which corresponds histologically with the pith of the visceral bars in the first stage of the Salmon ("Salmon's Skull," Plate 1, p. 113), it contains a cavity which soon closes (see p. 389).

The branchial arches do not chondrify; the auditory sacs are not yet cartilaginous; but both these and the other special sense-capsules soon acquire a coat of thin hyaline cartilage.

Between this stage and the next there is a large difference, but the foundations are already laid, and although the building rises almost as rapidly as "an exhalation"—for the development and metamorphosis of the cells is only slower than the formation of dew—yet from what we have just seen we are enabled easily to interpret what we are about to see.

Third Stage. Embryos of the Snake, from $1\frac{3}{4}$ to $2\frac{1}{4}$ inches long.

The metamorphosis of the head of the Snake's embryo, whilst the body has not doubled its length, is very remarkable.

The flexure of the head upon itself is now almost lost (Plate 28, figs. 1–5); thus the brain segments lie almost in a straight line (fig. 5); the mid brain (C^2) forms a very projecting, bulbous "crown."

From that part, over the hind brain (C^3), the head sinks rapidly, for the cerebellum is very small. The skin is now well developed. At present the fore brain does not lie flat down upon the floor of the skull, but rises, as in those types that have a septum to the orbital region.

From the primary vesicle (C^1) the hemispheres (C^{1a}) have budded and developed, and from these have budded the olfactory lobes (C^{1b}) which now lie in front, supplying fibres to the inner walls of the nasal sacs: these sacs have gained their position in front.

The hind brain swells downwards very much, and, like the fore brain of the Amphibia, causes the beams of cartilage on which it rests to bend outwards; we thus get a "posterior basi-cranial fontanelle" in this type equal to the anterior basi-cranial fontanelle (or pituitary space) in the Amphibia (figs. 6 and 8, *p.b.c.f.*).

The space between the hind and mid brain is now a clean fissure, whose convexity is forwards; it is occupied by a membranous "tentorium."

As in the early stages the mid brain is lifted up on high, and thus the fore and hind brains are only separated by the tentorium.

A membranous floor is now placed beneath the hollow pituitary body. I cannot trace any communication between that hollow and the interior of the budding infundibulum, at present (fig. 5, *py.*, *inf.*).

Leaving for awhile the internal structure of the head, we see a very notable outward form. This may be said to be the *Rhynchosaurian stage.*

The earliest condition of my second stage of the Fowl's head is very similar to this, and this is the essential form which is retained in that generalized Lizard, the New Zealand *Hatteria,* and in the Chelonians generally.

The head of typical "Lacertians" and sub-typical "Anguians" (as I shall show in future papers) is precisely like the Snake's head at this stage; it is the common primæval dragon-form, and is also the fundamental form of the head in "every winged fowl after his kind."

The flexure of the head now principally shows itself in the hooking downwards of the *beak,* and the setting of the head on to the arched and hunched neck.

The nasal sacs occupy a considerable space in front of the brain, their outer and inner openings are complete (figs. 3, 4, *e.n.*, *i.n.*); they are roofed by a pair of swelling hollow reniform cartilages that lie back to back, and are confluent below with the confluent fore part of the trabeculæ (figs. 5 and 6, *ol.*, *tr.*).

We thus have, already, a small septum nasi (*s.n.*), which forms an obtuse angle with the distinct interorbital tracts of the trabeculæ (figs. 5 and 6), and which is terminated in front by a hooked flap (figs. 6, 9, *c.tr.*); this flap is formed by the confluence of the "cornua trabeculæ," it grows into the fronto-nasal fold, and is, in reality, the rudiment (combined into one process) of the foremost visceral arch (Plate 27, figs. 2, 4, 6, *n.f.p.*).

In the obtuse angle between this process and the interorbital, free trabeculæ, on each side of and below the septum nasi, we have the large nasal glands (fig. 5, *n.g.*); each gland is enclosed afterwards in two bony capsules—the septo-maxillary and vomer.

Below the glands and septum there are now well developed pre-palatal folds of skin; the inner nostrils meet behind these and appear as one opening under the fore brain (fig. 4, *i.n.*).

Above the palatal structures, outside, we have the huge, almost perfect, eye-ball (*e.*);

between it and the maxillopalatine bar part of the *first cleft* (*l.cl.*) is clearly seen towards the "inner canthus."

Above and behind the junction of the maxillopalatine and mandibular regions, therefore above and behind the angle of the mouth *at this stage,* we see the form of the ear-capsule, whose cutaneous investment is very thin, above. Behind the angle of the jaw, and below the ear-ball, the first *post-oral* cleft is still visible, and behind it the other three, the hyo-branchial and branchial clefts, are still evident as chinks fast closing in (figs. 1, 2, *cl.* 1–4).

It will be seen that the angle of the lower jaw (figs. 1, 2, *mn.*, *mk.*) has already gained a position so far backwards as to be directly beneath the involution of the ear-sac; it looks now directly backwards. In the last stage this angle was placed between the eye and ear, and looked forwards and upwards (Plate 27, fig. 3, *mn.*).

So rapidly has the face taken on that peculiar metamorphosis which results in the formation of the adult Snake's mouth—a mouth which opens *behind the head,* at a distance equal to half the length of the head (Plate 32, figs. 1 and 2).

The immense eye-balls are acquiring sockets, the membrano-cranium adapting itself to their form; but they are too independent of the skull to need more than a casual notice here.

The ear-balls, on the contrary, are becoming part of the skull; they are chondrified now, and much elongated (figs. 6–8).

In the younger embryo (fig. 6) we see the auditory capsules marked off from the investing mass (*iv.*) by younger cells; they reach from the glossopharyngeal nerve (9) to opposite the entrance of the internal carotid arteries (*i.c.*).

Between the capsules and the narrow anterior part of the investing mass there is a large rounded notch; over this membranous space the Gasserian and facial ganglia lie.

The space between the notch and the hind margin shows two important structures below; the first of these is a small pouch containing otoconial granules: this is the first appearance of the cochlea (*cl.*)—first, practically, in the Vertebrata, as well as in the individual embryo.

Behind this ingrowing pouch there is an opening, and to the membrane stretched across this opening the small hyoid cartilage is attached: the membranous space is the fenestra ovalis; the hyoidean rudiment will be the columella (figs. 6, 7, *hy.*).

Partly occluding the notch in front of the cochlea is an ear-shaped flap of cartilage, widest behind, and having its crescentic or concave edge looking outwards; this is the small infero-lateral alisphenoid (*al.s.*).

In front of the optic nerve in the older embryo (fig. 8), a patch of thickening tissue shows the rudiment of a similarly independent orbito-sphenoid (*o.s.*); the independence of these tracts is characteristic of the Ophidia; as yet I have seen no other instance.

Looking now at the side view of the ear-capsule in the younger embryo (fig. 7), we see that the two post-oral cartilages lie on its side; the second is attached, the first lies loosely against it: the meaning of this will soon be apparent.

The form and relations of the ear-sacs are clearly shown in an upper view of the skull of the more advanced embryo (fig. 8).

These large capsules nearly meet above the hind brain, and are separated there by a right-angled tract of cartilage, the super-occipital (*so.*).

The great size and the alteration of form is due to the rapid growth of the semi-circular canals. The two anterior canals (*a.sc.*) bound the great fontanelle; each posterior canal (*p.sc.*) runs alongside the super-occipital cartilage, and the horizontal canal (*h.sc.*) bulges outwards over the fenestra ovalis, and forms the outer edge of the cranium.

All the ampullæ are outside, but those of the anterior and horizontal canals are close together, whilst that of the posterior canal is behind; the tube of this and of the foremost meet above the brain at less than a right angle.

The chondro-cranium is well developed behind, but dwindles down to almost nothing in front of the pituitary body; distinct filiform trabeculæ, and small orbito-sphenoids, make up all there is of cartilage in the fore half of the skull.

The parachordal tracts or investing mass (*iv.*) are large and united; they are forming the occipital condyle (*oc.c.*) behind; have the notochord, short and twisted, between and above them in their hinder half, and have a large oval deficiency in front.

This is what was spoken of just now as the "posterior basi-cranial fontanelle;" there is a cartilaginous bridge both in front of and behind this space.

On each side the cartilage grows out into hooked processes, in front of which we have the carotid foramen (*i.c.*), and the commencement of the trabeculæ (*tr.*).

The narrow bridge of cartilage which divides the two fontanelles (*p.b.c.f.*, *py.*) from each other, is bent forwards; the pituitary body dips immediately in front of it.

But, as in Amphibia, the pituitary space is much larger than the pituitary body; it is, here, a long spatula in form, with an emarginate blade and a pointed handle.

This space is bounded by the trabeculæ (*tr.*) and is caused by their form and relation; for from the post-pituitary bridge (which is formed by their apices), and the carotid foramina, they run at first outwards, then gently inwards, then they are parallel, and lastly they converge, unite, and after a short, free, single tract, they become two-crested, and unite also with the descending olfactory laminæ (figs. 5, 6, 8, 9, *tr.*, *s.n.*, *ol.*).

The olfactory roofs have already been described; but before turning to the facial rods, I must notice that a long styloid tract of granular tissue is wedged between the trabeculæ, and reaches the pituitary body by its broad hinder end; this is the tissue which is ready to become the parasphenoid (*pa.s.*).

The rudiments of the curious palatal basket-work are already apparent (fig. 4, post-oral arches removed); the granular tissue is just ready to develop "osteoblasts," but no hyaline cartilage precedes the bony tracts that soon appear.

Two bands on each side unite in front as an arch by means of a key-stone process; this key-stone is the old fronto-nasal process, and continues above the trabecular cornua (figs. 4, 5, 6, 9, *c.tr.*), and below the tissue which becomes the single pre-maxillary (fig. 5, *px.*).

From this, the bowed outer edge contains a pith which becomes the maxillary, and the inner, in like manner, the rudiment of the palatine bones, whose ascending process is prefigured by the lobes that constrict the inner nasal passage (*i.n.*).

The outer and inner bands are united obliquely behind by a tract which becomes the transpalatine, and the hind part of the inner band developes the pterygoid.

The two folds in front of the inner nostril contain the tissue that becomes the right and left vomers; the circular marking behind the opening, is the pituitary body (*py.*).

The mandible (figs. 2 and 7, *q.*, *mk.*) is now composed of two cartilages: a suspensory piece or quadrate, and the articulo-Meckelian rod.

The former has an anterior blunt process above, the stunted pedicle; and a longer otic process (fig. 7, *pd.*, *ot.p.*).

As in the Salmon, the segmentation of the rod is such as to make a hinge between the two parts with a long angular process to the lower segment.

This lower piece (*ar. mk.*) is a long sigmoid rod, thickest above, and then uniformly terete; the axes of the two segments are coincident, and their direction downwards and forwards forms a very acute angle with the basis-cranii.

The hyoid fold (*hy.*) has cartilage only in its upper third; this is a rib-like piece with a rounded capitulum, a solid tuberculum, and a sickle-shaped shaft, whose convexity is backwards.

The hind edge below has a crest ready to separate from the main part.

The capitulum is attached to the membrane of the fenestra ovalis (*st.*), which is beginning to chondrify continuously with the hyoid rib-like rod; the meaning of these parts will be seen in the next stage.

Fourth Stage. Embryo Snakes, $2\frac{1}{2}$ *to* 3 *inches long.*

In the last stage the chondrocranium was perfect, and granular tracts along the palate and face were beginning to ossify.

Yet only the hind part of the hind brain was covered with cartilage, and the fore part of the cranial floor had merely two rounded threads of cartilage supporting it; all the rest was membrane.

About five-sixths of the Snake's skull is made up of subcutaneous, supplementary bones; is dermo-skeletal, in fact.

This stage shows the origin of most of the endoskeletal centres of the limited chondrocranium, and nearly every centre whose origin is in a mere membranous tract.

The shape of the chondrocranium was perfect in the last stage, so that the main work to be done now is to show its histological transformation.

The first bony part to be noticed is the *cephalostyle* or *bony* sheath of the cranial notochord. It is perfect now, and has begun to infect the cartilage right and left, so as to widen the tract that becomes the broad basi-occipital (Plate 29, fig. 5, *bo.*).

A considerable synchondrosis separates the basal centre from the lateral occipital patches (*eo.*); these are creeping round the foramen magnum, and show themselves on the top (Plate 29, figs. 3–5; Plate 30, fig. 1, *eo.*).

Each is seen to be perforated for the hypoglassal nerve (12), in front of and within the posterior condyloid foramen (*p.c.f.*); and they reach to the foramina for the glosso-pharyngeal and vagus (9, 10), which emerge between the ear-capsule and basal plate.

Where the trabeculæ pass into that plate—the investing mass—there on their outer margin a thin film of bone may be seen (Plate 29, fig. 5, *bs.*): these films are the symmetrical basisphenoids; this deposit takes place exactly where the larger parasphenoid of the Bird grafts itself upon the cartilage.

In some specimens I found a thin ectosteal film over the anterior ampulla (Plate 29, fig. 4, *pro.*): this is the beginning of the prootic.

Besides these ossifications, certain changes in the rest of the chondrocranium have to be noticed.

The small hyoid cartilage now looks downwards and backwards, and the "tuberculum" is behind the "capitulum;" both are attached to the stapedial plate, and the whole structure is cartilaginous: it is the "columella" (*co.*).

The thin hinder edge of the hyoid is now distinct as a heart-shaped stylo-hyal (*st.h.*).

The alisphenoid (Plate 29, figs. 3 and 5, and Plate 30, fig. 1, *al.s.*) is now applied by its dilated ends to the basal plate, and the concave margins of the two conspire to make a large oval foramen for the 5th nerve.

But the first and second branches, only, pass through this double notch; the mandibular (third) branch escapes behind the alisphenoid, between it and the prootic region of the ear-sac: this has to do with the backward route it takes in company with the lower jaw.

The 7th or facial nerve passes through the front of the ear-capsule; close behind the third branch of the 5th and a little behind the 7th the 8th nerve enters the ear (Plate 30, fig. 1; 5, 7, 8).

The triangular tracts that lie in front of the optic nerve (2, *os.*) are now becoming cartilaginous; they are the small, *free* orbito-sphenoids.

The quadrate hinges on to the articular region of the mandible by a condyle like that of the *humerus* for the *ulna;* it forms with it an obtuse and then an acute angle (Plate 29, figs. 3, 4, *q*, *ar.*).

The re-entering angle of this joint makes the gape, which is getting further and further backwards; a membrane bone, the squamosal (*sq.*), intervening between the quadrate and the skull.

The quadrate (*q.*) is becoming ensheathed with bone; the upper part, like a Mammalian supra-scapula, and the condyle, being soft.

On the left side in fig. 4 a rudiment of the pedicle can be seen, but the main part at top is the otic process.

The articular region of the mandible is ossifying; the rod in the middle is flattened somewhat, and then runs in an arcuate manner to the free end.

The bones formed in non-cartilaginous tracts have now to be described.

In the fore half of the basis-cranii there is a sharp style of bone; it is wedged between the trabeculæ, and broadens out under the pituitary space; this is the parasphenoid (Plate 29, fig. 5, and Plate 30, fig. 1, *pa.s.*): it is seen in longitudinal section in the latter figure.

The infero-lateral regions of the cranium have acquired two bony tracts, the frontals and parietals (*f.*, *p.*); the frontals are rapidly growing round the fore part of the hemispheres (Plate 30, fig. 3, *f.*), whilst the parietals (fig. 5, *p.*) have a lower position.

The nasals (Plate 29, figs. 3, 4, and Plate 30, fig. 2, *n.*) lie on the inner side of the nasal roofs, and dip deep down between them.

The pre-maxillary (*px.*) is seen in front of these capsules, and below the recurrent trabecular cornua.

Over the large nasal gland a thin bone, the septo-maxillary (Plate 30, figs. 1, 2, *s.mx.*), is stretched; it ascends both up the side of the septum nasi and also towards the nasal wall.

Below the gland we see the vomer (*v*); it is hollow to receive it; and between the two bones on the under surface there is a section of the principal upper labial (*u.l.*).

In the transverse section (fig. 2, *mx.*) the fore part of the maxillary is cut through; it is a thin lath, and is seen also in the next two sections (figs. 3 and 4); it is shown from the side in Plate 29, fig. 3, *mx.*

Three bones have appeared in the inner palatine fold—namely, the palatine, transpalatine, and pterygoid: these are endoskeletal bones. There are also bony tracts outside the nasal roofs: these are the prefronto-lachrymals (*pf.*); another pair is seen in older specimens, namely, the post-orbitals.

The palatines (Plate 29, fig. 3, and Plate 30, figs. 3–5, *pa.*) are also thin laths of fibrous bone; they are rounded in front, pointed behind, and have in front of their middle a thin flat coiled process, the orbital or ethmopalatine process, which arches over the nasal canal (Plate 30, fig. 3).

The transpalatine (Plate 29, fig. 3, *t.pa.*) is a binding bar between the marginal and sub-marginal series; it is a cuneiform bar passing obliquely so as to attach the zygomatic process of the maxillary to the pterygoid.

The pterygoid (*pg.*) is a slender style, and it belongs, morphologically, to the mandible; it binds the quadrate to the palatine.

Another splint binds the quadrate to the auditory capsule (Plate 29, fig. 4, *sq.*): this is the squamosal; it is sickle-shaped, with the point directed forwards.

This bone already is serving as a loose swinging point for the descending and retreating mandibular suspensorium.

The description of this stage may be fitly ended by a short notice of what transverse sections reveal of the form and relation of the elements of the chondrocranium, besides what has just been noted, namely, the sectional views of the membranous bony tracts.

The 1*st section* (Plate 30, fig. 2) shows the junction of the coalesced trabeculæ with the inner laminæ of the nasal roof to form the septum nasi (*s.n.*).

This section shows the utter simplicity of the olfactory cartilages, and how that the huge nasal gland and its bony capsules—dish and cover—fill all the lower part of the nasal canal.

These canals are nearly surrounded by cartilage, for the wall turns inwards, and then there is a floor piece formed by the upper labial (*u.l.*).

In the 2*nd section* (fig. 3) the total arrest of the fore skull is shown; there is nothing in the region of the hemispheres and eyeballs save the pair of elegant, rounded, solid trabecular rods (*tr.*).

So, also, in the 3*rd section* (fig. 4). Here, however, we see two curious palatal papillæ, such as I long ago showed to exist in the mouth of the Tadpole, and which abound in the mouth of the *Ammocœtes-larva* of the Lamprey. In that case they are the rudiments of the armature of the sucking disk.

In the 4*th section* (fig. 5) we have the broadest region of the hemispheres (C^{1a}) cut through, just where the carotid arteries (*i.c.*) enter, and where the pituitary body (*py.*) is grafting itself upon the floor of the cavity of the infundibulum (*inf.*).

As the trabeculæ are bifurcated by the burrowing of the internal carotids, they appear in section as two cartilages on each side; here the parietals (*p.*) form *floor* and *wall* to the brain cavity.

In the 5*th section* (fig. 6) the mid brain is seen lying upon the hind brain (C^2, C^3); the anterior ampullæ are cut through (*a.s.c.*), and the wall of the ear-capsule is seen to be imperfect on its inner face. This open part is the internal meatus.

The section was made through the hind part of the "posterior fontanelle," and the basioccipital bone (*bo.*) had, in this case, just reached that open space; there we see the deep concavity of the basal plate, and the swelling of the hind brain.

Then, also, Meckel's cartilage (*mk.*) is cut through, and also the angular and surangular (*ag.*, *s.ag.*); the floor of the face has been cut away, but the roof of the throat is shown.

In the 6*th section* (fig. 7) we see the huge *optic lobes* resting on the hind brain (C^2, C^3, C^{3a}); the section has been made through the middle of the ear-sac and the apex of the notochord.

Here the basal plate is seen to be a large slab of cartilage, hollow above, convex below, and having the notochord lying upon it; it is quite distinct from the ear-capsule on one side.

The razor has cut through the anterior and horizontal canals (*a.sc.*, *h.sc.*), and the vestibule (*vb.*); the columella (*co.*) and Meckel's cartilage, with the same splints as in the last (*mn.*), are severed.

In the 7*th section* (fig. 8) the perfect occipital ring is cut through, close behind the ear-capsule (*au.*); the space is more than half an oval, the floor sinking downwards; here, thin, ectosteal laminæ are forming the basi- and exoccipital bones (*bo.*, *eo.*).

The most projecting part of the ear-capsule (*au.*) contains the ampulla of the posterior canal.

Fifth Stage. Embryo Snakes, $4\frac{1}{2}$ to $5\frac{1}{2}$ inches long; a few days before hatching.

Within a short time the development of the bony centres has been so great as to give, in many cases, the form, to each ossification, which is to be permanent.

This is especially the case in those bones which have no cartilaginous groundwork; those that have are still separated from each other by large tracts of cartilage (Plate 31, figs. 1–2; in fig. 1 the upper part of the skull has been cut away).

When a strong osteo-cranium, like that of a Snake, has to be formed, the ossifications soon blot out old morphological landmarks. It is indifferent to bone-tissue what variety of the connective-tissue series it shall get its matrix from: it is as if the *clay* and the *straw* laid out on the original ground-plan should indifferently be made into bricks and tiling.

We see here that the posterior basal fontanelle (*p.b.c.f.*) has grown to its utmost size; but the apex of the egg-like outline is taken off by the extension of the basioccipital (*bo.*) into the membrane. That ossification is very large and heart-shaped, the "cephalostyle" lying on its concave or upper surface; it extends its bony growth into the substance of the bilobate condyle (*oc.c.*).

The exoccipitals (*eo.*) are extending, but the chondrocranium keeps growing also; therefore there is still one large tract unossified between the median and lateral bony centres.

The lateral bones are riddled with holes, and these have added to them others that extend the series to the pituitary space.

Posterio-externally, we have the "posterior condyloid foramen" (*p.c.f.*); then within and in front of that the anterior (12), for the hypoglossal nerve, which now, for the first time in our ascent, has become a *cranial* nerve.

For the substance of the parachordal cartilages now gathers to the mid-line, to form the emarginate *single* condyle; this double tract, in many Urodeles, forms an intercalary vertebra—the odontoid ("Skull of Urodeles," p. 575).

Near to the ear-capsule and its fenestræ we see the foramina for the vagus (10) and glossopharyngeal (9); these holes have a bony tract between them, as in most Batrachia.

The super-occipital centre is ossifying; this, however, will be best seen in the next stage (figs. 3, 4, 6, *so.*).

The two films of bone that begin outside the carotid opening are now one large bone: the basisphenoid (*bs.*). This bone now occupies the roots of the trabeculæ, but stops in front, opposite the end of the parasphenoid (*pa.s.*), and at the beginning of the narrow interorbital region.

The bony matter grows off the cartilage, both laterally and behind, into the contiguous membrane; the carotids are en-ringed completely by bone; this is the foremost bone of the cranium, proper, at this stage, for the orbito-sphenoids (*os.*) are still soft, and no presphenoid ever appears.

The alisphenoidal cartilages (*al.s.*) have coalesced with the basal part, and also with the auditory capsules; they are ossifying, and the bone is both notched and perforated by the trigennial nerve (5).

The main periotic centre, the prootic (*pro.*), is now enclosing the fore part of the capsule; the other two are not visible. These will soon appear.

The frontals and parietals (*f.*, *p.*) are now just gaining the top of the head, and the parasphenoid (*pa.s.*) is well ossified; it is flat below and keeled above, fitting in between the trabecular rods.

The septo-maxillaries, vomers, and labial cartilage (fig. 2, *s.mx.*, *v.*, *u.l.*) are seen on each side and below the septum nasi (*s.n.*); these parts will be fully described in the adult stage.

The single pre-maxillary (*px.*) is edentulous; it has ascending, lateral, and palatine portions.

The maxillaries (*mx.*) are arcuate rods, pointed and bladed behind; the palatines (*pa.*) are hatchets with a coiled blade and an out-turned, pointed handle; the pterygoids (*pg.*) are arcuate and flat, with a spindle-shaped outline; on their outer edge the handle of a small trowel-shaped bone, whose blade is gapped, is attached; the broad end rests on the maxillary. This is the transpalatine (*t.pa.*).

The columella (*co.*) begins to ossify about this time; this part and the mandible will be described in the next or *ripe* stage.

Sixth Stage. Ripe Embryos of the Snake, 6½ inches long.

This is a very important stage; the osseous centres are all present, and none of them ankylosed to others; the synchondrosial tracts are mere clearly defined divisional lines.

From this stage to the adult few changes of any importance, except in size, occur. It is a strong skull even now; the changes afterwards go to make it one of the strongest known; this is the case in this and other small Ophidia, but in the gigantic kinds the skull is a marvel of solidity, there being much ankylosis, whilst the bony substance is like ivory.

The roof-bones have reached the top of the skull, but the fontanelle is still large (Plate 31, figs. 4 and 6); a side view of the interior (fig. 3) of the skull, whose septum nasi has been removed, will display most of the cranial characters.

Nearly all the ossified chondrocranium lies behind the eyes; this is almost entirely *occipito-otic.*

The basioccipital (figs. 3–5, *bo.*) is lozenge-shaped and hollow, like the bowl of a spoon. Upon its outer edges there rest three bones—namely, the exoccipitals (*eo.*), the opisthotics (*op.*), and the prootics (*pro.*); its fore edge is attached by the medium of a band of cartilage to the basisphenoid (*bs.*).

The condyle, which is practically single, although morphologically double, has both this basal and the two lateral bones entering it.

The foramen magnum is finished above by the lateral bones, or exoccipitals (*eo.*); they also send backwards from their upper edge an imbricating process to rest upon the "atlas."

The 9th, 10th, and 12th nerves (9, 10, 12) pierce these bones below; their front margin turns outward in such a manner as to clamp the hind part of the auditory mass, and they thus form a deep salcus on the inner face, the auditory mass swelling out in front of the out-turned exoccipitals (fig. 3, *eo.*, *op.*).

The key-stone of the arch or superoccipital (*so.*) lies on their fore margin; it is a transverse slab, with five sides: the foremost side is emarginate in a concave manner, finishing the rounded outline of the great upper fontanelle (*fo.*).

This roof-slab just touches the opisthotic (fig. 6, *op.*); is imbricated by the epiotics (*ep.*), and is joined to its own side plates (*eo.*) by synchondrosis.

Here we behold a *normal* and *primary* condition, not seen in the hot-blooded "Sauropsidan" Bird—namely, the freedom of the occipital bones from all participation in lodging the labyrinthic canals of the ear; *they* are confined to their own periotic region and bony centres.

The great bones of the otic mass are *three* only, not *five* as in Osseous Fishes. For in the Snake the head is made into a terete wedge; all snags, projections, buttresses, and the like are smoothed down. Thus there is no *endoskeletal* post-frontal ("sphenotic"), nor lateral ear-eave (*tegmen*) over the horizontal canal, to be ossified by the "pterotic."

These three bones are divided by a fine line of cartilage which is bifurcated above, so that it is triradiate (figs. 3, 4).

The huge semicircular canals (*a.sc.*, *h.sc.*, *p.sc.*) have long ago so dominated the whole capsule as to force it into conformity with their arches and swelling ampullæ; the vestibule and cochlea are relatively small.

Also the bony walls being merely the ossified cartilaginous capsule, with but little periosteal deposit, keep the very form, outside, of the elaborately ornate contents.

The foremost of the three, the prootic (*pro.*), is equal in size to the other two; and the opisthotic (*op.*) is the larger of these; the smallest of the three, the epiotic (*ep.*) contains no ampulla; the prootic has two, and the opisthotic one, of the pouches.

The prootic bone encloses the ampullæ of the anterior and of the horizontal canals; it runs along their pipes, and reaches to the middle of the horizontal and to the last third of the anterior canal.

Below, its anterior part rests upon the small alisphenoid, and behind that it runs down and rests equally upon the basisphenoid and basioccipital, wedging in between them. The prootic forms the back to the foramen for the third part of the trigeminal nerve, and encloses the passages of the 7th and 8th (figs. 3, 4; 5^3, 7, 8); it ensheaths the fore face of the cochlear pouch, and forms the front margin of the fenestra ovalis (*cl.*, *f.o.*).

The opisthotic (*op.*) ensheaths the back face of the cochlea, forms the back margin of the fenestra ovalis, and forks over the fenestra rotunda nearly enclosing it (fig. 5, *op.*, *f.r.*); but this part is left unfinished, and the *fenestra rotunda* does not open into the cochlea.*

The opisthotic takes in the ampulla and three-fourths of the arch of the posterior canal, which runs up in its hinder edge, close to the exoccipital; its lower margin barely reaches the basioccipital.

The epiotic (*ep.*) is wedge-shaped as seen from without (fig. 4); pyriform as seen on the inner side; and oblong as seen from above.

It takes in the contiguous *fourths* of the anterior and posterior canals, and their common "sinus;" its inner margin, above, slightly overlies the superoccipital; its anterior edge is flanked by a process of the parietal (*p.*) now making the roof.

The sub-reniform stapedial plate is one, both in its cartilaginous, and in this its ossified condition, with the columella (*co.*). I shall describe it with that element.

The alisphenoid (*al.s.*) is planted upon the basisphenoid (*bs.*) and props up the prootic; it is square now, having closed upon the first passage (for the first and second divisions of the 5th nerve, 5^1, 5^2), and is notched behind for the third branch (5^3). These two passages are similar to the *foramen ovale* and *foramen rotundum* of human anatomy; but the *foramen rotundum* here takes two parts of the nerve.

The basisphenoid (*bs.*) does not trespass further forwards along the trabeculæ than to the front of the circumscribed pituitary space; the bone has floored this space, and a cavity for the pituitary body exists above this secondary floor.

Just above the carotid passage (*i.c.*) the parietal (*p.*) is seen mounting up and creeping over the prootic; on the inside (fig. 3) this bone projects strongly inwards in front of the auditory capsule and behind the optic lobes.

The frontal (*f.*) is meeting its fellow above, but there is a considerable space unossified between it and the parietal laterally; at the bottom of this fenestra the small trilobate orbito-sphenoid (*os.*) is seen above, and a little in front of, the optic nerve (2). The highest part of the carinate parasphenoid (*pa.s.*) is seen under and between the optic passages; it is pointed in front and spatulate behind.

The septum nasi (fig. 3) has been removed, and the olfactory cartilage (*ol.*) is seen from the inside; postero-inferiorly we see a crescentic section of the reniform nasal gland (*n.g.*) covered by the septo-maxillary (*s.mx.*), floored by the vomer (*v.*), and having a large labial (*u.l.*) attached to its duct.

Behind the vomer we see the inner nostril, and in the front of the face the premaxillary (*px.*) is cut through.

The cartilaginous parts in front of the pituitary space have not altered since the last stage, and only alter in size afterwards.

There is nothing new to mention in the palato-maxillary series of bones, but the mandible is at an instructive stage.

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* In a large Boa, from Demerara, I find no *inter-fenestral* bar of the opisthotic; in *Python sebæ* it exists, as in our native Ophidia.

The suspensorium is now, in the closed mouth, placed at less than a right angle to the "ramus;" besides this, it is now firmly attached by its upper edge to the outer face of the hinder half of the long sub-falicform squamosal (fig. 7, *q.*, *sq.*).

Inside the middle of the squamosal there is a small sub-oval scale of bone, the supratemporal: here it is only distinct for a time, but in Lizards it is so permanently.

The quadrate (*q.*) is flat and fan-shaped above, and terete below: the upper part is still soft: the lower end only at the condyle.

The articulare (*ar.*) has almost ossified the whole of the long angular process, and runs forward over the middle of the Meckelian rod (*mk.*).

At the middle and from thence, forward, that rod is wholly unossified, and in this type the middle of the mandible is its "coronoid region;" there the coronoid bone meets the splenial (*cr.*, *sp.*); they are attached by enlarged ends, the point of one looking forwards, and of the other backwards.

The dentary (*d.*), surangular (*s.ag.*), and angular (*ag.*) are seen in the same figure: the latter on the inside mainly; the other two principally on the outside.

But the backward extension of the huge mandibular arch has thrust it past the hyoid rudiment; this is the cause of the very peculiar position of the stylo-hyal and columella (*st.h.*, *co.*) on the inner face of the quadrate.

The third branch of the 5th nerve, the facial nerve, and the columella, take the same backward direction, as a correlate of the extreme backward extension of the gape.

Notwithstanding the small size of the arrested hyoid arch, it has become segmented into the two normal pieces; the columella (minus the stapedial plate) answering to the hyo-mandibular, whilst the stylo-hyal is the upper part of the familiar "stylo-ceratohyal" bar.

The true stapedial portion of this compound columella (figs. 4, 5, 7, 8, *st.*) has its posterior margin almost straight, the fore edge being well curved: its inner face is concave (fig. 8).

The shaft (*co.*) is gently sigmoid; the terminal third is unossified: this is the "extra-stapedial part," and it has no supra-stapedial spur.

The form of the original rod (Plate 28) is scarcely changed, but the direction is backwards instead of forwards; the capitular part is below and forwards, the tubercular above and backwards.

There is no distinct *membrana* nor *cavum* tympani, and at present the stylo-hyal cleaves to the columella.

This element is reniform (fig. 8, *st.h.*), and its middle and convex portion is covered with a scabrous ectosteal plate; this and the extra-stapedial are jammed between the skull and the quadrate (figs. 4 and 7).*

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* This explains what for many years was an enigma to me—namely, that in every ophidian skull I examined there was to be seen a thin scale of bone adherent to or coalesced with the inside of the quadrate, above its middle. The arrested stylo-hyal is stowed away there, permanently; faint, and functionless.

Seventh Stage. Snakes six weeks old, $7\frac{2}{3}$ inches long.

Being informed by Professor HUXLEY that there was cartilage in the distal parts of the Snake's hyoid or lingual region, I made diligent search both in early stages and in the adult: the evidence obtained was negative.

Knowing well that feeble parts are often late in budding forth, and early in fading, I considered it to be the best chance for finding hypo-hyal or hypo-branchial rudiments in the Snake to look for them in well-grown young.

I thus take this for my *seventh stage*: there has been plenty of time for such cartilages to appear, but none for their vanishing: the result is the same; I can find no hyaline cartilage whatever between the distal parts of the mandible and the larynx; all the floor of the mouth with the forked tongue and its sheaths are membranous.

The other parts need have no further notice; there need be no intermediate stage for them between the ripe young and the adult.

Eighth Stage. Skull of the adult Snake.

There are several reasons why the skull of the adult Snake should be worked out exhaustively,—in the light of the history of its development.

One reason is, that the "Ophidia" are a large and important order of the Sauropsida, and to know one is to know all, so uniform are they, save in easily understood specializations.

Beginning at the Amphisbænæ and Anguians, if we run up the Lacertian Order, we shall find that their skull also can be read off in the light of that of the Snake.

The skull of other groups also, fossil or recent, will be more intelligible when once this kind is known; for the Snake's skull is at once extremely simple, and yet marvellously specialized: some things in it stand almost still, whilst others run to the utmost stretch of their morphological tether.*

The skull of the adult Snake is irregularly oblong, flat above in its fore part, and sub-carinate below; but the hinder part is gently convex both above and below (Plates 32 and 33).

The orbital and occipito-otic regions are about equal; the nasal region is about half the length of the others.

The gently bi-convex, transverse, occipital cincture articulates with the atlas by a transversely oval condyle (Plate 32, fig. 2, *oc.*, *c.*); above, the arch overlaps the vertebra in an imbricated manner (fig. 1).

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* Most solid in its *cranial*, yet the Snake's skull is, of all others, the most elastic and mobile in its *facial* parts; no foot must bruise its head, for it is doomed to go on its belly all the days of its life, yet its throat must be, practically, "unhidebound;" devouring, as it does, prey, whose girth is many times its own.

The basioccipital (*bo.*) is an irregular lozenge, transversely placed; it is slightly emarginate in front, and mammillate for muscular attachment.

It articulates with the basisphenoid in front (Plate 32, fig. 2, *bo.*, *bs.*), with the exoccipitals postero-laterally; in front of these (Plate 33, figs. 1, 2, *pro.*, *op.*) it receives the prootic and opisthotic.

With the hind part of the basisphenoid (Plate 33, fig. 1), it forms a very considerable hollow for the medulla oblongata.

The strong sides of the arch, the exoccipitals (*eo.*), run their lower angle into the condyle (*oc.c.*), whilst above the foramen magnum they meet and push out the superoccipital wedge (*so.*); they have coalesced with the opisthotics; the 9th, 10th, and 12th nerves pass out (Plate 33, fig. 2) close to the lower line of their ankylosis.

The superoccipital also seems to be much larger than in the young; this arises from its coalescence with the right and left epiotics (*ep.*).

It is now a five-sided, transverse roof-bone, its outer or epiotic angles being somewhat convex (Plate 32, fig. 1; Plate 33, fig. 2).

The parietals (Plate 33, fig. 1, *p.*) scarcely overlap the superoccipital; the junction is by a harmony-suture, exactly where the prootic (*pro.*) begins.

This main auditory bone has also coalesced with a true cranial element—the alisphenoid (*al.s.*); yet the tri-radiate suture between the three periotics is persistent; each element of the capsule is distinct from the other two, and yet each is fused to a neighbour-bone of the skull wall.

Here a distinct cartilaginous sense-capsule is ossified by three centres, which disclaim, as it were, their true affinity to each other, and lose themselves in the cranial wall.

In the Mammals, in sharp contrast with these transformations, the capsule chondrifies with but slight separateness from the cranial wall; and then these three centres ossify the capsule, and completely enucleate it from the skull proper. This is best seen in Shrews, the lesser Bats, and the Whale-tribe.

In the Bird these parts behave as in the Reptiles, but with this difference—namely, that the opisthotic and epiotic are very small, so that the labyrinth of the ear grows into the occipital arch.

The *otic* elements first unite with their *cranial* neighbours, and then everything is ankylosed; all the morphological writing is blotted out in that Class.

These lateral and upper cranio-auditory plates are marked with the form of the enclosed labyrinth, as may be seen by comparing their wavy surface with the form of the earlier cartilaginous capsule.

Infero-laterally, these plates are riddled with holes, mostly for the passage outwards of the cranial nerves.

In the crescentic chink below the opisthotic region (*op.*) the exoccipital is perforated for the 12th, 10th, and 9th nerves; these can be seen on their emerging below, but better in the inside; for externally they are partly hidden by that angle of the opisthotic which contains the hinder ampulla (*p.sc.*).

Above the exit of the 12th there is a small passage for a vessel; this hole is evidently the "foramen condyloideum posticum."

In front, on the inner side (Plate 33, fig. 1; 5), the large foramen ovale is reniform, and the space is large where the Gasserian ganglion gives off the *trigemini;* but a buttress of bone outside (fig. 2), formed by junction of the horns of the alisphenoid into a ring, causes the first and second branch to divide off, and leaves the third to take a recurrent course.

This it does (fig. 2; 5_3); and in some Snakes, especially the gigantic kinds, the bone is deeply grooved for this large mandibular nerve.

Behind the foramen ovale there are two larger and one smaller passage; the first of the larger holes transmits the facial nerve (7); the largest hole is for the 8th, in its passage to the membranous labyrinth.

A directly lateral view outside (fig. 2) does not show the holes for the 9th, 10th, and 12th nerves; but behind the hole for the facial (7th) we see two rather large passages: these are the fenestra ovalis and fenestra rotunda. In the figure the columella (*co.*, *st.*) is dislocated purposely; it has become relatively small, and its shaft a mere prickle.

As we saw in the ripe young (Plate 31), two spurs of the opisthotic stand upon the edge of the skull floor; the first of these divides the fenestræ from each other, and the other forms the back part of the fenestra rotunda, and is clamped to the fore edge of the exoccipital.

In the *dry* adult skull the fenestra rotunda is seen to have but a slight relationship to the membranous labyrinth; for the cochlear bud does not grow, the stapedial plate closes in the passage to the vestibule, and the fenestra rotunda opens into the cavity of the skull.

The lozenge-shaped alisphenoid forms both a wall and an eave to the second part of the foramen ovale; it also forms a double prop on which the swelling prootic rests, the "wing" itself resting obliquely upon its own base—the basisphenoid (*bs.*).

This latter is now a *compound* bone; it was made from the two proper basisphenoidal centres, and has gained a bulging floor and a large carinate rostrum from the parasphenoid (*pa.s.*).

But neither the parasphenoid nor the basisphenoid directly floor the cranial cavity; the arrest of the chondrocranial elements is correlated with a very curious undergrowth of the *roof* bones (*f.*, *p.*).

The frontal meets its fellow below, and the parietal nearly (fig. 1); the pituitary body is let down through a longitudinal chink between the floor-plates of the parietal, and there it finds a large empty room, "a world too wide" for so small a body.

By reference to the earlier stages this is easily understood, for the roots of the trabeculæ were planted on to the "investing mass" far apart, and no floor existed until the parasphenoidal blade grew backwards beneath the unfloored space.

Not only has the front fontanelle, or false floor, thus gained a good floor of bone, but the hinder gaping space is floored also (see earlier figures, *p.b.c.f.*).

Now, (Plate 32, fig. 2, *bo.*, *bs*, and Plate 33, figs. 1, 2, *bo.*, *bs.*) the basioccipital and the basisphenoidal plates have supplied what was wanting by periosteal growths; between these two large slabs there is a persistent spheno-occipital suture.

The post-pituitary part of the basisphenoid (Plate 33, fig. 1, *bs.*) shelves down behind the post-clinoid ridge, obliquely roofing the sub-cranial cellar; this it does completely, behind.

But in front, as before said, the two parietals fail to meet below the mid brain (fig. 1); they, however, send strong tentorial ingrowths, which run from the front of the auditory mass into the floor.

Above (Plate 32, fig. 1, *p.*), the parietals are completely confluent along the *sagittal* line; there they are somewhat depressed in this small kind, but in the gigantic kinds there is a parietal crest as strong as that of the Hyæna or the Tiger.

Also behind the coronal suture (which is persistent) the parietal grows inwards, filling up the space above, between the mid and fore brain (Plate 33, fig. 1, *p.*).

Above the beginning of the rostral part of the sphenoidal beam there is a large optic fenestra, as in the Batrachia, much larger than the optic nerve, and above this there is a post-orbital fenestra between the orbital regions of the parietal and frontal (Plate 33, figs. 1 and 2).

This is partly filled in by the small trifoliate orbito-sphenoid (*os*), which cleaves to the hind edge of the frontal descending plate.

The parasphenoidal rostrum ascends to meet the orbito-sphenoid in the angle between the diverging optic nerves, then for the rest of its length the crest is lower (Plate 33, figs. 1, 2, 3, *pa.s.*).

Both externally (fig. 2) and in section (fig. 3) the trabeculæ (*tr.*) are seen as unossified rounded rods, as far as the parasphenoid goes, that is, up to the olfactory region.

As in front of the ear-mass the parietals grew inwards, so behind the olfactory walls the frontals do also, but more perfectly (Plate 33, figs. 1 and 4, *f.*), for they are fitted, face to face, at the mid-line.

A recess on each side receives the narrow end of the fore brain, and under these lobes (C^1) the olfactory crura (1) pass to their destination (fig. 4).

There the frontals spread and articulate with each prefronto-lachrymal (*p.f.*), but further back they completely enclose the brain, as in certain Mammals (Plate 33, fig. 3, *f.*).

The sectional view (fig. 3) shows well the curious architecture of the Snake's skull: the roof and side walls are made by the frontals, the former flat, the latter concave and slanting inwards; the frontal suture (Plate 32, fig. 1, *f.*) is persistent. An eave is formed by the prefronto-lachrymals (*p.f.*), and a strong foundation by the crested parasphenoid (*pa.s.*), burrowed on each side by the trabeculæ (*tr.*): here the whole strength of this strong building is derived from membranous materials external to the endo-cranial elements.

The post-orbital region has an additional eave-tile, a post-orbital scale bone (*pt.o.*), which cleaves to the post-frontal projection of the large parietal bone—a remarkable state of things certainly, but the parietals are prepotent in the Snake's skull.

The foremost *fourth* of the skull belongs to the olfactory region: the essential parts here are most simple, the superadded things are as curiously complex as anything to be found in Vertebrate morphology.*

The fore end of the Snake's skull is composed of the following elements, viz.:

(*a.*) The coalesced vertical part of the trabeculæ, and

(*b.*) The nasal roof-cartilages (sense-capsules) grafted thereupon.

(*c.*) The short confluent trabecular cornua,—the rudiment of a terminal visceral arch (first pre-oral).

(*d.*) Attached to, or partly coalesced with these cornua, two pairs of upper labial cartilages.

(*e.*) Nine membrane-bones, of which the odd one, the premaxillary, is the splint of the cornua, the nasals or splints of the olfactory roof, the prefronto-lachrymals or splints of the postero-lateral region of the nose, and the septo-maxillaries and vomers, which are related to the middle wall of the nasal capsules.

(*f.*) Lastly, there are the huge nasal glands, that are encapsuled in the last-mentioned bones.

(*a.*) The form and relations of the septal portion of the trabeculæ are shown in the lateral view of the bisected skull (Plate 33, fig. 1, *s.n.*), and in the series of transverse sections (Plate 33, figs. 6–14, *s.n.*).

This wall is highest where the olfactory cartilages are united to it, behind, and lowest in front when the roof cartilages overlap it (fig. 6).

(*b.*) The nasal cartilages (*ol.*) are quite simple; I find no turbinal outgrowths in them whatever.

They are only imperfectly covered by bone (Plate 32, fig. 1); they turn inwards below in some degree, especially behind (Plate 33, fig. 4, *n.f.*), forming at the beginning of the septum a partial floor.

They are baggy in front (Plate 33, fig. 5), and notched externally for the nostril (Plate 32, figs. 1 and 3, and Plate 33, figs. 2 and 5).

(*c.*) The recurrent cornua are very short and have a median rudiment of the prenasal cartilage (Plate 32, fig. 3, *c.tr.*, *p.n.*); I found, in that specimen, the left cornu confluent with the first upper labial ($u.l^1.$).

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* My study of these structures in the Snake began ten or twelve years ago, during which time I have had frequently to refer to them in the descriptions of their homologues in other types, especially in the Birds, so far above them. The reason of the delay as to this paper has been the slow incoming of embryos young enough for my purposes.

Now, however, I can show a sort of practical standard, towards which the Ganoid and Teleostean Fishes and the Amphibia ascend, and from which the higher Reptiles and the Birds take their start. Of course I speak of the use of this Ophidian standard as an arbitrarily practical matter, and I do not wish to suggest anything except in a general way as to the actual descent of these Vertebrate types.

(*d.*) These labials are shown *in situ* in the same figure, which shows the nasal roofs from below, with the underlying parts.

The first labial is pointed in front: this is free or coalesced with the corresponding trabecular cornu, it then becomes wider, diverges, and becomes pedate and sub-bifurcate.

This first labial keeps along the suture, between the septo-maxillary above, and the vomer below (Plate 32, figs. 2, 3, and Plate 33, figs. 2, and 5–9, *u.l*^{1}., *sm.x.*, *v.*); it reaches the opening between the two bones for the duct of the nasal gland (*n.g.*).

The second upper labial (*u.l*^{2}.) is a broader cartilage; its external part, which acts as a valve to the opening for the duct, is oblong, but the band is carried inside the capsule, and then ends in a rounded lobe (Plate 33, fig. 10, also figs. 2, 9, 11, 12, 15, and 16; and Plate 32, figs. 2, 3).

(*e.*) The azygous bone attached to the nasal region is the premaxillary (*px.*): it has a somewhat angular anterior margin, a short, blunt nasal process (Plate 32, fig. 1, and Plate 33, figs. 1, 2), an edentulous edge, and two short, rounded palatine processes (Plate 32, fig. 2).

The nasals (*n.*) imperfectly cover the roof, their upper surface (Plate 32, fig. 1) is roughly triangular, and they send down a vertical plate between the cartilages, the right and left plates lying back to back; these plates are deeper than the septum nasi (Plate 33, figs. 1 and 6–12, *n.*, *s.n.*).

A stout shell of bone, having a lozenge-shaped outline, lies over the outside of the olfactory cartilages; this is the prefronto-lachrymal (*p.f.*). It has the character, and supplies the place of both those bones; by this the orbital rim is finished in front, and to the lower process of this bone the maxillary is attached, especially in venomous Snakes.

If this bone had grafted itself upon the cartilage, it must have been called the prefrontal or ecto-ethmoid; it is, however, as free as the ordinary pre-orbital or lachrymal.

Nothing that has come under my notice in cranial morphology shows a more curious or a more elegant architecture than the four *pre-palatal* bones now to be described.

The upper pair are the so-called inferior turbucals of older authors; but the Snake has no inferior turbucals, and when these exist, they are not membrane-bones, but cartilages, soft, or more or less ossified, that grow as outgrowths from the inner face of the nasal wall, and run from the inside of the outer nostril downwards and backwards to the "choana" or "middle nostril."

Those outgrowths are largely developed in Birds and Mammals, and I find a rudiment of them in *Chelone mydas*, and in Lizards.

But these bones form a floor to the nostrils; they are found in both Urodeles and Anourous Batrachia; and if those of the Lizards—the *Varanians* especially—be compared with that pre-orbital bone of Ganoid and Siluroid Teleostean Fishes, which lies directly beneath the nasal, and above the maxillary, the homology will at once be seen.

There they are mere muco-dermal bones, enclosing part of the net-work of the mucous glands of the head, where the "lateral line series" breaks up into rows. In the Amphibia and Reptiles they are specialized—just as other dermal bones are specialized—and assist in walling-in and flooring the more and more perfect nasal labyrinth.*

Here, in the Snake all the *slime-glands* appear to be concentrated into one on each side (Plate 33, figs. 9–12, *n.g.*), and these glands find a place under the olfactory pouches—involutions of the antero-inferior regions of the face.

Moreover, the bone which I seize upon in the Siluroid *Clarias capensis* as the forerunner of the reptilian septo-maxillary is the "os-terminale" of the *sub-orbital* series of slime-bones, as the "nasal" of the fish is the "os-terminale" of the *super-orbital* series.

Seen from the inside (Plate 33, fig. 16, *s.mx.*) the septo-maxillary is a flat wedge, broad behind and pointed in front; this flat inner face is separated from the septum nasi by the thick sub-mucous stroma (Plate 33, figs. 6–14, *s.mx.*, *s.n.*).

Its upper surface is sinuous (figs. 6–15) for at its broadest part, where it forms a lid to the *vomerine dish*, containing the kidney-shaped gland, it shelves downwards, is gently convex and then rises into the nasal wall (figs. 9–11), and also grows round the side of the gland. In front (figs. 6–8) it is a mere splint; behind (figs. 13–15) it is a thick plate.

The vomer is related to the lower edge of the septum (figs. 7–14, *v.*); on its inner face it is flat, but has a larger and more irregular surface than the septo-maxillary (fig. 16, *v.*).

Below (Plate 32, fig. 3, *v.*) it shows three parts: an anterior spike; a middle bowl; and a hinder ear-shaped lobe. The bowl (see also Plate 33, figs. 2 and 15, *v.*) is notched for the duct of the gland and the second labial (see also figs. 10, 11).

Thus in some of the sections (figs. 11, 12) we apparently have two bones; these, however, are only parts without and within the notch.

Where the duct passes out (fig. 11, *d.n.g.*) there the gland is most accurately encapsuled; the septo-maxillary, as a *lid*, with its rising outer and inner edges, covering the gland, which is embraced by the curling laminæ of the vomer both without and on the inner side.

The copious illustrations of this pleasing piece of morphology will make the above description quite intelligible.

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* In the Herring and its congeners, the maxillary bone carries two bones on its back; the hinder of these looks forwards, and forms an acute angle with the free end of the maxillary: this is the jugal. The other forms an acute angle with the fore part of the bone, and almost a right angle with the jugal: this is the *pre-orbital*, which takes on the curious specialization in the air-breathers by which it becomes the "septo-maxillary."

(*f.*) The form and relations of the nasal gland (*n.g.*) are involved in the foregoing details :—

The parts remaining to be described are the bones that belong to the maxillo-palatine, mandibular, and hyoid arches; the latter has only two traces, the spike of the columella (*co.*) and the minute style-hyal scale (Plate 32, figs. 1, 2, *st.h.*); but the parts round the mouth are greatly developed and modified.

The palatines, transpalatines, and maxillaries are the three elements of the second pre-oral arch; the first of these (Plate 32, figs. 1, 2, *pa.*) is a slightly inbent rod loosely attached to the sides of the nasal region in front, and overlapping the apex of the pterygoid (*pg.*) behind.

Its ethmoid process (Plate 33, figs. 13, 14, *e.pa.*) is much flatter than in the early stage; it lies over the middle nostril.

The maxillary (*mx.*) is a narrow, arcuate, dentigerous bone; it bounds the gape, running forwards from the pre-maxillary to a point opposite the post-orbital region.

Lying on the jugal end of the maxillary is the blade of a curious hatchet-shaped bone, the transpalatine (*t.pa.*) : its handle lies on the outer edge of the middle of the pterygoid.

The mandibular arch is yoked on to the maxillo-palatine by means of the pterygoid (*pg.*): this bone only answers to the bony plate of the Salamander's or Frog's pterygoid; there is no corresponding cartilage budding forth from the quadrate.

This is a long falcate bar, widest in the middle, gently convex below, and slightly scooped above; it is obliquely attached to the under face of the end of the palatine, and then stretches backwards and outwards, clamping the quadrate above and inside its hinge, and reaching to the end of the long angular process of the mandible itself (*ar.*).

In the gigantic types (Python, &c.) there are large " basi-pterygoid processes," on which the pterygoid glides; here these are not distinct.

The suspensorium or quadrate (*q.*) is let down, backwards, by its own splint, the squamosal (*sq.*). This latter bone lies over the auditory mass, reaching to the parietal; it is oblong and rounded in front, and behind it is bevelled and faced, there, with articular cartilage.

The supra-temporal scale is now ankylosed to the squamosal.

The broad, oblique, top end of the quadrate (*q.*) glides over the articular face of the squamosal, and then lessening, this bone becomes a rib-like rod, with a cylindroidal condyle for the excavation in the articulare (*ar.*).

This latter bone has ossified much of Meckel's cartilage, and is invested with the long angular and surangular, both narrow and pointed at both ends (*ag.*, *s.ag.*).

There is a partial hinge on the ramus opposite the end of the maxillary, dividing the bar so as to leave only two-fifths in front and as much as three-fifths behind.

Externally, the dentary (*d.*) overlaps this part by its forks; but, on the inside, the splenial and coronoid (*sp.*, *cr.*)—the latter is the larger bone—meet each by a broad end, the suture being vertical; this is the anomalous position of the true *coronoid region;* the two bones are very similar, but their points look in opposite directions.

These two bones hide the shrivelled remains of Meckel's cartilage; none of these bones of the mandible are dentigenous except the dentaries, and only the palatines and pterygoids and maxillaries above.

Of the hyoid or second post-oral arch I can only find two rudiments, and these have lost their independence; the antero-superior or hyo-mandibular element is now the small columellar prickle on the oval stapedal plate (Plate 33, fig. 2, *co.*).

The other, or postero-inferior, piece is starved and useless; it is ankylosed to the inner face of the quadrate, towards the back of the upper third (Plate 32, fig. 2, *st.h*). This is all that remains of the stylo-cerato-hyal bar; at least I have failed to find any cartilage in the distal or lingual region.

Concluding Remarks.

I have carefully studied the skulls in the ripe young and the adult of Lacertians, Chelonians, and the Crocodiles, but time has not served for working them out from their early stages.

Materials are ready for the Lacertians and their sub-group, the *Anguians.* Early embryos of the Tortoise* and Crocodile are still wanting; yet this present piece of work, it is to be hoped, will be of considerable use. It will serve me as a lantern with two windows: letting light backwards upon the Ichthyopsida and forwards on to the nobler Reptiles; and it will light up even the winged Fowls that, in their perfectness, seem to have exhausted the possibilities of the Sauropsidan type.†

Description of the Plates.

PLATE 27.

Fig. 1. First stage. Side view of Embryo of Common Snake (*Tropidonotus natrix*), whose total length was $\frac{3}{4}$ inch. × 17 diameters.

Fig. 2. Head of the same, severed and slightly bent back (straightened); lower view. × 17 diameters.

Fig. 3. Second stage. Side view of the head and neck of an older Embryo, measuring about 1 inch in length. × 16 diameters.

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* Since the above was written, I have received from Sir Wyville Thomson several *large*, and from Mr. Moseley many *small*, embryos of this important type.

† The use which I have made from time to time of the Snake's skull may be seen by reference to various papers on the Structure and Development of the Bird's Skull in the Transactions of the Royal, Linnean, Zoological, and Microscopical Societies. In the Snake's skull, *ichthyic elements* are curiously specialized; in that of the Bird the same parts re-appear, but in most remarkable metamorphic combinations.

Fig. 4. The same from below (not straightened). × 16 diameters.

Fig. 5. The same from above. × 16 diameters.

Fig. 6. Palatal view of severed head of same. × 16 diameters.

PLATE 28.

Fig. 1. Third stage. Side view of the head of an Embryo, $1\frac{3}{4}$ inch long. × 8 diameters.

Fig. 2. The same dissected. × 8 diameters.

Fig. 3. The same from above. × 8 diameters.

Fig. 4. The same; palatal view, with lower arches removed. × 8 diameters.

Fig. 5. Longitudinally vertical section of the same. × 12 diameters.

Fig. 6. Palatal view of same dissected. × 12 diameters.

Fig. 7. Ear-capsule and arches of same, side view. × 12 diameters.

Fig. 8. Upper view of chondrocranium of an older Embryo, measuring $2\frac{1}{4}$ inches long. × 12 diameters.

Fig. 9. Nasal capsule of same, front view. × 12 diameters.

PLATE 29.

Fig. 1. First stage. Vertical section of head of youngest Embryo, $\frac{3}{4}$ inch long. × 17 diameters.

Fig. 2. Second stage. Vertical section of head of Embryo, 1 inch long. × 12 diameters.

Fig. 3. Fourth stage, $2\frac{1}{2}$ to 3 inches long. Side view of skull with nasal sac removed to show septum nasi. × 15 diameters.

Fig. 4. Upper view of a skull of a somewhat older Embryo with all the sense-capsules *in situ*. × 12 diameters.

Fig. 5. Lower view of chondrocranium (same as fig. 3), with eyeballs and nasal sacs not drawn. × 12 diameters.

PLATE 30.

Fig. 1. Longitudinally vertical section of a skull—(fourth stage continued). × 12 diameters.

Fig. 2. Transversely vertical section of a similar skull through the middle of the nasal region. × 16 diameters.

Fig. 3. A *second* section through fore part of eyeballs. × 16 diameters.

Fig. 4. Part of a *third* section through the middle of the eyeballs. × 16 diameters.

Fig. 5. A *fourth* section between eye and ear. × 16 diameters.

Fig. 6. A *fifth* section through fore part of ear-capsules. × 16 diameters.

Fig. 7. A *sixth* section through their middle. × 16 diameters.

Fig. 8. A *seventh* section between ear-capsules and foramen magnum. × 16 diameters.

PLATE 31.

Fig. 1. Fifth stage, $4\frac{1}{2}$ to 5 inches long. A horizontal section of the skull showing the floor, from above. × 10 diameters.

Fig. 2. The same stage and size, showing skull with palatal bones, from below. × 10 diameters.

Fig. 3. Sixth stage. Longitudinally vertical section of skull of ripe Embryo, $6\frac{1}{2}$ inches long. × 10 diameters.

Fig. 4. Hinderpart of same skull, seen from the outside. × 10 diameters.

Fig. 5. Part of same view. × 20 diameters.

Fig. 6. Same skull, hind part, seen from above. × 10 diameters.

Fig. 7. Mandible and columella of same; inner view. × 10 diameters.

Fig. 8. Another view of columella. × 20 diameters.

PLATE 32.

Fig. 1. Last stage. Adult Snake, upper view of skull and arches. × 5 diameters.

Fig. 2. The same from below. × 5 diameters.

Fig. 3. Nasal sacs and vomerine bones, from below. × 5 diameters.

PLATE 33.

Fig. 1. Separated cranium of same seen in section.* × 5 diameters.

Fig. 2. Side view of same. × 5 diameters.

Fig. 3. Transverse section of same, orbital region. × 5 diameters.

Fig. 4. Another section, ethmoidal region. × 5 diameters.

Figs. 5 to 14. A series of transversely vertical sections through nasal region of same, from the nostrils to the orbits. × 12 diameters.

Fig. 15. Vomerine bones and cartilages of left side, outer view. × 5 diameters.

Fig. 16. Those of right side, inner view. × 5 diameters.

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* The round space seen on *op.*, in this figure, is accidental.

XIII. *Addition to Memoir on the Transformation of Elliptic Functions.*

By A. Cayley, *Sadlerian Professor of Mathematics in the University of Cambridge.*

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I have recently succeeded in completing a theory considered in my 'Memoir on the Transformation of Elliptic Functions,' Phil. Trans., vol. 164 (1874), pp. 397–456—that of the septic transformation, $n=7$. We have here

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{\alpha-\beta x+\gamma x^2-\delta x^3}{\alpha+\beta x^2+\gamma x^2+\delta x^3}\right)^2,$$

a solution of

$$\frac{\mathrm{M}dy}{\sqrt{1-y^2\,.\,1-v^8y^2}}=\frac{dx}{\sqrt{1-x^2\,.\,1-u^8x^2}},$$

where $\frac{1}{\mathrm{M}}=1+\frac{2\beta}{\alpha}$; and the ratios $\alpha:\beta:\gamma:\delta$, and the uv-modular equation are determined by the equations

$$u^{14}\alpha^2=v^2\delta^2,$$
$$u^6(2\alpha\gamma+2\alpha\beta+\beta^2)=v^2(\gamma^2+2\gamma\delta+2\beta\delta),$$
$$\gamma^2+2\beta\gamma+2\alpha\delta+2\beta\delta=v^2u^2(2\alpha\gamma+2\beta\gamma+2\alpha\delta+\beta^2),$$
$$\delta^2+2\gamma\delta=v^2u^{10}(\alpha^2+2\alpha\beta);$$

or, what is the same thing, writing $\alpha=1$, the first equation may be replaced by $\delta=\frac{u^7}{v}$, and then, α, δ having these values, the last three equations determine β, γ and the modular equation. If instead of β we introduce M, by means of the relation $\frac{1}{\mathrm{M}}=1+2\beta$, that is $2\beta=\frac{1}{\mathrm{M}}-1$, then the last equation gives $2\gamma=u^3v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right)$; and α, β, γ, δ having these values, we have the residual two equations

$$u^6(2\alpha\gamma+2\alpha\beta+\beta^2)=\ v^2(\gamma^2+2\gamma\delta+2\beta\delta),$$
$$\gamma^2+2\beta\gamma+2\alpha\delta+\beta\delta=v^2u^2(2\alpha\gamma+2\beta\gamma+2\alpha\delta+\beta^2),$$

viz., each of these is a quadric equation in $\frac{1}{\mathrm{M}}$; hence eliminating $\frac{1}{\mathrm{M}}$, we have the modular equation; and also (linearly) the value of $\frac{1}{\mathrm{M}}$, and thence the values of α, β, γ, δ in terms of u, v.

Before going further it is proper to remark that, writing as above $\alpha=1$, then if $\delta=\beta\gamma$, we have

$$1-\beta x+\gamma x^2-\delta x^3=(1-\beta x)(1+\gamma x^2),$$
$$1+\beta x+\gamma x^2+\delta x^3=(1+\beta x)(1+\gamma x^2),$$

and the equation of transformation becomes

$$\frac{1-y}{1+y}=\frac{1-x}{1+x}\left(\frac{1-\beta x}{1+\beta x}\right)^2,$$

viz., this belongs to the cubic transformation. The value of β in the cubic transformation was taken to be $\beta=\frac{u^3}{v}$, but for the present purpose it is necessary to pay attention to an omitted double sign, and write $\beta=\pm\frac{u^3}{v}$; this being so, $\delta=\beta\gamma$, and giving to γ the value $\mp u^4$, δ will have its foregoing value $=\frac{u^7}{v}$. And from the theory of the cubic equation, according as $\beta=\frac{u^3}{v}$ or $=-\frac{u^3}{v}$, the modular equation must be $u^4-v^4+2uv(1-u^2v^2)=0$, or $u^4-v^4-2uv(1-u^2v^2)=0$.

We thus see *à priori*, and it is easy to verify that the equations of the septic transformation are satisfied by the values

$$\alpha=1,\ \beta=\ \ \frac{u^3}{v},\ \gamma=\ \ u^4, \delta=\frac{u^7}{v},\ \text{and}\ u^4-v^4+2uv(1-u^2v^2)=0\,;$$
$$\alpha=1,\ \beta=-\frac{u^3}{v},\ \gamma=-u^4, \delta=\frac{u^7}{v},\ \text{and}\ u^4-v^4-2uv(1-u^2v^2)=0\,;$$

and it hence follows that in obtaining the modular equation for the septic transformation, we shall meet with the factors $u^4-v^4\pm 2uv(1-u^2v^2)$. Writing for shortness $uv=\theta$, these factors are $u^4-v^4\pm 2\theta(1-\theta^2)$, the factor for the proper modular equation is $u^8+v^8-\Theta$, where

$$\Theta=8\theta-28\theta^2+56\theta^3-70\theta^4+56\theta^5-28\theta^6+8\theta^7$$

[viz., the equation $(1-u^8)(1-v^8)-(1-uv)^8=0$ is $u^8+v^8-\Theta=0$], and the modular equation as obtained by the elimination from the two quadric equations in fact presents itself in the form

$$(u^4-v^4+2\theta-2\theta^3)^2(u^4-v^4-2\theta+2\theta^3)^2(u^8+v^8-\Theta)=0.$$

Proceeding to the investigation, we substitute the values

$$\alpha=1,\ \beta=\tfrac{1}{2}\left(\frac{1}{\mathrm{M}}-1\right),\ \gamma=\tfrac{1}{2}u^3v^3\left(\frac{1}{\mathrm{M}}-\frac{u^4}{v^4}\right),\ \delta=\frac{u^7}{v}$$

in the residual two equations, which thus become

$$\begin{aligned}\frac{1}{\mathrm{M}^2}(1-v^8) \qquad &+\frac{2}{\mathrm{M}}(1-uv)^3(1+uv)\\ &+\left\{(1-u^8)-4(1-uv)\left(1+\frac{u^7}{v}\right)\right\}=0,\end{aligned}$$

$$\begin{aligned}\frac{1}{\mathrm{M}^2}\left\{-u^2v^2(1-uv)^3(1+uv)\right\}+\frac{2}{\mathrm{M}}&\left\{u^2v^2(1-u^8)+\frac{u^3}{v}(1+u^2v^2)(u^4-v^4)\right\}\\ &+\left\{\frac{u^{14}}{v^2}+6\frac{u^7}{v}(1-u^2v^2)-u^2v^2\right\} \qquad =0,\end{aligned}$$

the first of which is given p. 432 of the 'Memoir.' Calling them

$$\left(\mathrm{a},\ \mathrm{b},\ \mathrm{c}\between\frac{1}{\mathrm{M}},\ 1\right)^2=0,\ \left(\mathrm{a}',\ \mathrm{b}',\ \mathrm{c}'\between\frac{1}{\mathrm{M}},\ 1\right)^2=0,$$

we have

$$\frac{1}{\mathrm{M}^2}:\frac{2}{\mathrm{M}}:1=\mathrm{bc}'-\mathrm{b}'\mathrm{c}:\mathrm{ca}'-\mathrm{c}'\mathrm{a}:\mathrm{ab}'-\mathrm{a}'\mathrm{b},$$

and the result of the elimination therefore is

$$(\mathrm{ca}'-\mathrm{c}'\mathrm{a})^2-4(\mathrm{bc}'-\mathrm{b}'\mathrm{c})(\mathrm{ab}'-\mathrm{a}'\mathrm{b})=0.$$

Write as before $uv=\theta$. In forming the expressions ca′−c′a, &c., to avoid fractions we must in the first instance introduce the factor v^2, thus

$$\begin{aligned}v^2(\mathrm{ca}'-\mathrm{c}'\mathrm{a})=v\{v(1-u^8)-4(1-\theta)(v+u^7)\}&\{-\theta^2(1+\theta)(1-\theta)^3\}\\ -\{u^{14}+6u^6\theta(1-\theta^2)-v^2\theta^2\}&\{1-v^8\},\\ =-\theta^2(1+\theta)(1-\theta)^3\{v^2(-3+4\theta)&+u^6(-4\theta+3\theta^2)\}\\ -\{u^{14}+6u^6(\theta-\theta^3)-v^2\theta^2\}&(1-v^8)\ ;\end{aligned}$$

but instead of θ^2v^2 writing u^2v^4, the expression on the right hand side becomes divisible by u^2; and we find

$$\begin{aligned}\frac{v^2}{u^2}(\mathrm{ca}'-\mathrm{c}'\mathrm{a})=-(1+\theta)(1-\theta)^3\{v^4(-3+4\theta)+u^4(-4\theta^3+3\theta^4)\}\\ -\{u^{12}+6u^4(\theta-\theta^3)-v^4\}(1-v^8),\end{aligned}$$

and thence

$$\begin{aligned}-\frac{v^2}{u^2}(\mathrm{ca}'-\mathrm{c}'\mathrm{a})=u^{12} \qquad\qquad &+u^4(6\theta-10\theta^3+11\theta^4-6\theta^5-8\theta^6+10\theta^7-4\theta^8)\\ +v^4(-4+10\theta-8\theta^2-6\theta^3&+11\theta^4-10\theta^5+6\theta^7)+v^{12},\end{aligned}$$

and similarly we have

$$\begin{aligned}\frac{v^2}{u^2}(\mathrm{bc}'-\mathrm{b}'\mathrm{c})=u^{12}(5-5\theta+4\theta^2-5\theta^3+2\theta^4)&+u^4(9\theta-16\theta^2+\theta^3+10\theta^4+\theta^5-16\theta^6+9\theta^7)\\ +v^4(2-5\theta+4\theta^2-5\theta^3+5\theta^4),&\end{aligned}$$

$$\begin{aligned}\frac{v^2}{u^2}(\mathrm{ab}'-\mathrm{a}'\mathrm{b})=u^4(\theta+\theta^3-\theta^4) \qquad &+v^4(2-5\theta+4\theta^2+3\theta^3-10\theta^4+3\theta^5+4\theta^6-5\theta^7+2\theta^8)\\ +v^{12}(-1+\theta+\theta^3)\ ;&\end{aligned}$$

say these values are

$$u^{12}+pu^4+qv^4+v^{12},\ \lambda u^{12}+\mu u^4+\nu v^4,\ \rho u^4+\sigma v^4+\tau v^{12}.$$

The required equation is thus

$$0=(u^{12}+pu^4+qv^4+v^{12})^2-4(\lambda u^{12}+\mu u^4+\nu v^4)(\rho u^4+\sigma v^4+\tau v^{12}),$$

viz., the function is

$$\begin{aligned}&u^{24}\\ &+u^{16}(2p-4\lambda\rho)\\ &+u^8\ (2q\theta^4+p^2-4\lambda\sigma\theta^4-4\mu\rho)\\ &+\quad\ (2\theta^{12}+2pq\theta^4-4\lambda\tau\theta^{12}-4\mu\sigma\theta^4-4\nu\rho\theta^4)\\ &+v^8\ (2p\theta^4+q^2-4\mu\tau\theta^4-4\nu\sigma)\\ &+v^{16}(2q-4\nu\tau)\\ &+v^{24},\end{aligned}$$

or say it is

$$=(1,\ b,\ c,\ d,\ e, f,\ 1\between u^{24},\ u^{16},\ u^8,\ 1,\ v^8,\ v^{16},\ v^{24}).$$

Supposing that this has a factor $u^8-\Theta+v^8$, the form is

$$(u^{16}+\mathrm{B}u^8+\mathrm{C}+\mathrm{D}v^8+v^{16})(u^8-\Theta+v^8);$$

and comparing coefficients we have

$$\begin{aligned}\mathrm{B}-\Theta\ &=b,\\ \mathrm{C}-\Theta\mathrm{B}+\theta^8\ &=c,\\ \mathrm{D}\theta^8-\Theta\mathrm{C}+\mathrm{B}\theta^8&=d,\\ \theta^8-\Theta\mathrm{D}+\mathrm{C}\ &=e,\\ -\Theta\ \ +\mathrm{D}\ &=f,\end{aligned}$$

where Θ has the before-mentioned value

$$=(8,\ -28,\ +56,\ -70,\ +56,\ -28,\ +8\between\theta,\ \theta^2,\ \theta^3,\ \theta^4,\ \theta^5,\ \theta^6,\ \theta^7);$$

from the first, second, and fifth equations, $\mathrm{B}=b+\Theta$, $\mathrm{C}=c+\Theta\mathrm{B}-\theta^8$, $\mathrm{D}=f+\Theta$; and the third and fourth equations should then be verified identically. Writing down the coefficients of the different powers of θ we find

$$\begin{array}{rl} 2p= & 0+12\quad\ 0-20+22-12-16+20-8(\theta^0\ .\ .\ .\ \theta^8)\\ -4\lambda\rho= & 0-20+20-36+60-44+36-28+8\qquad ,,\\ \hline b= & 0-\ \ 8+20-56+82-56+20-\ \ 8\quad 0\qquad ,,\\ \Theta= & 0+\ \ 8-28+56-70+56-28+\ \ 8\quad 0\qquad ,,\\ \hline \therefore\ \mathrm{B}= & 0\quad\ 0-\ \ 8\quad\ 0+12\quad\ 0-\ \ 8\quad\ 0\quad 0\qquad ,, \end{array}$$

that is

$$B=-8\theta^2+12\theta^4-8\theta^6\,;$$

and in precisely the same way the fifth equation gives

$$D=-8\theta^2+12\theta^4-8\theta^6.$$

We find similarly C from the second equation: writing down first the coefficients of p^2, $2q\theta^4$, $-4\lambda\sigma\theta^4$, and $-4\mu\rho$, the sum of these gives the coefficients of c; and then writing underneath these the coefficients of $B\Theta$ and of $-\theta^8$, the final sum gives the coefficients of C: the coefficients of each line belong to $(\theta^0, \theta^1, \ldots \theta^{16})$.

$$\begin{array}{l}
0\quad 0\quad 36\qquad 0-120+132+\ 28-316+361-\ 20-340+396-144-112+164-80+16\\
\qquad\qquad\quad -\ \ 8+\ 20-\ 16-\ 12+\ 22-\ 20\qquad 0+\ 12\\
\qquad\qquad\quad -\ 40+140-212+140+\ 80-188+168-\ 92-\ 64+176-164+80-16\\
\quad -36+64-\ 40+\ 60-\ 72+\ 28\qquad 0+\ 68-100+\ 36\\
\hline
0\quad 0\quad\ 0+64-208+352-272-160+463-160-272+352-208+\ 64\qquad 0\qquad 0\qquad 0\\
0\quad 0\quad\ 0-64+224-352+224+160-392+160+224-352+224-\ 64\qquad 0\qquad 0\qquad 0\\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad -\ \ 1\\
\hline
0\quad 0\quad 0\qquad 0+\ 16\qquad 0-\ 48\qquad 0+\ 70\qquad 0-\ 48\qquad 0+\ 16\qquad 0\qquad 0\qquad 0\qquad 0
\end{array}$$

that is

$$C=16\theta^4-48\theta^6+70\theta^8-48\theta^{10}+16\theta^{12},$$

and in precisely the same way this value of C would be found from the fourth equation. There remains to be verified only the fourth equation $(D+B)\theta^8-\Theta C=d$, that is

$$2\theta^8(-8\theta^2+12\theta^4-8\theta^6)-\Theta C=(2-4\lambda\tau)\theta^{12}+(2pq-4\mu\sigma-4\nu\rho)\theta^4,$$

and this can be effected without difficulty.

The factor of the modular equation thus is

$$u^{16}+v^{16}+(-8\theta^2+12\theta^4-8\theta^6)(u^8+v^8)+16\theta^4-48\theta^6+70\theta^8-48\theta^{10}+16\theta^{12},$$

viz., this is

$$\begin{aligned}
&(u^8+v^8)^2+(-4\theta^2+6\theta^4-4\theta^6)2(u^8+v^8)+16\theta^4-48\theta^6+68\theta^8-48\theta^{10}+16\theta^{12},\\
&=(u^8+v^8-4\theta^2+6\theta^4-4\theta^6)^2,\\
&=\{(u^4-v^4)^2-4\theta^2(1-\theta^2)^2\}^2
\end{aligned}$$

that is

$$\{u^4-v^4-2\theta(1-\theta^2)\}^2\{u^4-v^4+2\theta(1-\theta^2)\}^2;$$

or the modular equation is

$$\{u^4-v^4-2\theta(1-\theta^2)\}^2\{u^4-v^4+2\theta(1-\theta^2)\}^2(u^8+v^8-\Theta)=0;$$

viz., the first and second factors belong to the cubic transformation; and we have for the proper modular equation in the septic transformation $u^8+v^8-\Theta=0$, or what is the same thing $(1-u^8)(1-v^8)-(1-\theta)^8=0$, that is $(1-u^8)(1-v^8)-(1-uv)^8=0$, the known result; or as it may also be written $(\theta-u^8)(\theta-v^8)+7\theta^2(1-\theta)^2(1-\theta+\theta^2)^2=0$.

The value of M is given by the foregoing relations

$$\frac{1}{M^2}:\frac{2}{M}:1=\lambda u^{12}+\mu u^4+\nu v^4:-(u^{12}+pu^4+qv^4+v^{12}):\rho u^4+\sigma v^4+\tau v^{12};$$

but these can be, by virtue of the proper modular equation, $u^8+v^8-\Theta=0$, reduced into the form

$$\frac{1}{M^2}:\frac{2}{M}:1=7(\theta-u^8):14(\theta-2\theta^2+2\theta^3-\theta^4):-\theta+v^8,$$

viz., the equality of these two sets of ratios depends upon the following identities,

$$\begin{aligned}&(-\theta+v^8)(u^{12}+pu^4+qv^4+v^{12})+14(\theta-2\theta^2+2\theta^3-\theta^4)(\rho u^4+\sigma v^4+\tau v^{12})\\&\qquad=\{-\theta u^4+(1-\theta)(-4-\theta+5\theta^2-\theta^3-4\theta^4)v^4+v^{12}\}(u^8-\Theta+v^8),\\&-7(\theta-u^8)(\rho u^4+\sigma v^4+\tau v^{12})-(\theta-v^8)(\lambda^{12}+\mu u^4+\nu u^4)\\&\qquad=\{(2\theta+5\theta^2+3\theta^3-2\theta^4-2\theta^5)u^4+(2+2\theta-3\theta^2-5\theta^3-2\theta^4)v^4\}(u^8-\Theta+v^8),\\&-2(\theta-2\theta^2+2\theta^3-\theta^4)(\lambda u^{12}+\mu u^4+\nu u^4)+(u^8-\theta)(u^{12}+pu^4+qv^4+v^{12})\\&\qquad=\{u^{12}+\theta(1-\theta)(3+5\theta+3\theta^2)u^4-\theta v^4\}(u^8-\Theta+v^8),\end{aligned}$$

which can be verified without difficulty: from the last-mentioned system of values, replacing θ by its value uv, we then have

$$\frac{1}{M^2}:\frac{2}{M}:1=7u(v-u^7):14uv(1-uv)(1-uv+u^2v^2):-v(u-v^7),$$

which agree with the values given p. 482 of the 'Memoir,' and the analytical theory is thus completed.

XIV. THE CROONIAN LECTURE.—*On the Structure of the Stylasteridæ, a Family of the Hydroid Stony Corals.*

By H. N. MOSELEY, *F.R.S., Fellow of Exeter College, Oxford, late Naturalist on board H.M.S. 'Challenger.'*

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[PLATES 34–44.]

INTRODUCTION.

IN the Proceedings of the Royal Society, No. 172, 1876, I published a preliminary note on the present subject, and gave a short account of the results which I had arrived at from a somewhat hurried examination of the material at disposal. After this short account had been written, I devoted my time during the remainder of the homeward voyage of H.M.S. 'Challenger' to the further study of the structure of the Stylasteridæ, and the preparation of drawings illustrating it. I have supplemented this work by additional work in England, and the results are embodied in the present paper. The main part of the specimens of Stylasteridæ, from the study of which the anatomical details were determined, was obtained at a single haul of the trawl-net taken on February 14th, 1876, in lat. 37° 17′ S., long. 53° 52′ W., off the mouth of the Rio de la Plata in a depth of 600 fathoms. The specimens then obtained included six genera of the family of the Stylasteridæ. They were in most excellent preservation, although they had been slowly raised from the bottom, and in all the genera but one the generative organs were in full development. It was the examination of this set of specimens which first convinced me that the Stylasteridæ were Hydroids and not Anthozoans, a fact which I had already been led to suspect from the structure observed in the case of a species of *Astylus* obtained from 500 fathoms off the Meangis Islands, and that of a *Cryptohelia*, a short reference to which was given in a paper "On the Structure and Relations of certain Corals" (Proc. Roy. Soc., No. 64, 1875, p. 64, and Phil. Trans., Vol. 166, Pt. I., 1876, p. 116). I have examined also other specimens of Stylasteridæ obtained by the dredge and trawl of the 'Challenger' in various parts of the world, and a few specimens from those obtained by the United States dredging expeditions, which have been generously placed at my disposal by Mr. ALEXANDER AGASSIZ and Count de POURTALES of the Museum of Comparative Zoology of Cambridge, Massachusetts.

Literature of the Subject.

The family Stylasteridæ was formed by the late Dr. GRAY in his "Outline of an

Arrangement of Stony Corals" (Ann. and Mag. Nat. Hist., vol. xix., 1847, p. 127). The family was made to contain the genus *Stylaster* alone, and was thus characterized :—

" Coral minutely porous, cells deep, cylindrical, with six grooves, each ending in a pore and a central style."

M. EDWARDS and HAIME placed *Stylaster* in a sub-family, Stylasteraceæ, from which however they excluded *Errina* and *Distichopora*, although they included *Axohelia*, which is a Madracis.

Count de POURTALES, in his "Deep Sea Corals" ('Illustrated Catalogue of the Museum of Comparative Zoology at Harvard College,' No. 4, 1871, p. 33), writes as follows :—

" Professor VERRILL first recognized the close affinity of *Distichopora*, *Errina*, and *Stylaster* (Bull. Mus. Comp. Zool., No. 3, 1864). In his 'Notes on the Radiata' (Trans. Conn. Acad., vol. i., 1870), he adopted a suggestion of mine to make a distinct family of the Stylasteridæ, which he places in his sub-order Oculinacea, both of us overlooking the fact that GRAY had already established it."

POURTALES, struck by the porous nature of the cœnenchym of the coralla of the Stylasteridæ and other points in the hard structure which he observed, removed the Stylasteridæ from amongst the imperforate corals and ranged them next to the Eupsammidæ. He fully recognized many strong points of affinity which rendered the family a natural one, but failed to ascertain the true character of the organisms because he had not opportunity to examine their soft structures.

The coralla of several species of the family have been known to science from early times. The earliest known species, according to M. EDWARDS and HAIME, seems to have been *Stylaster flabelliformis*, the *Corail blanc* of SEBA (Thesaurus III., p. 204, Pl. 110, fig. 10, 1758), whilst *Stylaster roseus* and *Distichopora violacea* were described under the general genus *Madrepora*, by PALLAS, in 1766.

GRAY gave the name *Stylaster* to the genus in 1831 (Zool. Miscell., p. 36), and described the genus *Errina* in 1835 (Proc. Zool. Soc., 1835, p. 35).

Distichopora was named by LAMARK.

Allopora by EHRENBERG in 1834.

Cryptohelia was described by M. EDWARDS and HAIME in 1849.

POURTALES has added a new genus to the family, viz., *Pliobothrus*, as one of the results of the United States deep sea dredging operations, whilst I here add four further genera, viz., *Sporadopora*, *Astylus*, and *Spinopora*, dredged by H.M.S. 'Challenger,' and *Labiopora*, wrongly described by GRAY as a Bryozoon, under the name *Porella*. The only extant account of the soft parts of any Stylasterid is that by G. O. SARS, of the animals of *Allopora Norwegica*.*

SARS kept a succession of living specimens of the coral in fresh seawater, but never

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* G. O. SARS. Bidrag til Kundskaben om Dyrelivet paa vore Havbanker. Forh, i Videnskabs. Selskabet i Christiana, 1872, p. 115.

got the animals to expand themselves so as to raise themselves above the level of the stellate openings. Nevertheless he saw clearly with lenses the tips of the opaque white tentacles in the angles between the so-called incomplete septa, which tips were usually more or less bent inwards towards the centre. He also saw deep down in the bottom of the calicle a similarly opaque white knot-shaped projection. This was all that could be seen in the fresh living animals. Specimens were, however, preserved in spirit and subsequently examined, and the conclusion was come to that the animal was essentially different from the rest of corals, and probably did not belong at all to the Anthozoa, but rather to the Hydrozoa.

By means of lucky breakings through of the stony-hard but nevertheless porous coral, SARS was able to obtain some little view of the general form of the polyps, and their relation to the coral. The true polyp body, he says, lies at the bottom of the central cavity of the calicle. It is very small, almost hemispherical in form, and provided with an apparently protusible beak or proboscis, which is sharply defined and blunt-conical in form, and on which no mouth opening was observable. At the circumference of .the head of the polyp proceed out the narrowly cylindrical tentacles which correspond in number to the in-foldings of the calicle. Their lower region is inserted in the interseptal canals, while their upper parts project free from the foldings in between the so-called septa, and usually bend with their bluntly rounded ends towards the centre. No distinct connexion between the different polyp cavities was to be observed. They all seemed completely closed below. But it is to be remarked that the whole upper lining part of the coral is highly porous. Often there were to be found outside real polyp cavities in the inner mass of the coral near the surface ; small cavities apparently everywhere closed, wherein were included one or several spherical bodies (eggs ?). Portions of the coral were decalcified in acetic acid. The organic basis remaining preserved to a considerable extent the form of the coral, and was composed of a tolerably regular network of apparently fibrous tissue in which were embedded numerous small elliptical nematocysts. The body of the polyp could be prepared out with considerable ease from this mass in connexion with its several tentacles, which under the microscope showed themselves beset all over with extremely small tightly packed nematocysts.

Although SARS thus suspected the affinity of the Stylasteridæ to the Hydroida, his results were insufficient to demonstrate the fact, since he could obtain no satisfactory information concerning the generative structures of the coral which he studied, and he failed entirely to detect the compound nature of the cyclo-systems of *Allopora*, since he regarded the dactylozooids as the tentacles of the gastrozooid, or body of the polyp, as he terms it. He, however, determined a great deal which was of high value. He was the first to make any observations on the structure of the soft parts of the Stylasteridæ, and is as yet the only naturalist who has watched a Stylasterid in the living condition.

He concludes his account with the following words, which show that he was not certain as to the true nature of *Allopora*, although he considered there were strong grounds for removing it from amongst the Anthozoa :—

"Af det allerede anførte synes imidlertid med sikkerhed at fremgaa, at vi her have for os en Dyreform der i mange væsentlige Punkter afviger fra Anthozoerne hvortil den maaske slet ikke engang kan henføres."

Methods.

A brief examination of some of the soft structures of certain of the Stylasteridæ was made while they were in the fresh condition, and especially of the various elements of *Sporadopora dichotoma* and of the female gonophores of *Cryptohelia;* but since the trawl-net by which most of the specimens available for examination were obtained came up late in the day, very little unfortunately was able to be done in this way.

Portions of the corals were preserved by means of chromic acid, osmic acid, absolute alcohol, and glycerine, and they were subsequently decalcified and examined in the usual manner by means of sections. In cutting the sections, a method described by MILCHALKOVICS 'Arch. für mikroskopische Anatomie,' ii. Bd. 3$^{\text{tes}}$ Hft., p. 386, was adopted and found to yield most successful results. The method is especially valuable for cutting fine sections of structures, the parts of which are loosely held together, and where it is desirable to maintain the exact relations in position of parts which in the sections otherwise become entirely disconnected from one another. A strong jelly composed of equal parts of glycerine and gelatine is used as an imbedding substance. It permeates the tissues and takes the place of the hard calcareous supporting structures which have been removed by the acid. The sections are mounted in glycerine, and the imbedding substance which is left in situ in the sections becomes perfectly transparent; in fact, almost invisible in this fluid.

I now proceed to a detailed description of the structure of the several genera of the Stylasteridæ.

Each of the members of the family is composed of hard inert calcareous parts, or "corallum," and soft living structures. In the case of each genus the structure of the hard parts will be first treated of, and then that of the soft parts. The latter consists of cœnosarc, zooids, and gonophores, and will be described under these several headings in each case. A full description of all details will be given in the case of *Sporadopora*, which will be first accounted for, and in the cases of the other genera only those points in which they differ from it will be dwelt upon.

Terms.

In all Stylasteridæ two forms of zooids are present. One form has a mouth and gastric cavity, the other is devoid of these and has a purely tentacular function.

For the former the term "gastrozooid" is here adopted, and for the latter that of "dactylozooid."

The pore in the corallum occupied by the gastrozooid is termed "gastropore," and that of the dactylozooid "dactylopore."

In the more highly differentiated Stylasteridæ the pores are arranged in regular circular systems simulating the calicular systems of the Anthozoans in appearance. These systems are termed "Cyclo-systems."

STRUCTURE OF THE HARD AND SOFT PARTS IN THE SEVERAL GENERA OF THE STYLASTERIDÆ.

(1.) GENUS *SPORADOPORA* (H. N. M.).

This genus, hitherto unknown, I described in the Proceedings of the Royal Society, vol. clxxiii., 1876, p. 94, under the name *Polypora*, but as I have been informed by Mr. ETHERIDGE, junr., that the name *Polypora* is already in use, I here change it to *Sporadopora*, which refers to the irregular scattering of the pores over the surface of the corallum. The genus is founded on a single species, *Sporadopora dichotoma*, obtained on one occasion only by the 'Challenger' off the mouth of the Rio de la Plata in 600 fathoms.

Corallum of Sporadopora dichotoma.

The corallum or hard calcareous structure in this Hydroid occurs in the form of stout upright stems, which branch with tolerable regularity dichotomously to form a flabelliform expanse. The stem is usually nearly circular in section towards its base, but becomes compressed above in the plane of the fan, whilst the branches and branchlets forming the fan itself are very much flattened, so as to be more oblong than oval in transverse section. The number of branchings is few in number, only four or five at most. The flattened branches and branchlets coalesce at their adjacent margins. A figure of a well-grown but partly broken example is given on Plate 34, fig. 2, reduced to half the natural dimensions.

Sometimes the stems are somewhat bent and irregular, as are also the flabellate expanses which they support. The height of the largest specimen obtained is about $5\frac{3}{4}$ inches, and the breadth of the fan about 5 inches. The diameter of the stem at its base is about 1 inch; in slenderer specimens $\frac{1}{2}$ inch to $\frac{3}{4}$ inch. In one large broken and dead specimen the stem is 2 inches in diameter. The corallum is dense and heavy, and when macerated out from a living specimen is of a pearly white, and smooth and glistening in appearance (Plate 34, fig. 1). The surface is pierced by deep pores, which are simply circular in outline and of two kinds, large and small, and are scattered irregularly over it. The larger pores or gastropores are less numerous than the smaller. They are deep, reaching nearly to the centra laxis of the branch or stem on which they are situate, and contain a deep-seated, long, and slender style. The smaller more numerous pores, the dactylopores, are thickly dispersed between the larger ones. They have no style. The pores are usually more abundant on one face of the coral flabellum than on the other; indeed, large areas of what may be called the back of the stem are often devoid of pores altogether.

The appearance of the surface of the corallum as seen by reflected light under a low magnifying power is shown in Plate 35, fig. 2. The surface presents slight irregular undulations. Its texture is somewhat like that of loaf sugar, being composed of closely apposed minute glistening white granules. The margins of the mouths of the dactylopores are often slightly raised above the general surface.

The older pores of the corallum are very deep, and as may be seen in longitudinal sections of the branches or stems (*t.o.*), commence deep down within the stem near its axis, and bend outwards on all sides to the surface of the branch with a nearly uniform curve. The coralla of all Stylasteridæ are traversed in all directions by a system of freely anastomosing and branching canals. In the case of *Sporadopora*, these canals are especially abundant and form comparatively close meshworks, hence the whole corallum is spongy and excessively porous when seen in section (Plate 35, fig. 1). The corallum may, with most truth, be said to be built up of a series of hard partition walls, intervening between and enclosing a highly complex system of tortuous canals and cavities. The meshwork formed by these canals is closer and smaller towards the surface of the corallum, more open and with wider meshes in the deeper regions. In the deeper regions the main canals, as will be seen from the figure, follow more or less the curved directions taken by the walls of the pores on their way towards the surface. There is no main system of canals in the axis of the stem connecting all the zooid cavities. The deep canals become more or less filled up, and the only connexion between distant zooids is by the more superficial living meshworks. In some places irregular cavities of some extent occur amongst the smaller canals, as beneath the ampulla (Plate 35, fig. 1, G). At the very surface, the canal reticulation is very fine indeed.

The pores are cylindrical pits sunk in the spongy mass of the corallum, and their walls are perforated all over by the openings of numerous canals. At their bottoms their cavities pass off into a few large main canals of the meshwork. The styles of the gastropores are very long, and can be traced deep into the axes of the branches of corallum, they having become elongated as the growth of the pores and corallum required it. In their deeper regions, these slender styles show a surface composed of a few dentate ridges (Plate 35, fig. 1, S) only, whilst in their upper and functionally active region they terminate in a long brush-like mass, composed of complicated branchings of fine and delicate calcareous spicules. At the base of this brush-like part of the style, a very thin calcareous partition or "tabula" (Plate 35, fig. 1, T) is sometimes present, stretched across the pore cavity at right angles to its axis. Sometimes two or three such tabulæ are present in a single gastropore, placed at successively deeper intervals. In some instances, two tabulæ occur close together in a pore, one above the other. These tabulæ are so excessively thin that I considered them at first to be membranous, but I have been unable to dissolve them by the use of very strong alkalis, and I am now convinced that they are calcareous. They do not seem to occur in all the gastropores, and I have not observed them in any instance in the dactylopores. The dactylopores vary much in size, as will be seen from the figures.

Spheroidal cavities occur excavated in the corallum at a very slight depth from the surface. These contain the gonophores in the recent state of the coral, and may be called ampullæ. They are in this genus entirely buried beneath the surface, whereas in most genera of Stylasteridæ they project above it often to a very conspicuous extent. They communicate with the exterior when mature, by means of small slit-like apertures placed at the bottoms of small irregularly shaped depressions which are to be seen with some difficulty scattered over the coral surface (Plate 35, fig. 2, G G). Only male specimens of *Sporadopora* have been obtained as yet. No doubt, in the case of ampullæ containing female gonophores, a comparatively wide opening in the surface of the corallum is formed to allow of the escape of the fully formed planula.

The actual tissue of the corallum must be in *Sporadopora* and in most other Stylasteridæ excessively dense and compact, since the masses formed by it, although, as described, excavated by canals in all directions, are heavy.

In the older parts of the stems and their bases, the corallum appears to become compact and stony, and crystalline in fracture by obliteration of the canals and pores. In some specimens, portions of the surfaces of the stems which have once been dead have undergone rejuvenescence by the spreading of a thin layer of living coral over them from adjacent healthy regions.

The dead coralla are overgrown by a Flustra and other Bryozoa, and form bases of attachment to large masses of other Stylasteridæ, such as *Errina labiata*.

Since the calcareous meshwork is closer at the surface of the corallum, its meshes must necessarily become enlarged by re-absorption as growth proceeds. Cavities also such as those of the ampullæ must be filled up as the corallum grows. The irregular cavities existing beneath the ampullæ in some cases, as shown in Plate 35, fig. 1, probably represent spaces occupied in an earlier condition of the coral by gonophores. Sometimes also old ampullar cavities remain unfilled up, situate beneath the more superficial and active ones.

The tissue of the corallum is very like that of *Millepora* in histological structure, but appears somewhat more granular in texture, and less fibro-crystalline than it.

Soft structures of Sporadopora dichotoma. (Plate 36.)

Cœnosarc.—The tortuous canals and pores by which the coralla of all the Stylasteridæ are traversed, are occupied in all the genera alike in the living condition of the coral by a series of meshworks of correspondingly branching, twisting, and anastomosing canals, which compose the cœnosarc or common body of the compound organism in each case. In *Sporadopora*, only a comparatively thin layer on the surface of the coral is occupied by living soft structures. These living structures are separated from the non-living deeper masses of the corallum by the action of acids, and then appear as a sheet of soft tissue composed of cœnosarcal meshwork zooids and gonophores, which may be called the living lamina.

The canals of the cœnosarc are composed of a very thin and transparent membranous wall, covered on the outer surface by a layer of ectoderm cells, and on the inner lined by endoderm cells. In general structure the canals closely resemble those of the cœnosarc of *Millepora* as described and figured by me. ("On the Structure of Species of *Millepora* occurring at Tahiti, Society Islands," Phil. Trans. Roy. Soc., 1877, p. 9, Plate 3, fig. 16).

The ectoderm layer covering the cœnosarcal canals varies much in thickness, being thickest in the more superficial parts of the cœnosarcal meshwork. I was, unfortunately, unable to examine its structure in the fresh condition, because the trawl by which the specimens of *Sporadopora* and of most of the other genera were obtained came up late in the day, and the short daylight available sufficed only for the investigation of more important matters.

Although a definite cell structure is not to be made out everywhere in the ectoderm of the cœnosarc, as for example in the surface layer of the coral, it seems probable from the appearances presented by specimens hardened in osmic acid, that such characterizes it throughout. The layer investing the canals is mainly composed (Plate 44, fig. 13) of transparent inflated nucleated cells which vary in size, so that the stratum is in some places two cells thick, in others only one. Amongst these cells occur nuclei and certain cells in which nematocysts of two kinds to be presently described are contained in various stages of development.

The calcareous matter of the corallum must be secreted by this ectodermal layer of the cœnosarcal canals, but I have not been able to observe how this takes place, or whether by means of any particular form of cell.

In the membranous layer of the canals no structure was detected. The endodermal lining of the canals is composed of abundance of spheroidal pigmented cells, containing a nucleus and granules of pigment of various sizes, and closely similar in appearance to those occurring in *Millepora*. The pigment in the present species is of a brick-red colour. Besides these cells, smaller transparent, colourless, spheroidal cells occur in the endodermal layer, and also free pigment granules and effete pigment cells, devoid of granular contents (Plate 44, fig. 14). The arrangement of these several constituents of the endoderm within the lining of the canals was not determined. No doubt in all the Stylasteridæ the inner surface of the canals is, as usual, ciliated, although cilia were not able to be made out in any case, owing to the action of reagents on the tissues.

As will be seen by reference to Plate 36, the cœnosarcal canals form in *Sporadopora* a very complex network, which brings, by means of the freest anastomoses in all directions, the several members of the compound organism into complete circulatory connexion with one another.

The interspaces in the meshwork occupied in the recent condition of the coral by hard masses of the corallum, are larger and wider in the deeper regions of the cœnosarc than nearer to the external surface. For here the meshwork is much closer, and the mass of soft living tissue much greater in proportion to the calcareous structures secreted by it, than is the case in the deeper regions. Further, the deeper canals are of greater calibre than those nearer the surface. Towards the deepest regions of the cœnosarc the canals are shrunken and atrophied, and pass off into effete and almost dead fragments of tissue, which form the inner boundary of the living lamina.

The largest trunks of the meshwork are those which proceed directly from the bases of the zooids and gonophores. These are soon reduced in size by branching, and are lost in the general anastomosis.

Around the sacs containing the zooids the canals of the cœnosarc have a special radiate disposition (Plate 43, fig. 3). The radial canals occupy an area circular in outline, extending all around the outer sides of the sacs of the zooids. They pass directly inwards radially, from the margins of the areas where they take origin from the general meshwork to join themselves on to the walls of the zooid sacs, towards the centres of the areas. They branch but seldom on their course, and then only towards their outer origins, where they not uncommonly bifurcate.

As may be seen from the figure, the radial canals, which lie at successive depths from the surface, do not correspond in any way in position with those above or below them, but are quite irregular as far as radial disposition is concerned. In vertical sections, however, of the living lamina (Plate 36), these radial canals are seen to succeed one another at tolerably regular intervals, in vertical disposition, with a somewhat regular series of interspaces between them.

This radial disposition of the canals is less marked around the sacs of the larger dactylozooids than around those of the gastrozooids, and is hardly apparent around those of the smaller dactylozooids. Traces of it are to be seen around the sacs of the gonophores, as at G′, Plate 36. Although towards the periphery of the area occupied by them these radial canals contain endoderm cells, and appear similar in structure to the other canals of the cœnosarc, towards their inner extremities, where they join the zooid sacs, they become diminished in size, and often appear as mere slips of transparent tissue, having a fibrillate appearance.

Muscular filaments, continued from ovoid muscular cells embedded in the walls of the zooid sacs, pass outward along the radial offsets, and are attached to them in the region about the mouths of the sacs (Plate 43, fig. 3, R M).

Attached to the radial offsets, and often extending over the interspaces between adjacent ones, slips of a fine transparent membrane, containing minute nuclei and striated in appearance as if composed of fine fibrillæ, are constantly to be seen, but they seem to occur at altogether irregular intervals, and only towards the more superficially lying parts of the zooid sacs (Plate 43, fig. 3 ; Plate 36, A A).

A continuous layer of ectodermal tissue extends over the outer surface of the coral. No distinct cell structure was made out in this surface layer in *Sporadopora*, although such probably exists, as it was clearly seen in the case of the surface layer of *Errina labiata*. The layer bridges over the gaps in the superficial meshwork of the cœnosarcal canals, and portions of it close in the mouths of the sacs of the zooids when the zooids are in the retracted state. Over the mouths of the sacs and radial canals of the retracted gastrozooids these special parts of the surface layer appear as discs of membrane, with very small apertures in their centres, and showing a radial fibrillation diverging from these central openings, which seems as if caused by contraction of the tissue in order to close the aperture.

Embedded in the surface layer are numerous nematocysts of two kinds, larger and smaller. These are figured Plate 43, fig. 9. The larger nematocysts are of the form of cylinders,. very slightly bent. Their ends from which the threads are shot are bluntly pointed, whilst their opposite extremities are rounded. The thread at rest is coiled up spirally within the cell, in the usual manner (Plate 43, fig. 9, *a*). The emitted thread has an elongate enlargement upon it near the cell, which is beset with a spiral of spines (*a′*). The remainder of the thread is simple. These larger nematocysts have a length of about ·0016 of an inch.

The smaller kind of nematocysts are of an ovoid form, slightly flattened on one side, and, like the larger kind, more pointed in shape towards the end from which the thread emerges. They measure ·00064 of an inch in length. They have a small bladder-like enlargement on their emitted threads, but it is, as far as was ascertained, devoid of spines.

In both kinds of nematocysts the threads are shot out, not in a line with the length of the cell, but at a slight angle to this, and in continuation of the curves of the cells.

Thread cells of almost exactly similar structure to these two occur in all the genera of Stylasteridæ, the soft parts of which are described in the present paper.

The nematocysts are developed in transparent cells of the ectoderm, which always contain a nucleus of finely granular appearance. The young nematocyst is seen developing within the cell with the nucleus lying beside it, and in proportion as the nematocyst increases in size and maturity, the nucleus diminishes in bulk (Plate 43, fig. 9, *c*, *d*, *e*).

Nematocysts of both kinds are to be seen in abundance in all stages of development in the ectodermal cells of the more superficial regions of the cœnosarcal meshwork. Both larger and smaller nematocysts are present in abundance, scattered in the superficial layer of the ectoderm.

The larger form of nematocysts also occur in well differentiated nematophores, which occur disposed irregularly amongst the zooids in the superficial region of the coral

(Plate 36, N N). The nematophores are irregularly semicircular in vertical section, with the flat side of the semicircle coinciding with the surface of the superficial layer of the ectoderm. Except on this flat side they are bounded by a membranous wall, which forms a sac open above. The open mouth of the sac is crammed with nematocysts of the larger kind, closely packed side by side, with their pointed ends directed to the surface. The cells are so closely packed that, in a section of the superficial layer taken parallel to the surface through the nematophores, no interstices can be seen between them (Plate 43, fig. 3, N). The lower part of the cavity of the nematophore is filled with nuclei and parent cells of the nematocysts. The nematophores, as viewed from the surface of the superficial layer, are seen to have an irregular outline, showing a tendency to be somewhat oblong, with curved boundaries.

No triple-spined nematocysts, such as those occurring in *Millepora* and in most other Hydroids, were detected as existing in any of the Stylasteridæ. The two kinds described as occurring in *Sporadopora dichotoma* appear to be present in all members of the family, with very slight variations in form indeed.

Zooids.—The zooids in *Sporadopora dichotoma* are of two kinds, dactylozooids and gastrozooids: the former occupying the smaller, and the latter the larger, style-bearing pores, already described as characteristic of the corallum.

Dactylozooids.—I have named the mouthless zooids of the Stylasteridæ "dactylozooids" because, although they are invariably destitute of tentacles, they are reduced to the condition of simple tentacles themselves, and evidently perform a tentacular function.

The dactylozooids are closely similar in form and structure in all the genera of Stylasteridæ hitherto examined, and differ only in dimensions. They are simple, elongate, conical bodies, just like the ordinary tentacles of Hexactinians in form, and are devoid of mouth or any opening to the exterior. They are attached to, and, when unprotruded, retracted within membranous sacs or sheaths, which rest within the corresponding pores of the corallum. In *Sporadopora*, the sacs of the zooids, the walls of which are shown in longitudinal section in Plate 36, F F, are composed of a transparent membrane, derived from the ectoderm, and continuous with its surface layer. The membrane has numerous fine nuclei dispersed in its substance, and is strong and tough. It is lined on its inner surface next the cavity of the sac by a layer of small transparent cells, which are shown in the figure cited above.

On their outer surfaces, the walls of the zooid sacs are abutted on by the peculiar radial offsets of the cœnosarcal meshwork already described. These offsets appear to lose their tubular character as they near the walls of the sacs, and, as far as was seen, no openings occur in the sac walls communicating by means of these radial canals with the cœnosarcal circulation, although such an arrangement was supposed to exist when the first hasty examination of specimens was made.

The sacs are attached to the bases of the zooids, being continuous in those regions with the ectodermal covering of the zooids. They closely invest the retracted zooids, and are thus cylindrical in form in their deeper parts; whilst above the level of the retracted zooids they contract gradually in diameter, to terminate at the surface of the coral in small openings, which are usually seen to be quite closed by contraction of the surrounding superficial membrane in hardened specimens. The sacs lie loose within the pores of the coralla; that is, they are smaller in diameter than their containing calcareous cavities, but they are held in place by the attached radial offsets of the cœnosarc, which issue from the numerous openings in the walls of the pores to join on to them (Plate 35, fig. 1, G Z).

The dactylozooids of *Sporadopora* vary much in size, the smaller being of less than half the dimensions of the largest. They are elongate-conical in form, and are composed of an ectoderm, endoderm, membranous and muscular layers. They have an axial tubular cavity within, which communicates directly at their bases with the larger deeply-situate canals of the cœnosarcal meshwork.

The ectoderm forms, in the retracted zooids, a thick external layer, which is thrown by the contraction of the zooid into a series of transverse folds (Plate 36, D Z). No doubt, in the expanded condition of the zooid the ectoderm would appear much thinner. The outer suface of the layer is thickly beset with nematocysts of the smaller variety, which are so closely packed side by side, with their pointed ends outward, that in the retracted zooid no interstices between them are to be made out (Plate 43, fig. 2, E). Beneath this armature of nematocysts the main thickness of the ectodermal layer is composed of finely granular matter filled with ovoid nuclei and nematocysts, in various stages of development. No definite cell-structure could be determined in the layer, but fine lines, having a radial disposition in transverse sections of the zooid, seemed to indicate that the layer is composed in reality of somewhat prismatic-shaped cells, disposed in it radially to the central axis of the zooid.

At the inner surface of the ectodermal layer is a layer of very distinctly differentiated muscular slips, which have a longitudinal disposition (Plate 43, fig. 2, M; Plate 36). These muscular slips do not form a quite continuous layer, being separated from one another, as appears in transverse section, by a definite series of intervening intervals. These muscles are fine, and difficult to detect towards the tips of the zooids, but increase in thickness towards their bases. In these regions of the zooids they are extremely conspicuous, and spread out in a thick layer over the large main vessels of the cœnosarc in immediate connexion with the bases of the zooids, passing beneath the ectoderm of these canals, and being inserted into their walls. The muscles act evidently as the retractors of the zooids. Since they are more highly developed in the case of the gastrozooids, they will be further described when these are under consideration.

United with the muscular layer and inseparable from it, is a layer of membrane which is continuous with the membranous layer of the cœnosarcal canals, and forms a complete sac within the zooids. This basement membrane shows, in the contracted zooids, a transverse striation (Plate 43, fig. 6), which was at first supposed to indicate the existence of a layer of circular muscular fibres crossing the described longitudinal muscles. No definite circular fibres could however be detected, and the appearance is probably due to contraction of the membrane.

The inner surface of the membrane is lined by endodermal cells. In the contracted zooid, these form a layer two, three, or four cells thick. The cells are globular, clear, and transparent, and contain a nucleus (Plate 43, fig. 2; Plate 36). On the actual inner surface of the layer, bounding the zooid cavity, is a layer of cells similar in character to, but much smaller than, those composing the main mass of the layer. No doubt the inner surface of the cavity of the zooid is ciliated in living condition; cilia were, however, not detected. Towards the base of the zooid cavity, the transparent cells are replaced in the endoderm by the spherical pigmented cells, which are the principal constituent of the endoderm of the cœnosarc.

The dactylozooids have a tendency to be attached by their bases to one side of the bottoms of their sacs, rather than to the lowest extremities of the sacs. When this is the case, as in Plate 3, D Z, the zooid in the retracted condition is partly doubled up upon itself, and not merely drawn directly in. The main retractor muscles, however, pass almost directly downwards to their insertion into the cœnosarcal canals. In consequence of this arrangement the bottoms of the sacs are, when it occurs, pulled somewhat to one side. This form of attachment of the dactylozooids occurs mostly amongst the larger examples, no doubt because their greater length requires such an arrangement in order to allow of more complete retraction by the aid of the doubling of the zooid. This tendency to lateral attachment in the dactylozooids, as occurring in *Sporadopora*, where the zooids are diffusely scattered over the coral surface, is of interest because the same tendency is shown by the dactylozooids in nearly all the Stylasteridæ; and in some, as in *Cryptohelia*, *Allopora*, &c., it is the normal and only method of attachment.

Gastrozooids.—The gastrozooids in *Sporadopora dichotoma* are cylindrical in form, with four short tentacles set on to the body equi-distantly in a single whorl. Above the line of origin of the tentacles rises the dome-like hypostome, which in the retracted condition of the zooids has a height equal to that of about one third of the entire height of the zooid body.

The zooid in its inferior region is circular in section, but superiorly in the region where the tentacles are given off and in that of the hypostome, it assumes the form in section of a rectangle with the corners rounded off and the sides indented, the tentacles being situate at these corners of the rectangle.

Within the zooid is a wide gastric cavity, into the axis of which, in the retracted condition of the zooid, the calcareous style of the gastropore protudes for two-thirds of the height of the cavity (Plate 36, *St*).

The mouth at the summit of the hypostome appears when viewed from above as a cruciform opening leading directly to the gastric cavity. The gastric cavity communicates by tubular offsets with the axial cavities of the tentacles, and at its base it becomes at its periphery continuous with the cavities of four large canals. These canals subdivide almost immediately into smaller trunks which anastomose with the general cœnosarcal meshwork.

The gastrozooids are structurally composed of the same number of layers as the dactylozooids. The ectoderm forms on these zooids a somewhat thinner layer than on the dactylozooids. Definite cell structure was not made out in it. It is, however, full of nuclei, and is no doubt definitely cellular in the living condition. It is not, as in the case of the dactylozooids, thickly beset with nematocysts, but contains very few of these bodies (Plate 43, figs. 1 and 5).

On the inner surface of the ectoderm in combination with the basement membrane occurs a muscular layer which is very highly developed. The layer is composed of a series of longitudinally disposed muscular slips, which are set side by side with narrow interspaces, so as to form a thick layer (Plate 43, fig. 6). This layer is extremely thick and dense towards the base of the zooid, as will be seen from Plate 36, M, and becomes gradually thinner and less conspicuous towards the hypostome. The muscular slips are stout and closely set towards the base of the zooid, and prominent objects in transverse sections of it in that region (Plate 43, fig. 5), whilst they are widely separate and fine, and far less numerous towards the upper regions of the zooid (Plate 43, fig. 1, M), where little is to be seen but the transparent basement membrane. The muscular slips are composed of very distinctly differentiated cells which have mostly a fusiform shape (Plate 43, fig. 8), with the tails of the cells usually somewhat bent. Many cells are found to occur amongst the mass which are apparently in the act of division, two fusiform bodies being connected together by a string, or broad mass, of protoplasm. Such cells are so numerous that possibly a considerable proportion of the muscular elements remain permanently in this compound condition. The cells are closely fitted together side by side to form the muscular slips which, where most developed, have a breadth of three or four cells (Plate 43, fig. 7).

The longitudinal muscular slips pass from the bases of the zooids to spread out beneath the ectoderm of the four main canals of the cœnosarc, in which the cavities of the zooids terminate inferiorly.

Fused with the muscular layer, occurs, as in the dactylozooids, a continuous layer of membrane. This basement membrane is transparent, and the only structure which I have seen in it is a striation transverse to the longer axes of the zooids, which, as already stated in reference to the dactylozooids, I at first believed to give evidence of the existence of circular muscular fibres in the zooids. Such fibres I have however been unable to discover on closer examination.

Beneath the membranous layer lies the endoderm. This is composed, towards the upper region of the zooid and in the hypostome, of elongate ovoid cells with an inflated appearance, very transparent, each containing a small nucleus. These cells, as is well seen in transverse sections (Plate 43, fig. 1, G), are packed side by side to form the endodermal lining of the zooid, with their longer axes directed inwards, radially, towards the axis of the zooid, except towards the uppermost region of the zooid, where the direction of these cells is modified by the peculiar rectangular form assumed by the zooid. These elongate cells are closely similar to those occurring in a similar situation in other Hydroids, and there can be little doubt that they are gastric in function. It will be observed that they do not occur in the endoderm of the mouthless dactylozooids. Towards the base of the zooid cavity these cells become shorter and shorter in length, until in the deepest regions they become mere small globular transparent cells, like those composing the endoderm of the dactylozooids. Towards the base of the zooid they are further overlaid by a layer of the pigmented endoderm cells, which form the endodermal lining of the general cœnosarcal meshwork. The lining of the cœnosarcal canals thus becomes continuous with that of the zooid cavity (Plate 36).

The calcareous style projecting up into the cavity of the zooid has reflected over it from its base a covering of ectoderm, and over that it is protected within the zooid cavity by a layer of ordinary pigmented endodermal cells (Plate 36, *St*).

The tentacles of the alimentary zooid of *Sporadopora dichotoma* were the only ones amongst those of all the Stylasteridæ which I was able to observe in the fresh condition, and time did not allow of more than a cursory glance at these even. It sufficed, however, to show that the tentacles are, as in the case of *Millepora*, knobbed at their tips (Plate 43, fig. 4), and that their stems display the usual characteristic transverse segmentation of the endoderm.

The knobs of the tentacles are ovoid in form and are densely beset with nematocysts of the smaller variety. The ectodermal layer of the stems of the tentacles contains few or no nematocysts.

Gonophores.—Although the soft parts of at least three different colonies of *Sporadopora dichotoma* were examined, these specimens proved all to be male. In all the specimens gonophores were very abundantly present. They occupy the ampullar chambers in the calcareous corallum already described (Plate 35, fig. 1, G). The male gonophores are ovoid bodies with their long axes directed at right angles to the surface of the coral. Sometimes only one such body is present in an ampulla, sometimes two or three. The outer extremities of the gonophores are sometimes drawn out into a short tail-like prolongation (Plate 36, G). The bodies vary considerably in dimensions. Often a gonophore which is fully mature and just ready to discharge its contents at the summit of its ampulla (as seen in Plate 36, G), has beneath it in the deeper part of the same ampulla an immature gonophore, around which latter the ampulla is less dilated.

The gonophores are composed of a spadix, which is extremely conspicuous in the fresh condition of the tissues, because it is full of red endodermal cells, and thus deeply pigmented, and a mass of testis cells or spermatozoa. The spadix is cylindrical in form, with a rounded extremity. It occupies the axis of the deeper region of each gonophore. It thus forms the core of the spheroidal body, the remainder of the mass of which is composed of spermatozoa or the cells from which they are developed in various stages of advancement. These cells and spermatozoa are contained within a fine and transparent but tough membrane, which invests the whole body of the testis, being derived from the ectoderm. I believe that a layer of the ectoderm invests the spadix within the testis, but am not certain. I could not determine from which layer the spermatic cells are developed.

The spermatozoa are developed in the same manner as in other Hydroids. In Plate 36, G, is figured the usual mass of small spermatic cells in an unripe testis. Above this is a ripe testis which is shown as not cut right through its axis, it being bent over a little from the perpendicular to the surface. Hence the spadix is not seen in the section.

The ripe spermatozoa (Plate 43, fig. 12) have conspicuous heads which are elongate bodies curved into a bow shape. They are compressed and flattened in the plane of the curve, so that though broad and conspicuous when viewed on the flattened sides, they appear almost linear when seen on edge. At the extremity of the head where the tail is attached, a small rounded vesicle was observed in all cases to be present.

The bases of the gonophores are continuous with large canals of the cœnosarcal meshwork, the endoderm of the spadix being continuous with that of these canals.

(2.) GENUS *PLIOBOTHRUS* (POURTALES).

The genus *Pliobothrus* was formed by Pourtales (Bull. Mus. Comp. Zool., Cambridge, Mass., No. 7) to include specimens dredged by the United States Coast Survey off the coast of Florida, in from 100 to 150 fathoms. Pourtales rightly placed the new genus amongst the Hydroids, but, judging from the structure of the hard parts alone, associated it with *Millepora*. Count Pourtales, however, most kindly placed at my disposal specimens of *Pliobothrus symmetricus* preserved in spirit, and in excellent condition; and these have yielded tolerably complete evidence as to the structure of the soft parts. Moreover, the two small specimens transmitted to me proved to be of opposite sexes. I have observed both sexes only in the case of one other genus of the Stylasteridæ, namely, *Cryptohelia*. The structure of the soft parts of *Pliobothrus* proves the coral to belong undoubtedly to the Stylasteridæ. The specimens of *Pliobothrus symmetricus* examined by me were dredged off Florida Reef, in 100 to 300 fathoms.

Corallum of Pliobothrus symmetricus.

The corallum is described and figured by Pourtales ('Deep Sea Corals,' Ill. Cat. Mus. Comp. Zool., Harvard Coll., Cambridge, Mass., 1871, p. 57, Plate iv., figs. 7 and 8). He describes three kinds of pores as existing in the corallum. In reality, there are only two kinds of true pores present, viz., the larger circular-mouthed gastropores and the smaller dactylopores, which open at the summits of short tubular projections from the general surface of the corallum. The third kind of pores is described by Pourtales as very small linear disposed over the whole cœnenchyma, and arranged in rows. These are merely spaces between the trabeculæ of hard tissue forming the cœnenchym of the corallum, and are occupied by canals of the cœnosarcal meshwork in the recent condition of the coral. They do not contain any form of zooid. It is to be noted that in *Pliobothrus tubulatus,* a second species (POURTALES, *l.c.*, p. 58), the projecting tubules of the tubulated pores are much longer than in the case of *P. symmetricus,* and thus form a stepping-stone in the series towards the condition existing in *Errina.* The pores of both kinds in *Pliobothrus* are devoid of styles. The gastropore cavities are tubular in form for a short depth from the surface, and then expand suddenly into a wide basin-shaped chamber, which lodges the similarly formed base of the gastrozooid, and from the margins of which proceed numerous large canals running mostly to the bases of neighbouring dactylopores. The corallum is very coarsely porous, otherwise the finer structure is much as in *Sporadopora.* The ampullæ are, as in *Sporadopora,* buried beneath the surface of the corallum. POURTALES remarks on them as "occasional round cavities found in the centre of the branch, filled with a yolk-like substance contained in a membrane."

Structure of the Soft Parts. (Plate 41, fig. 2.)

The cœnosarcal meshwork of *Pliobothrus symmetricus* is very like that of *Sporadopora* in general arrangement, as will be seen by reference to the figure (Plate 41, fig. 2). The tubes composing it are, however, much finer and smaller in diameter, and the components generally of the coral are on a smaller scale.

There is the usual surface layer of ectoderm present, and the nematocysts which occur are of the two forms found in the whole of the Stylasteridæ. The offsets of the cœnenchymal meshwork, which join the sheaths of the gastrozooids, show only a very indefinite trace of the radiate arrangement which is so marked in *Sporadopora.* A trace of the arrangement does, however, exist (Plate 41, fig. 2, X X).

The gastrozooids are devoid of tentacles. In the contracted condition they consist of an upper cylindrical portion (Plate 41, fig. 2, Z), and a wider saucer-shaped basal region, to join the margins of which the lower part of the cylindrical portion gradually widens out inferiorly. Canals are given off from the margin of the basal saucer of the zooid all round, and pass to join the general cœnenchymal meshwork; but no canals at all are given off from the rounded under surface of the zooid. The upper surface of the cylindrical portion of the zooid is nearly flat, and is occupied by the mouth of the zooid, which is a cruciform slit bounded by elongate gastric endoderm cells, closely similar to those described as existing in *Sporadopora.*

The dactylozooids are simple elongate-conical bodies devoid of mouths, with a minute structure closely similar to that of the corresponding zooids of *Sporadopora.* In the retracted condition they are thrown into a series of transverse folds, which are indicated by fine transverse lines in the figure (Plate 41, fig. 2, T Z, T Z). The zooids appear to be retracted directly within their sheath, and not to be attached on one side of their base.

The gonophores are contained in ampullæ, which are often sunk deep within the corallum; and it is not apparent by what means the large mature planulæ find their way to the exterior. I have not had sufficient material at command to determine whether the ampullæ, as they enlarge, come gradually to communicate with the surface of the corallum by means of absorption of the intervening hard tissues. It seems probable that they may do so.

The ova are solitary, one only being developed in each growing ampulla. Each ovum is developed within the cup of a cup-shaped spadix (Plate 41, fig. 2, O). The ovum is provided with a germinal vesicle and spot. It is covered by a fine layer of ectoderm, which is reflected over it from the surface of the spadix. It is not patent how fertilization takes place—that is to say, how the spermatozoa find their way to the sometimes deeply-seated ova. As the ovum advances in development and increases in size, the spadix enlarges with it (Plate 41, fig. 2, G). Subsequently, however, in later stages, the spadix appears not to increase further; and when in relation with a nearly fully-developed planula appears proportionately small.

The nearly mature planula (Plate 41, fig. 2, P) is a large object of an ovoid form, with a long diameter greater than the extreme width of the gastrozooids. Its ectoderm and endoderm are plainly differentiated. The endoderm is composed mainly of oil-cells, but contains also a few fully formed nematocysts of the larger variety. The ectoderm, a thick layer, shows the characteristic striation vertical to the outer surface of the planula, the striæ being composed of granules and nuclei arranged in linear groups. As far as could be ascertained from the scanty material at command, it appeared that the ectodermal layer is formed in development by the process of delamination. No trace of an invagination in the embryo was observed.

In very advanced stages the planulæ become folded to a slight extent, as in the case of those of *Errina labiata* (Plate 37), in order to accommodate their length within the ampullæ.

The male stocks of *Pliobothrus symmetricus* are in every way similar in structure to the female, with the exception that they bear male gonophores instead of female in their smaller ampullæ.

The male gonophores (Plate 41, fig. 3) are sacs containing a number of small ovoid bodies, which contain spermatozoa or sperm-cells in various stages of development. The exact structure of these smaller bodies, and of their relations to the endoderm, were not determined.

(3.) GENUS *ERRINA* (GRAY).

The genus *Errina* was formed by GRAY to contain the *Millepora aspera* of LINNÆUS and ESPER. GRAY gave a short diagnosis of the genus in the Proc. Zool. Soc., 1835, p. 85, from specimens in the British Museum; and this was supplemented by SAVILE KENT, in a paper published in the same journal for 1871, p. 282, by further reference to the same specimens. A specimen dredged by H.M.S. 'Challenger' off the mouth of the Rio de la Plata in 600 fathoms, is clearly referable to this genus, but represents a new species, for which the name *Errina labiata* is adopted.

Corallum of Errina labiata. (Plate 34, fig. 7.)

The corallum occurs in the form of arbuscular multi-ramified masses, which have an extreme height, in the specimens obtained, of about 5 inches. The mass of branches and branchlets has a tendency to form an irregularly flabellate expansion, which in the largest specimen obtained has a breadth of about 4 inches. The main stems, which are irregularly oval in section, being flattened in the plane of the flabellate expansion, have a longer diameter of about two-thirds of an inch. They, as well as the remainder of the corallum, are composed of a compact, hard, glistening, white, calcareous tissue. At their bases, this tissue spreads over and encrusts objects to which the coral mass is adherent. In one specimen obtained, the support thus fastened on is a large dead mass of *Sporadopora dichotoma*. The main stems have a surface which appears smooth and even to the naked eye, but when magnified is seen to be scored in all directions by small more or less tortuous canals, which in the recent state contain the superficial ramifications of the cœnosarcal meshwork. In specimens in which certain regions of the main stems are dead and somewhat corroded, these scorings of the surface are much more conspicuous than on the recently living regions, and give the surface a roughly engraved appearance. The finer branches have a tendency to develope mostly on one face only of the flabellate expansion, one face of the main stems being frequently devoid of such branches. The branches and branchlets are nearly circular in section, and have an hirsute or finely spinous appearance. This appearance is due to their being beset all over their surfaces with small nariform projections, the wide openings of which are all turned towards the tips of the branches. These nariform projections vary much in form, being often drawn out into tubes opening by a slit-like mouth on the side next the tip of the branch, and frequently coalescing, especially towards the tips of the branches, so that two or three of the projections have a common base.

These projections are the prolongations of the walls of the dactylopores beyond the main surface of the corallum. Their cavities, the pores, are simply tubular without any style, and extend for a short distance into the mass of the branch, on which they are situate in an oblique direction, in continuance of the oblique inclinations of the nariform projections. The dactylopore projections are very numerous and closely set towards the tips of the branchlets, more widely scattered upon the surfaces of the branches and almost absent on the main stems.

Scattered over the surfaces of the branches and branchlets are the mouths of the gastropores, which are tubular cavities larger than the dactylopores, but with a similar oblique direction towards the axes of the branches, and are provided with a calcareous style, with a finely dentate surface (Plate 37, S T). The mouths of the gastropores are irregularly circular in outline, their margins being frequently broken and indented by the confluence with the pore cavities of the superficial channels of the surface of the corallum. The gastropores are frequently situate beneath the bases of the dactylopore projections, so as to be covered by these as by a projecting lip; and in places the margins of the gastropores themselves are drawn out into scale-like lips, though these lips are nearly always fused with nariform projections of contiguous dactylopores. Gastropores are frequently to be seen occurring isolated and solitary on the branches.

The ampullæ are, in this genus, conspicuous bodies, since they appear as hemispherical projections from the surfaces of the branches of about the size of a mustard-seed. In vigorous specimens they are closely crowded together in masses on both sides of the branches and branchlets in various regions of the flabellum. The ampullæ commence as small cavities in the surface layer of the corallum of the branches, and gradually enlarging in accordance with the development of the ovum contained in each, project more and more, until those containing mature, or nearly mature, planulæ appear as the conspicuous projections above described. A hemispherical cavity, excavated in the surface of the corallum, corresponds with each ripe ampulla, but the excavation is usually not deep enough to render the entire ampullar cavity spherical in form. The cavity has rather the form of a sphere with one side flattened somewhat. In accordance with the gradual expansion of the ampullar cavity, its outer wall, which is finely reticular in structure, becomes thinner and thinner until, no doubt, it at last breaks away entirely for the escape of the imprisoned planula. The empty hollows remaining after this process is complete are abundantly present on the surfaces of the branches, and are often to be seen remaining on the older regions of the main stems, although in these older regions there is a tendency to obliteration, by interstitial calcareous deposit, of all pores and ampullæ.

The mass of the corallum is composed, as in other Stylasteridæ, of hard calcareous tissue, permeated in all directions by meshworks of canals. The canals generally are, in the present genus, larger in proportion to the size of the zooids than in most other forms (Plate 37), and the meshworks formed by them are comparatively widely open. The main canals have a general tendency to traverse the axes of the stems and branches, spreading out at an inclination corresponding with that of the pore cavities towards the surfaces. This arrangement necessarily results from the mode of growth. In the older regions of the stem the corallum becomes more compact and stony by obliteration of many of the canals, but the main canals appear never to become entirely obliterated, even very low down towards the bases of the stems.

Soft structures of Errina labiata. (Plate 37.)

Cœnosarc.—The cœnosarcal meshwork in *Errina labiata* is more widely open in its structure than in *Sporadopora dichotoma* (Plate 37). Hence the mass of soft structures separated from the corallum by decalcification is comparatively soft and less able to maintain the original form of the corallum. In the present species, however, in all the actively living branches it is not, as in *Sporadopora dichotoma*, a mere surface layer of the coral which is living supported by dead corallum below, but the deeper canals of the cœnosarc retain their vitality even to the very axes of the branches. The general arrangement of the cœnosarcal canals is seen in Plate 37. Closer meshworks compose the mass near the surface, and in deeper regions the canals are larger and form wider and longer meshes, and constitute an axial system of main canals by which the various distant zooids are brought into relation with one another. Around the sacs of the gastrozooids an irregular radial arrangement of the canals immediately adjoining the sacs is to be observed, representing the more regular radial disposition described as existing in *Sporadopora dichotoma.*

The histological structure of the cœnosarcal canals is closely similar to that occurring in those of *Sporadopora.* The endodermal pigmented cells are of a light brick red colour, and hence the entire coral in the recent state is thus coloured. The pigment is, however, soluble in alcohol, and thus quickly extracted in specimens preserved in that fluid, but it is insoluble in glycerine. A continuous superficial layer is present on the surface of the coral, as shown in Plate 37, and it is composed of polygonal nucleated cells (Plate 11, fig. 10).

Errina is the only genus of Stylasteridæ in which the definite cellular structure of the surface layer of the ectoderm could be determined, although no doubt a similar structure exists in that of all the species of the family.

In places, the cells composing the layer appear to overlap and sometimes to form a double layer, as seen in the figure. Possibly this appearance is due to the action of reagents.

Nematocysts of two kinds, larger, and smaller occur, and of the usual forms. The larger are mostly gathered into thickly set masses or nematophores (Plate 37, N N), but occur also scattered, or in twos or threes, within the surface layer (Plate 44, fig. 10, N). These scattered nematocysts have the appearance of lying within the polygonal cells composing the surface layer (Plate 11, fig. 10), as is the case in *Hydra viridis,* as shown by F. E. Schultze.*

The smaller nematocysts occur scattered in the surface layer (Plate 44, fig. 10, N), and thickly set in the tentacles of the gastrozooids and outer surfaces of the dactylozooids.

Zooids.

Dactylozooids.—These are simple elongate mouthless conical bodies closely similar to those of *Sporadopora* but somewhat more attenuated in appearance (Plate 37, D Z). They are attached to the bases of sacs which line the cavities of the nariform dactylopores of the corallum, the walls of which sacs are continuous in structure with the surface layer of ectoderm.

Gastrozooids.—These are cylindrical in form (Plate 37, G Z), with a rounded conical hypostome and four tentacles set in a single whorl at its base. The tentacles are in

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* "Über den Ban und die Entwicklung von *Cordylophora lacustris.*" Leipzic: W. Engelman, 1871. Taf 6, fig. 10, *s. rr.*

the contracted condition clavate in form. The base of the zooid rests on the style of the containing gastropore, which in the retracted condition of the zooid appears to project into the gastric cavity to a considerable distance as in *Sporadopora.* The ectodermal covering of the gastrozooids is composed of transparent ovoid cells (Plate 44, fig. 4), which form a layer resting upon a substratum containing numerous nuclei and bounded by the basement membrane. The gastric endodermal lining of the zooids is composed of elongate cells of closely similar nature to those occurring in *Sporadopora.* The mouth appears, when closed, as a crucial slit; four main canals usually lead from the base of the zooid cavity to the cœnosarcal meshwork.

Fresh zooids are added to the colony by means of buds arising from the surface layer of the cœnosarc at points where this is joined by offsets of the superficial canals of the cœnosarcal meshwork. Such a bud is represented in Plate 37, D. The part of the superficial layer immediately surrounding the bud is depressed and forms the sac of the zooid.

Gonophores.—Only female examples of *Errina labiata* were obtained for examination. The female gonophores are closely similar in structure to those already described as occurring in *Pliobothrus symmetricus;* but there is this great difference—that whilst in *Pliobothrus* the ampullæ and their contained ova and planulæ remain until maturity immersed in the corallum beneath its surface, in *Errina* the ampullæ project more and more above the surface as development proceeds.

The spadix in *Errina labiata* is at first cup-shaped (Plate 37, S), the walls of the cup being composed of a very thick layer of endoderm. The cavity of the cup is directed towards the surface of the coral, and within it rests the large single ovum with its distinct germinal vesicle and spot. Each ampulla contains invariably only one spadix and ovum. The ovum is covered over in the cup by a reflection of the ectodermal investment of the spadix (Plate 44, fig. 4).

The stages of yelk division were not detected. The ovum becomes developed into the condition of a planula within the ampulla. As development proceeds, the embryo becomes gradually greatly increased in size, and assumes a form corresponding to that of the containing ampulla already described. As the process proceeds, the spadix becomes divided at its margin into a series of lobes, which lobes sub-divide, branch and unite to form a network, and encroach over the surface of the embryo until more than half of the proximal surface of the embryo becomes thus embraced by the reticulate cup of the spadix (Plate 37, S′).

The ectodermal layer of the embryo seems to be formed from the general mass by delamination. No trace of any process of invagination was observed; but all stages were seen which would appear to prove that the ectoderm layer is gradually differentiated at the surface from the outer elements of the mass. The ectodermal layer when first observable as distinct, appears finely granular in structure, whilst the abundant endodermal mass is composed in large part of highly refracting oil-globules. The ectoderm, as development proceeds, shows a striation directed perpendicularly to the surface of the embryo all over, and this condition is very conspicuously marked in the fully-developed planula (Plate 37, E C). The mature planula is elongate-ovoid in form, and is slightly folded once upon itself in order to accommodate itself to the confined space within the ampulla. The layer of ectoderm described as investing the surface of the ovum and derived from the spadix, persists as a covering of the mature planula until set free (Plate 37, B).

In fine sections of mature planulæ the fine structure of the ectoderm and endoderm is well seen. The ectoderm forms a thick layer composed of alternately placed transparent and opaque tracts disposed vertically to the surface of the planula. The more opaque tracts contain numerous nuclei and thread cells in process of development. The dark tracts fuse together towards the inner region of the layer, and form a continuous mass full of nuclei which rests upon the basement membrane, as yet little differentiated but still clearly indicated (Plate 44, fig. 9, B).

When the planula is viewed from the surface the transparent areas of the ectoderm are seen to be enclosed by the opaque tracts which spread round them: a condition more clearly marked in the case of the planulæ of *Cryptohelia.*

The endodermal mass of the planula is composed of much granular matter, in which are embedded numerous small transparent cells and nuclei, also oil-globules of various sizes, and many nematocysts in various stages of development (Plate 44, fig. 9, E N).

(4.) GENUS *SPINIPORA*, Gen. Nov. (H. N. M.)

Amongst the other Stylasteridæ obtained off the mouth of the La Plata in 600 fathoms, was a single specimen of a form to receive which I have made a new genus, *Spinipora.* It is closely allied to *Errina,* but shows sufficient differences in the structure both of the hard and soft tissues to warrant its being placed, at present at least, in a separate genus.

Corallum of Spinipora echinata.

The corallum (Plate 34, fig. 3) is in the form of a single irregularly cylindrical stem, bearing at its summit, in the only specimen procured, a couple of similarly shaped branches. The base of the stem is somewhat swollen, and encrusts the object to which it adheres. The whole surface of the corallum is thickly beset with spinous projections which being all inclined towards the tips of the branches stand out beyond the main surface of the stem to a distance of as much as one-tenth of an inch, the diameter of the stem itself being about three-tenths of an inch. The spines are spout-like in form, more or less conical, with the ends usually truncated, and their upper surfaces—that is, those turned towards the tips of the branches—channelled out into deep and wide grooves. The grooves usually commence on the surface of the spines as slits, and widen out to terminate at the truncate ends of the spines in wide spout-like mouths. The groove-like excavations are continued as tubular cavities for a short distance into the axes of the spines, beyond the slit-like commencement of the grooves. The grooves are the cavities which are occupied by dactylozooids—are, in fact, the dactylopores, which are here excavated within long projecting spines, and are widely open on one side for nearly their entire length. The small continuation of the groove within the axis of each spine represents the normal dactylopore.

Two kinds of dactylopores occur in the present form: the larger ones already described, and much smaller pores, which are mostly placed on the bases of the spinous processes, but occur also more sparingly on the general surface of the stem. These smaller pores often have the sides of their mouths slightly raised above the surface which they perforate.

The main surface of the stems and branches of the corallum is grooved by short canals, which are just open to the surface and run short courses, being never much branched and usually crooked (Plate 35, fig. 4). These channels correspond with those described as occurring in *Errina*, and are occupied in the recent condition of the coral by the most superficial reticulations of the cœnosarcal meshwork.

Lying in deep depressions between the bases of the spinous projections are the gastropores, which are deep pits with circular mouths, at the margins of which dactylopores of the smaller kind frequently open. The gastropores are provided with styles, which are very deeply situate and have brush-like tips, and are much like those of *Sporadopora*, but not so elaborately branched. The substance of the corallum of *Spinipora echinata* is hard and compact in structure, and white.

Soft Structures of Spinipora echinata. (Plate 38.)

Cœnosarc.—The cœnosarc consists of the usual reticulation of canals (Plate 38), offsets of which pass into and ramify within the dactylopore spines as at B, Plate 38. There is a well-developed continuous surface layer of ectoderm, which invests the spinous processes and entire surface of the coral, and feebly maintains, in decalcified specimens, the form of the corallum. The layer is, as in other genera of the family, continued into the pores of the corallum to form the sacs of the zooids. The nematocysts are closely similar to those of *Errina*.

Dactylozooids.—These are of two forms, larger and smaller. The larger dactylozooids are attached by elongate bases along nearly the whole lengths of the bottoms of the groove-like dactylopore cavities. The ends of these elongate bases nearest the coral stems assume a cylindrical form, and are continued into the pore-like prolongations of the grooves, to become continuous with canals of the cœnosarcal meshwork. In Plate 38, two dactylopore spines, B B, are shown as cut open in order to exhibit this arrangement. The pore-like continuations of the dactylopore grooves are lined by continuations of the surface layer representing the zooid sacs. The free parts of the dactylozooids spring from the elongate attached parts not far from the tips of the spines. In the contracted condition they appear as short, stout, bluntly conical bodies, which are slightly curved and bent inwards towards the coral stem, and at the same time directed towards its upper extremity. Since the larger dactylozooids were all found in the described condition in spirit specimens, it would appear that they are incapable of being retracted to a greater extent. The pores are certainly not deep enough to allow of their entire retraction within them, and the mode of attachment of the bases would not allow of such retraction. No doubt the zooids, when active and expanded, are long and attenuated, and the long spines on which they are borne are very possibly to be regarded as contrivances for giving them a long reach. A tendency to attachment by the side of the base, within the zooid pore, has been already noticed as occurring in the dactylozooids of *Sporadopora dichotoma.*[*] It is here the normal condition, and much more fully completed. A closely similar method of attachment and retraction of the dactylozooids occurs in all the genera of Stylasteridæ, which form regular cyclosystems of zooids.

The smaller dactylozooids are simple bluntly-conical bodies, of less than one-third the size of the larger form. They occupy the smaller dactylopores, and are retracted directly within these when at rest (Plate 38, D D).

Gastrozooids.—These are cylindrical in form, with a dome-like hypostome and six apparently simple conical tentacles, set on in a single whorl. The zooids are, as usual, retracted within sacs lining their pores. The tentacles in the retracted condition of the zooids are doubled together over their hypostomes, with their tips bent inwards and downwards towards them. The zooid bases terminate in four large canals of the cœnosarcal meshwork, and are firmly united to the styles of the pores.

Gonophores.—No generative elements were discovered in the single specimen of this coral obtained for examination.

(5.) GENUS *STYLASTER* (GRAY).

The genus *Stylaster*, which gives its name to the family Stylasteridæ, was established by Gray, in 1831, for the reception of *Stylaster roseus*, the old *Madrepora rosea* of Pallas, and *Oculina rosea* of Lamark and others. The species the structure of which is here to be described was obtained off the mouth of the La Plata. It appears to have been hitherto undescribed.

Corallum of Stylaster densicaulis, *sp. n.* (H. N. M.)

The corallum (Plate 34, figs. 5, 5A) is flabellate in form, with a very stout main stem and branches, which make with one another angles of from 25° to 30°. The main stem and larger branches are oval in section, the longer diameter of the ellipse being at right angles to the plane of the flabellum. The stem and branches give off numerous comparatively small and short ramifying branchlets from their lateral margins. Occasionally as an exception a branchlet is given off from one of the faces of the flabellum at right angles to it, thus distorting its fan-like form.

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[*] See p. 437.

The pores are arranged in regular symmetrical cyclo-systems, a circular group of dactylopores surrounding in each system a single centrally-placed gastropore. The pores of both kinds occur only arranged in these systems in this species.* The cyclo-systems so closely simulate in appearance the calicles of ordinary Hexactinian corals, that the genus *Stylaster* and its allies, such as *Allopora* and *Cryptohelia*, have hitherto been placed amongst the Oculinidæ. The cyclo-systems in the present species appear as small cylindrical masses of calcareous matter, which have a somewhat greater diameter at the free extremity than at the base. In the growth of the coral new systems bud off from the sides of the older cylinders, at the tips of the branchlets. The cylinders thus newly formed have their axes at right angles to those of the old systems to which these are attached, but in the same plane with them, which is also that of the entire flabellum. The branchlets of the corallum, already described as given off by the main stem and branches, are composed of zooid systems thus related to one another. In the more recently formed twigs the arrangement described is plainly apparent, and they have thus a zigzag appearance ; but in proportion as the branchlets are traced nearer and nearer to the stems from which they spring, this zigzag arrangement becomes more and more obliterated by deposit of cœnenchym, and in the older regions of the corallum, on the sides of the main branches and stem, the mouths only of the zooid systems remain unburied by the swollen dimensions of the support.

No pore systems occur on either of the flabellar faces of the stem or branches. Short branchlets, as well as single pore systems, are evidently swallowed up, to some extent, by the spread of cœnenchym and increase of the dimensions of the stem, and all stages of the process appear at the lateral margins of the stem near its base. But in order to secure an excessive strengthening of the stem, with the least amount of encroachment upon early-formed pore systems, the stem swells to the greater extent in the direction of its surfaces which correspond with the faces of the flabellum and bear no pore systems. Hence, as already described, it becomes oval in section, being flattened in a plane at right angles to that in which the younger branchlets are compressed.

The cyclo-systems are groups of zooid pores as already described, which have a regular symmetrical arrangement, a single gastropore in each system being surrounded by a circlet of dactylopores. The centrally placed gastropore in each system is a wide tubular cavity, with a circular transverse section. This pore is much deeper than its surrounding dactylopores, and has at its bottom a short stout style, with a brush-like conical tip (Plate 35, fig. 3, S). Just above the level of the top of the style is a circlet of small rough projections, which stand out from the wall of the gastropore, and contract its bore at this point.

* In another species of *Stylaster*, *S. granulata*, dredged off Ascension Island in 420 fathoms, small isolated dactylopores were observed to occur on the surface of the corallum, as a rare exception, apart from the pore systems. One such was observed situate on the side of a cylindrical cyclo-system, and two others at the margin of an ampullar prominence. These were very probably occupied, in the recent state of the coral, by small dactylozooids, the rudiments of those which, in an earlier stage of evolution of the Stylasteridæ, overspread the surface of the ancestral form, as in *Sporadopora*.

Around the mouth of the gastropore is a circlet of from about ten to fourteen dactylopores, arranged symmetrically at equal distances from one another and from the centre of the mouth of the gastropore. The mouths of these pores are elongated towards the axis of the gastropore, so as to open into, and become continuous with, the cavity of this latter pore (Plate 35, D Z, D Z). The openings of the dactylopores are continued down, as wide slits, for some distance on the upper part of the wall of the gastropore, so that the pores have, as it were, two mouths placed at right angles to one another and confluent with one another, the one opening to the exterior, the other into the cavity of the gastropore. The cavity of each dactylopore consists of a wide upper chamber in the region of the widely open mouth (Plate 35, T Z, T Z), and a narrow tubular continuation of this, which traverses the corallum in a direction parallel with that of the axis of the gastropore for about half the length of the latter. Against the outer wall of the pore is a small ridge-like excrescence, with an hirsute surface, which is the style of the dactylozooid (Plate 35, fig. 3, S′), and which is described by POURTALES as "a rudimentary septum in the shape of a hairy fringe" (POURTALES, *l.c.*, p. 34).

The dactylopores in each cyclo-system are separated from one another by thin plates of calcareous matter, which are directed inwards radially towards the axis of the gastropore (Plate 35, fig. 3, P), and which at first sight have all the appearance of the septa of hexactinian corals, and have hitherto been mistaken for such by observers. They are, however, composed each of two thin laminæ of dense calcareous matter, united by somewhat less compact calcareous substance, which is freely perforated by canals for the passage of offsets of the cœnosarcal meshwork. The thin laminæ are merely the juxtaposed walls of the adjacent dactylopores. These radially disposed plates, which may be termed pseudosepta, have their inner edges continued down the wall of the gastropore for a short distance beyond the margins of the mouths of the dactylopores, as well-marked vertical ridges, which soon become merged in the general surface in their course (Plate 35, fig. 3).

The cylindrical masses formed by each cyclo-system are sometimes flat, often gently rounded at the top. Their summits are irregularly circular in outline, but have an indented border, the indentations corresponding with the centres of the pseudosepta in position, and representing the intervals between the opposed dactylopore walls, which are here not obliterated by growth of cœnenchym.

The cyclo-systems, when viewed from above in a line looking directly into the mouths of the pores, show, in all essential particulars, the same structure as that which occurs in *Allopora profunda.* which is represented diagrammatically in Plate 35, fig. 13. The styles of the tenticular zooids, S S, appear as small projections in the interspaces between the pseudosepta, and were taken by POURTALES and others for septa of a second order.

The cyclo-systems have been described as circular in outline of summit, because this may be regarded as their normal condition ; but very many of them are distorted in various ways. One edge of the summit of the system is frequently elevated above the other, and this elevation is on the side of the same face of the flabellum in all the calicles; whilst the dactylopores, on the opposite margin of the system, are frequently more or less aborted. This condition forms a step towards that occurring in *Cryptohelia,* where all the cyclo-systems have their mouths turned towards one face of the flabellum. The cyclo-systems in the present species are also frequently elongated in a direction in the plane of the flabellum, and in the case of those systems which are placed at the sides of the main branches, parallel with the line of extension of these branches.

Besides being permeated completely by fine canals, the cœnenchym of the pore systems is excavated by numerous rather large lacunar cavities, especially near the base of the style and place of origin of an ampulla (Plate 35, fig. 3).

The ampullæ appear, on both faces of the branchlets, as conspicuous rounded prominences, set in groups, and often fused together into large papillated masses. They do not occur on the flabellar faces of the main stem or branches. They present internally a nearly spherical cavity, which communicates freely by openings with the canal systems of the cœnenchym (Plate 35, fig. 3).

Soft Structures of Stylaster densicaulis. (Plate 40.)

Cœnosarc.—The outer surface of the coral generally and of the cylindrical cyclo-systems, is invested by a continuous surface layer of cœnosarc (Plate 40). This layer dips down to line the dactylopores, and form the small tubulate sacs of the contained zooids, and also is reflected into the wide cavity of the gastropore, the inner lining of which is the greatly expanded sac of the gastrozooid, which zooid, deeply seated at the bottom of the sac, occupies a very small area of its space (Plate 40, A). Beneath the surface layer the cœnosarcal meshwork forms a fine reticulation of smaller canals, and a similar fine reticulation lies immediately beneath the lining membrane of the gastropore (Plate 40). In the walls of the cyclo-systems, between these two finer reticulations, a series of larger canals form an intermediately placed network, in which the branches have a general direction parallel to the axis of the gastropore, and form a direct communication between the bases of the dactylozooids and the large canals which spring from the bases of the gastrozooids. Offsets of this reticulation pass up into the canals in the interior of the pseudosepta. The three reticulations described are intimately connected together by abundant anastomoses. In Plate 40, B B, the interior of a zooid cyclo-system is represented with the sac of the gastropore and superficial lining network removed, in order to display the connexions of the deeper reticulation with the dactylozooids, and its general arrangement. The connexions of the reticulations with one another are well seen at the cut edges of the bisected zooid systems, as shown in the plate.

A tortuous and complicated mass of large canals springs from the bases of the gastrozooids at their margins, but not from their under surfaces. Some of these large canals turn almost immediately after springing from the gastrozooids upwards, through the wall of the zooid system, to join the main network already described as communicating with the dactylozooids. The remainder of the large canals form a tortuous reticulation, which passes down through the cœnenchym of the corallum, by the side of the immediately adjoining zooid system, to anastomose with the corresponding reticulation arising from the base of the gastrozooid of this latter. The walls of the ampullæ, as shown in the figure, are traversed by a fine reticulation of the cœnosarcal canals beneath their covering derived from the superficial layer of ectoderm.

Nematophores, composed of nematocysts of the usual larger form, are placed on the pseudosepta, between the dactylozooids (Plate 40, N N).

Zooids.—One form of dactylozooid and one of gastrozooid only is present.

Dactylozooids.—These, in the retracted condition, are short cylindrical bodies, with a rounded, blunt-conical, free extremity. They widen out towards their attached extremities, and are united to the sides of the dactylopores which are outermost in the systems, and to their styles, by elongate bases, which are drawn out below into narrow prolongations which join the cœnosarcal meshwork. The zooids are, in fact, attached in an almost precisely similar manner to that in which the dactylozooids of *Spinipora echinata* are fixed within their groove-like pores. The free cylindrical portions of the dactylozooids in the present species are bent upwards, so as to extend in the wide upper cavity of the dactylopore, in a direction parallel to that of the axis of the gastropore. They are seen thus projecting in the centrally placed zooid system represented on Plate 40, D Z, showing partly free above the inner margin of the dactylopore sac, partly seen through the transparent sac of the gastrozooid. A curved line, crossing them transversely, marks the point where the sac of the gastrozooid becomes bent over, and unites with that of the dactylozooid. The dactylozooid surfaces, as well as those of the tentacles of the gastrozooids, are thickly set with nematocysts of the usual smaller form.

Gastrozooids.—These are short and broad cylindrical bodies, somewhat contracted in diameter towards the middle of their length. They terminate above in a dome-like hypostome with the mouth opening at its apex, and are provided with a single whorl of light tentacles set on immediately below the hypostome. The tentacles are, in the contracted condition, very short and stout, with swollen, rounded knob-like extremities, which reach to a height only just exceeding that of the summit of the hypostome. At the margins of their bases the gastrozooids (Plate 40, G G) are drawn out into a series of large radially-disposed canals, which lead directly into the cavities of the zooids, and the further disposition of which has been already described. The immediate under surface of the gastrozooids is devoid of canal offsets, and is attached to the centrally placed style.

The histological structure of the zooids in the present species of *Stylaster* closely corresponds with that already described as existing in those of *Sporadopora dichotoma.*

The gastrozooids in the present form, and their tentacles, are so short that it seems improbable that these zooids are able to emerge from the summit of the gastropore in the expanded condition of the coral. The dactylozooids probably become, when active, long and filiform, and acting as tentacles bend inwards to supply the gastrozooid with food.

Gonophores.—Only male specimens of *Stylaster densicaulis* were obtained. Each male ampulla contains two or three ovoid gonophores, which are attached to large offsets of the cœnosarcal meshwork at one end of their longer axes. They have an internal spadix, and in finer structure seem to differ very little from those of *Sporadopora.* They are shown as seen through the transparent walls of the ampullar sacs in Plate 40, G G.

(6.) GENUS *ALLOPORA* (EHRENBERG).

To this genus I have referred a coral dredged off the mouth of the La Plata, on account of the very considerable irregularity with which the pore systems grow out from one another. The coral seems to represent a species hitherto undescribed, which I term *Allopora profunda.*

Corallum of Allopora profunda, *sp. n.* (H. N. M.)

The corallum (Plate 34, figs. 6, 6A) is composed of a stout stem, bearing numerous branches. The branches ramify to some extent in the same plane, so as to form a sort of flabellum; but this flabellum is curved considerably in the direction of its height, and its lateral margins are also bent over sharply towards the same curved face. The main stem has a sinuous course, and the branches are all more or less curved in direction. The stem and branches are oval in transverse section, being flattened in the plane of the flabellum. The corallum is white, and its surface is minutely granular. The pores occur in regular cyclo-systems; when young they project from the terminal branchlets in the form of small cylindrical masses, which are slightly expanded in diameter at the free extremity. These cyclo-systems show a tendency to a regular alternate arrangement, the base of each system abutting on the side of the preceding, and the axes of the systems being inclined to one another at an angle of about 45° in the general plane of the flabellum. This tendency is, however, to a large extent obscured by irregularities. One face of the flabellum, viz., that rendered convex by the curving of the whole mass, is entirely devoid of cyclo-systems; whilst a considerable number are scattered over the surface of the branches on the concave face. As the branches thicken by growth of cœnenchym, the cylindrical masses of the cyclo-systems become buried, and only their free ends remain in view, as the mouths of pore systems on the surfaces of the branches. Even these mouths become partially overgrown in the active regions of the coral, and in the older parts of the stem frequently obliterated. The cyclo-systems consist of a deep gastropore, provided with a style, and surrounded by from twelve to sixteen dactylopores. The dactylopores are provided with a small hirsute style, as in *Stylaster densicaulis.*

A diagrammatic view of a cyclo-system, as viewed from above the mouths of the pores, is given in Plate 35, fig. 13. The styles are supposed to be brought into view by deep focussing of the lens. The form and arrangement of the pores are almost exactly similar to that already described as occurring in *Stylaster*.

The very small ampullæ are spherical cavities, which are usually entirely sunk beneath the surface, but sometimes near enough to it in situation to raise upon it very small conical elevations, which easily escape notice, and are present only here and there. The ampullæ are present in abundance in the walls of the pore systems and at their bases.

Soft Structure of Allopora profunda.

Cœnosarc.—A surface layer of ectoderm covers the surface of the coral, as in *Stylaster densicaulis*, and is reflected into the pores to form the sacs of the zooids. The cœnosarcal canals form a fine superficial reticulation at the surface of the coral, beneath the surface layer, and spring from a deeper meshwork of larger canals which, as in the *Stylaster* already described, have a mainly longitudinal course within the thickness of the walls of the pore systems, parallel to the axes of the systems, and lead almost directly from the bases of the dactylozooids with which they anastomose to the large canal offsets given off at the periphery of the bases of the gastrozooids. At the inner surface of the gastropore are finer canals springing from this main meshwork, and from these spring a series of offsets which pass in a direction radial to the axis of the gastropore, to abut on and become united with the outer surface of the sac of the gastrozooid.

The radial offsets are disposed irregularly, at unequal distances from one another, and at all heights in the gastropore (Plate 39, R R). The inner ends of the radial offsets are often enlarged where they abut on the wall of the sac of the gastrozooid, and they are often forked at their outer extremities, where they spring from the cœnosarcal meshwork. They appear to be homologous with the radial canals already described as occurring in a similar connexion in *Sporadopora dichotoma* (Plate 43, fig. 3). In transverse sections of a zooid system, these radial offsets have much the appearance of mesenteries of an Anthozoan coral cut across, and in some sections they show a certain amount of regularity in disposition at the particular level selected for the cut (Plate 44, fig. 12).

These radial structures are here termed offsets, and not canals, because, although in some instances they appear to be similar in construction to the ordinary cœnosarcal canals, and usually show similar structure to these at their outer extremities, they usually consist towards their middles and inner extremities of simple bands of transparent fibrous tissue. The exact structure could not be determined, but it seems probable that these radial bands represent radially disposed offsets of the canal meshwork, which have become developed into fibrous organs with an elastic or muscular function, which is brought to bear on the zooid sac. In the case of the radial canals of *Sporadopora dichotoma*, distinct muscular elements were observed as forming part of their structure.

Zooids.

Dactylozooids.—The dactylozooids and their sacs in *Allopora profunda* are so closely similar in form and structure to those of *Stylaster densicaulis,* that they need no further description.

Gastrozooids.—The sacs of the gastrozooids in the present species differ from those in *Stylaster densicaulis* in being of smaller diameter in proportion to the dimensions of the pore cavities, and in being held in place by means of the radial offsets already described. It is possible that the wall of the sac of the gastrozooid lies nearer to the wall of the gastropore in the recent condition of the coral than is represented to be the case in Plate 39, G Z, and in Plate 44, fig. 12; but all the spirit specimens examined yielded a similar result when decalcified. The sac of the gastrozooid is, as usual, a reflection of the surface layer of the ectoderm. In the contracted condition it forms a long tubular cavity, somewhat narrowed in the region just above the gastrozooid and at the mouth, and dilated towards the centre. At its mouth, the marginal fold of the sac rises in the form of a flattened dome, somewhat above the level at which the openings of the sacs of the dactylozooids commence.

The gastrozooids are very deeply seated at the bottoms of their sacs. They are dome-like in form, with expanded bases. A whorl of tentacles, set on at some distance from the summit of the dome, marks the commencement of the large rounded hypostome. The tentacles are twelve in number, and are set on in a single whorl. They are elongate-ovoid in form. From the base of the zooid a series of radially disposed large canals pass outwards to be distributed, as in *Stylaster densicaulis,* a certain number of their branches forming a tortuous meshwork, offsets from which pass to join those of the gastrozooid of the adjacent system.

The structure of the zooid clyco-systems is clearly displayed in Plate 44, fig. 12, which is, in some respects, somewhat diagrammatic, but is taken from an actual transverse section. The section is taken above the level of the gastrozooid, which hence does not appear. The sac of the gastrozooid is, however, seen in section, together with its radial supports. The dactylozooids seen in section show the three layers of tissue of which they are composed, viz., ectoderm, basement membrane and muscular layer, and endoderm; and sections of their styles are introduced to show the position of these. The two finer reticulations of the cœnosarcal meshwork, inner and outer, are also well seen, with the larger canals in the interval between them, which at this height in the wall of the system are confined to the interspaces between the dactylozooids.

Gonophores.—Male examples only were obtained of the present species. The ampullæ are covered by the surface layer of the ectoderm, and the superficial reticulations of the cœnosarcal meshwork. Within, they contain a sac (Plate 39, G) in which are developed two or three gonophores of an ovoid form, which are attached to offsets of the cœnosarcal canals, and which show the usual elements characteristic of the various stages in the development of spermatozoa in the family, which elements are massed around a spadix, as in *Sporadopora*. The process of development was not closely followed in the present species.

(7.) GENUS *ASTYLUS*, Gen. Nov. (H. N. M.).

I have formed the above genus for a Stylasterid with regular cyclo-systems, dredged off the Meangis Islands in 500 fathoms. The coral is, like *Cryptohelia*, devoid of styles in both kinds of zooids, and differs in structure in no important particular from this genus, with the exception that it has no solid lid-like covering overhanging the mouths of the zooid pores. It possesses, however, a curious tongue-like process deeply seated in the calicle, which probably is the homologue of this lid.

Corallum of Astylus subviridis.*

The corallum (Plate 34, fig. 4) consists of a short stem, which breaks up into a few primary branches. These, with their slender secondary branches and branchlets, which are very few in number, ramify in the same plane, and form a small flabellum. The stem and branches are circular in transverse section throughout their length, except where distorted by the presence of zooid cyclo-systems upon them. They are composed of a hard and compact pearly-white calcareous tissue, the outer surface of which is marked by a series of conspicuous fine rounded ridges, which separated by shallow grooves follow the directions of the stem and branches with parallel course, each ridge preserving its integrity for a long distance, except on the pore-bearing face of the flabellum, where the ridges are interrupted by the prominent cyclo-systems. The branches are somewhat swollen at the points where cyclo-systems are attached to them. The whole corallum is, as in other Stylasteridæ, permeated by networks of canals. The axes of the branches are traversed by bundles of large main canals, which place the cyclo-systems in relation with one another.

The cyclo-systems are all, with one exception, which is evidently abnormal in the present specimen, placed on one face of the flabellum, with their axes at right angles to its plane. The systems appear as globose bodies, with flattened tops, which are much wider in diameter than the branches on which they rest, and stand out prominent and entirely free from one another, at regular intervals along the course of the branches. The globose appearance of the systems is due to their being each encircled by a broad prominent zone of confluent ampullæ, which zone has a rounded surface, rendered somewhat irregular by the occasional prominence of individual ampullæ. Immediately above this zone, the edge of the summit of each system appears as a delicate lamina, which slightly overhangs the outer wall of the system all around (Plate 35, fig. 8). The summits of the systems are circular in outline, with a series of

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* The Hydroid here named *Astylus subviridis* was referred to in my abstract paper on the "Structure of the Stylasteridæ" (Proc. Roy. Soc., 1876, p. 95), as "a *Stylaster* resembling *Cryptohelia*."

indentations in the marginal lamina, as in *Allopora profunda*, corresponding with the centres of the outer ends of the pseudosepta. A diagrammatic view of a cyclo-system, as viewed from above the mouths of the pores, is given on Plate 35, fig. 15.

The arrangement of the pores in the systems is closely similar to that in *Stylaster densicaulis* and *Allopora profunda*. There is a centrally-placed gastropore in each, which is surrounded by a ring of dactylopores with slit-like mouths. The gastropore in the present genus, however, appears in the form of two chambers, an upper and a lower, which communicate with one another by a constricted aperture. The upper chamber (Plate 35, fig. 8, G P) communicates with the exterior superiorly by a short tubular passage, bounded by the inner ends of the pseudosepta. The walls of the chamber are curved, so that, taken in conjunction with its upper prolongation, it is flask-shaped. At the base of the chamber its walls are curved inwards, so as to bound a horseshoe-shaped aperture, which leads to the lower gastropore chamber beneath. The aperture is rendered horseshoe-shaped by the projection from its margin on one side of a tongue-like process of calcareous matter, which is directed horizontally, with a slight upward curve across the aperture, reaching as far as its centre (Plate 35, fig. 8, B; fig. 15, A).

The tongue-like process is a solid calcareous structure of a bent conical form, with a rounded extremity. It is grooved on its under surface in the direction of its length, and springs from the margin of the wall of the upper chamber of the gastropore, which is thickened in this region by its stout roots. The process always points in a uniform direction, viz., in that of the length of the branch on which it is situate towards the tip of the branch. It thus has a similar direction to that of the lids of the cyclo-systems in *Cryptohelia pudica*. In this latter genus, a stout process of calcareous matter, prolonged from the support of the lid, forms a prominent ridge on the wall of the upper chamber of the gastropore in an homologous situation (Plate 35, fig. 7). It seems probable, therefore, that this tongue-like process in *Astylus* represents either a rudiment of a lid like that of *Cryptohelia*, which in an ancestral form protected the mouths of the whole of the zooids of each system, but is in *Astylus* withdrawn deep into the central cavity of the system, so as to protect the gastrozooid only; or that the reverse is the case, and that the condition in *Cryptohelia* represents a further development of that seen in commencement in *Astylus*.

The separation of the gastropore into two chambers by a constriction is already foreshadowed in *Stylaster densicaulis*, as has been described, by the circlet of excrescences which there form a prominent zone in the gastropore above the level of the tip of the style (Plate 35, fig. 3, A).

The wall of the upper chamber of the gastropore in *Astylus subviridis* terminates below in a thin margin, and behind the wall a cavity, continuous with that of the lower chamber of the pore, runs up to communicate by offsets with the tubular portion of the dactylopores. This cavity, in the recent condition of the coral, lodges the main upward-directed canal offsets of the gastrozooid.

The lower chamber of the gastropore is a cavity with a rounded bottom, which is excavated within the substance of the branch supporting the pore system. The cavity communicates with the upper chamber by the horseshoe-shaped opening, and with the dactylopore as already described. With adjacent cyclo-systems it communicates by means of the axial canals of the branches. There is no trace of a style at the bottom of the gastropore.

Around the mouth of the gastropores the mouths of the dactylopores appear as elongate slit-like openings, radially directed towards the axes of the systems. The outer peripherally-placed margins of these slits are rounded, whilst internally the slits join the cavity of the gastropore. The pseudosepta intervening between the dactylopores are, in origin, double laminæ, as in *Stylaster densicaulis*, but in the present form appear as thin plates, which have so regular a radial arrangement and so wide an extent that they simulate the septa of Hexætrinian corals more closely than do those of any other Stylasterid.

The inner extremities of the summit borders of the pseudosepta by their arrangement form a circular aperture leading to the cavity of the gastropore. There are from eighteen to twenty-one dactylopores in each cyclo-system. The upper wide slit-like chambers of the dactylopores are continued into small short tubular cavities below, as in *Stylaster densicaulis;* but these are entirely devoid of a style. The mouths of these tubular cavities are set in a circle, at the bottoms of the interspaces between the pseudosepta, at points about equidistant between the inner extremities of the pseudosepta and the outer margins of the chambers which they enclose (Plate 35, fig. 15).

The ampullæ are confined to the zones around the pore systems, and do not occur on the branches. Their cavities are usually kidney-shaped.

Soft Structures of Astylus subviridis.

The general arrangement of the soft structures is represented on Plate 41, fig. 1.

Cœnosarc.—The usual surface layer is present, which is continuous with the sacs of the zooids. A fine superficial reticulation of smaller cœnosarcal canals (Plate 41, fig. 1, S, S) extends over the surfaces of the branches and ampullæ, and coral generally, beneath the surface layer. The axes of the branches are occupied by meshworks of large canals, which lead from one cyclo-system to another, and place the whole of the systems in free communication with one another.

Large canals are given off from the periphery of the gastrozooids. Some of these communicate directly with the axial meshwork of canals, whilst another set passes upwards in the wall of each cyclo-system to join, after a certain small amount of ramification and anastomosis, the basis of the dactylozooid. From the surface of the meshwork of these latter canals, which adjoins the dactylopore cavity, a few transverse smaller canals are given off, which pass inwards radially to be attached to the wall of the pore-sac, and represent the more fully-developed "radial offsets," already described as occurring in *Allopora profunda* (Plate 41, fig. 1, R).

The ampullar sacs are embedded in a meshwork of offsets of the larger canals, and each of the gonophores is attached to one or more stout canal branches.

Stout offsets of the deeper canal meshwork traverse the interior of the pseudoseptal laminæ, and especially near the summits of the pseudosepta large tortuous branches pass radially outwards between the dactylozooid sacs, and, branching at their outer extremities, join the surface network at the margins of the cyclo-systems (Plate 44, fig. 2). Just over the outer extremities of each of the pseudosepta, at the margin of the top of each cyclo-system, and in the angles between the outer margins of the dactylopores, are situate ovoid nematophores. A single nematophore is placed in each above-described position. The nematophores are ovoid sacs, closely packed with about three tiers of nematocysts of the larger form, placed with their longer axes parallel to those of the containing sacs (Plate 44, fig. 2, N).

The endoderm of the soft parts in the present form were observed to have, in the fresh condition, a dusky green colour. The pigment is soluble in alcohol, and yields a green solution, which produces a well-marked absorption-band in the spectrum when examined spectroscopically. The position of this band was, however, unfortunately not determined.

Zooids.

Dactylozooids.—The dactylozooids have bases of closely similar form to those of the dactylozooids in *Stylaster densicaulis* and *Allopora profunda*, and are attached in the same manner within their sacs with the exception that they have no styles. The free portions of the zooids differ, however, from those in the species just mentioned, in that they are in the contracted condition, longer, more slender, and more gently tapered towards the extremities. Moreover, instead of being retracted within their sacs in a vertical position, *i.e.*, one parallel to the length of the sac, they are in the present form placed out of harm's way by being doubled down within the mouth of the sac of the gastrozooid (Plate 41, fig. 1, D Z, D Z).

Gastrozooids.—The sac of each gastrozooid is narrowed at the horseshoe-shaped opening, already described as leading, in the corallum, from the upper chamber of the gastropore, to the lower chamber in which the gastrozooid lies. The sac is reflected over the surface of the tongue-like process, and passing into the lower chamber, becomes attached to the zooid near the margin of its base. The tongue-like process projects in front of the mouth of the zooid, and must prevent the protrusion of the zooid, except in a crooked direction.

The gastrozooid itself is basin-shaped below, with a cylindrical mass above, the bottom of which gradually expands to join the margin of the basin. The cylindrical upper portion has a flat top perforated by the mouth, which is in the form of a crucial slit, and is abutted on by regularly disposed elongate gastric cells of the endoderm. Numerous large canals are given off from the periphery of the lower basin-shaped portion of the zooid, but none from the under surface of the basin. The disposition of these canals has already been described. The gastrozooids are devoid of tentacles.

Gonophores.—Only one specimen of the present form was obtained and it was of the male sex. The male gonophores appear as large rounded lobulated masses resting within the ampular sacs, and springing from stout offsets of the cœnosarcal meshwork, which pass into the sacs to reach them. Usually two tiers of ampullar sacs encircle each cyclo-system, being contained in the zone of ampullæ described in the account of the corallum.

The minute structure of the lobulated masses is shown in Plate 43, fig. 10. A membranous sac derived from the ectoderm and containing abundant nuclei in its tissue, S, lines the ampullar cavity and encloses the generative lobules. One or two large offsets of the canals of the cœnosarcal meshwork penetrate this sac, and with the ends of these the central mass of the generative structures is continuous. This central mass is composed of spherical nucleated cells filled with granules, and closely similar in appearance to the endoderm cells which line the cœnosarcal canals; and apparently the endodermal lining cells of the canals, from which the mass springs, are continuous with those composing its substance. All over the surface of this central mass of cells, which is invested with a thin layer of ectoderm, small globose sacs arise as buds, and gradually increase in size until they assume the form of the ovoid masses, which, being thickly set over the surface of the central mass, and hiding it from view, give to the active generative mass the lobulated appearance figured in Plate 41, fig. 1, G.

The young lobules when first formed appear as small rounded sacs with a thin wall of ectoderm, and containing a very few cells apparently derived from the main central mass. These cells become multiplied in number as the sac increases in size with progressive development. The sac as it enlarges becomes gradually pedicellate, and when mature is attached to the central mass by a narrow pedicle of some length. The walls of the pedicle are continuous with the ectodermal wall of the sac, which wall contains well-defined nuclei in its substance. Within the sac of the lobule a second sac, composed of a finer membrane, encloses the mature or developing generative elements. The wall of this inner sac is not prolonged into the cavity of the pedicle, but passing across its commencement shuts off the main cavity of the lobule from this latter.

The cells contained within the young lobule maintain a closely similar appearance to ordinary pigmented endoderm cells, until they have become multiplied into a large mass. On further increase they change their structure and appear as spherical perfectly transparent masses, each of which contains a large nucleus which becomes most intensely stained when treated with carmine. These transparent nucleated cells, which are closely similar in appearance to those figured by ALLMAN from the male gonophores of *Laomedea flexuosa*,[*] multiply further by division, becoming very minute but retaining the same structure (Plate 43, fig. 10, C).

From each of these minute cells a spermatozoon is developed. The head of the

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[*] ALLMAN, "Gymnoblastic or Tubularian Hydroids." Ray Soc., 1871, part 1, p. 65, fig. 316.

spermatozoon appears to be developed out of the nucleus of the cell, which as the process proceeds becomes first attached to the wall of the cell on one side, and is then gradually drawn out in the form of a curved elongate mass along the wall of the cell until it assumes the form of the head of the spermatozoon, being curled round within the cell nearly into a circle. The various stages in development are shown on Plate 43, fig. 11.

The mature spermatozoa were not observed in the fresh condition. Their appearance as seen in specimens hardened in alcohol is shown on Plate 43, fig. 11, *g*. They form closely felted masses within the ripe lobules, which masses do not entirely fill the cavities of the inner sacs of the lobules.

In the cavities of the pedicles of the more mature lobules a tissue containing a few transparent rounded cells was seen to be present. This may represent a spadix. No rounded spadix such as that occurring in *Allopora* is present in the interior of the lobules. The histological details were preserved with very great completeness in the present form when hardened in spirit and decalcified, so much so that Plate 43, fig. 10, might almost have been executed with a camera lucida from a fine section of a gonophore stained with carmine. It is, however, impossible to determine without close study of fresh material so difficult a problem as the determination whether the male elements are derived from the ectoderm or endoderm. The apparent development from endoderm cells, in the present instance, may be entirely misleading; the presence of hard skeletons in the Stylasteridæ unfits them for research on such points. It seems improbable that they differ in this respect from other Hydroids. E. VAN BENEDEN[*] has shown that in *Hydractinia* the spermatozoa are developed from the ectoderm, and G. v. KOCH[†] has observed the same fact in two species of Tubularia.[‡]

(8.) GENUS *CRYPTOHELIA* (M. ED. and H.).

A deep sea coral, dredged in many parts of the world by the 'Challenger,' is referable to the above genus, and although the specimens vary a great deal, seems not distinct from MILNE EDWARDS' and HAIME'S species, *C. pudica*. The specimens, the anatomy of which is here described, were dredged off the mouth of the La Plata.

Corallum of Cryptohelia pudica.

The corallum is well figured by MILNE EDWARDS and HAIME,[§] and described[||] by these authors as having the form of a small espalier tree, with all the branches comprised in the same vertical plane, and all the calicles turned to the same side.

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[*] Recherches faites au laboratoire d'embryologie de l'université de Liege. Vol. 1, 1876, p. 2.

[†] Mittheilungen über Cælenteraten. Morphol. Jahrbuch. Bol. II., 1876, s. 84.

[‡] Note added Sept. 24, 1878. The matter, however, seems variable or as yet undetermined.

[§] MILNE EDWARDS et J. HAIME. Ann. des Sci. Nat., 3^{e} sér., t. xiii., plate 3, fig. 1, 1850.

[||] Hist. Nat. des Coralliaires, Paris, 1857, t. ii., p. 127.

As far as the form and arrangement of the branches is concerned, the corallum of *Cryptohelia* differs in no important particular from that of *Astylus subviridis* which has just been described. The striæ on the surface of the branches are in the present form finer and run for shorter courses than in *Astylus subviridis*, and well marked prominent ridges are not formed between them.

Regular cyclo-systems are present in *Cryptohelia* which are all turned towards one face of the flabellum. Their mouths are not elevated above and isolated from the surfaces of the branches as in *Astylus*, but the branches swell vertically as well as horizontally where cyclo-systems are present, and the cœnenchym of the branch thus rises in a gradual curve to the level of the margin of each cyclo-system (Plate 35, fig. 7). There are from about fifteen to twenty-two dactylopores in each cyclo-system, in form and arrangement almost identical with those of *Astylus subviridis*.

The margin of one side of each cyclo-system is raised up into a stout projection, which is inclined slightly over the mouth of the system for a short distance. After running this inclined course the projection spreads out into a thin broad lamina, with a rounded border, which extends horizontally over the mouth of the cyclo-system and hangs as a lid or cover over its entire extent (Plate 35, fig. 7). The inclined portion of the projection is stout and thickened, and is strengthened by being continuous at its base with the adjacent cœnenchym of the branch. It is thickened to the greatest extent in the direction towards the centre of the cyclo-system, and so much so that its substance projects within the cavity of the upper chamber of the gastropore as a prominent ridge. This ridge becoming gradually less marked as it descends, is continued downwards to the margin of the aperture leading from the upper to the lower chamber of the gastropore, and appears as a prominent thickening of the wall of the upper chamber in this region (Plate 35, fig. 7, A). The dactylopores are aborted and absent in the region of the cyclo-system overgrown by the base of the lid. The lateral margins of this base are often grooved by dactylopores on either side, which have the appearance of having been pushed aside, as it were, by the growth of the projection. The thin horizontal lamina constituting the lid of the cyclo-system is often not quite smooth in surface but somewhat undulate or crumpled as it were. The lids are all directed with great regularity towards the tips of the branches on which the cyclo-systems to which they belong rest; the supports of the lids arising from the sides of the cyclo-systems nearest the origins of the branches.

The gastropores are divided into two chambers as in *Astylus*. In the present form the lower chamber is relatively smaller than in *Astylus subviridis*. Its communications with the dactylozooids are closely similar to those in *Astylus* (Plate 35, fig. 7). The opening between the two chambers in *Cryptohelia* is circular, not horseshoe-shaped as in *Astylus*.

Ampullæ occur only in connexion with the cyclo-systems in *Cryptohelia pudica*. In the female specimens examined by me, only one ampulla is developed in connexion with each system. It may lie on either side of the system, but not on the back of the flabellum. The ampullæ are rounded cavities of irregular form, which, when mature, are so large as to be as wide as the side of a cyclo-system, and occupy it entirely (Plate 42).

Numerous specimens of *Cryptohelia pudica*, the soft parts of which are not preserved, have several ampullæ developed in connexion with each cyclo-system. These are concluded to be male examples. The ampullæ are not nearly so large as in female specimens, and do not give evidence of their presence by forming swellings on the surface of the corallum.

Soft Structures of Cryptohelia pudica. (Plate 42.)

Cœnosarc.—This differs in structure in no essential particulars from that of *Astylus subviridis*. Similar axial canal systems are present in the branches and similar surface networks, but these latter are finer and more complicated in the present form than in *Astylus*. The lid and its support consists of a reflection of the surface layer of ectoderm, beneath which is a prolongation of the surface network of the cœnosarcal canals, and in the thicker portion of the stem of the lid run abundance of prolongations of the deeper and larger vessels (Plate 42, L). Rounded nematophore sacs, closely similar to those of *Astylus subviridis*, are dotted about over the upper surface of the lid, and, as in *Astylus*, a single one of these bodies is placed at the margins of each cyclo-system over the outer extremity of each pseudoseptum (Plate 42, N, N). The pigmented endoderm cells are coloured brick-red as in *Sporadopora dichotoma*.

Zooids.—The zooids of both kinds most closely resemble those of *Astylus subviridis*.

Dactylozooids.—These are elongate-conical in form, tapering to a point. Their bases are attached as in *Astylus subviridis*. In retraction the part of them nearest the base is doubled back in the wide slit-like chamber of the dactylozooid towards the periphery of the cyclo-system, and then the remainder of the zooid is bent over in the reverse direction and doubled down into the mouth of the gastrozooid (Plate 42, D, Z).

Gastrozooids.—These are flask-shaped and closely similar in structure to those of *Astylus subviridis*, being, like it, devoid of tentacles (Plate 44, fig. 1). The mouth appears at the flat summit of the neck of the flask as a cruciform aperture. The cavity of the zooid is lined with the usual gastric endodermal cells of elongate form, and the layer formed by these cells becomes, as in the gastrozooid of *Sporadopora dichotoma*, thinner as the base of the zooid cavity is approached. There is a thick investing ectoderm layer in the upper part of the zooid, between which and the endoderm layer is a well-marked layer of longitudinal muscular fibres, which fibres are, as in *Sporadopora*, continued for insertion along the main canal offsets of the base of the zooid (Plate 42, M).

The gastrozooid sac is attached just beneath the origin of the neck of its flask-shaped mass. The main canal offsets spring from the periphery of the rounded base of the zooid, with a radiating disposition (Plate 44, fig. 1). They curve upwards to be distributed, as in *Astylus subviridis*. The calcareous wall of the upper chamber of the gastropore lies in the interval between these canals and the outer surface of the sac of the zooid. No canals spring from the under surface of the zooid.

Gonophores.—A fragment of a male specimen of *Cryptohelia*, obtained off the Japanese coast, was examined in a cursory manner, and it was seen that, at first sight at least, it resembled *Astylus subviridis* in the structure of its gonophores. Unfortunately it was mislaid, and I have been unable to find it again.

All other specimens of *Cryptohelia* available for the examination of the soft structures proved to be female only. The ampullæ in *Cryptohelia* are occupied by thin walled sacs. Those in connexion with newly-formed cyclo-systems at the tips of the branches of the corallum are small, and contain only a few gonophores in early stages (Plate 42, G); but those attached to older systems are often of enormous relative dimensions, and appear as long reniform bodies (Plate 42, G′), which are almost as large as the masses of the cyclo-systems themselves in volume, and contain gonophores in all stages from the very earliest upwards, and one or two mature planulæ.

The early stages in the development of the ovum of *Cryptohelia* were examined in the fresh condition of the soft parts, without decalcification or use of spirit, the ampullæ being broken open and the gonophores removed from the freshly-dredged coral.

The earliest stage in the formation of a female gonophore observed is the massing together of a small quantity of the endoderm cells of one of the canals of the cœnosarc, which enter the gonophore sac (Plate 44, fig. 3). In the next stage observed, a cup-shaped spadix of endoderm cells is fully formed, the cup being attached to the cœnosarcal canals by a pedicle. In the hollow of the cup rests a fully formed ovum, with a well-defined germinal vesicle and spot, its main mass being composed of fine rounded particles. Only a single ovum is developed in relation with each spadix. A thin reflection of the ectodermal investment of the spadix covers the ovum within its cup (Plate 44, fig. 4, E).

The ova must be in some manner impregnated within the gonophore sac. As development proceeds the ovum increases in size, and the germinal vesicle and spot disappear, and the ovum appears entirely composed of thickly-set oily globules. At the same time the margin of the cup of the spadix, which increases in dimensions in accordance with the ovum, becomes divided into a series of small rounded lobes, about twelve in number, which embrace the lower part of the ovum. The cells composing the spadix and its lobes being coloured dark chocolate, the contained colourless ovum contrasts strongly with its support in appearance in the fresh condition of the structures.

The ovum, as it enlarges, becomes gradually drawn out into an ovoid form (Plate 42, G). On further development the margin of the growing spadix becomes fringe-like in appearance, the lobes composing it lengthening and becoming forked at their extremities (Plate 42, S P). The ovum in this stage is much dilated, and drawn out into an elongate ovoid form. Its contents are nearly transparent and highly refractive, but dotted all through their mass with sparsely-scattered oil globules of various sizes (Plate 42, OV). The reflection of the ectoderm at this stage still covers the ovum within its cup. A space is enclosed all round the gonophore by this ectodermal membrane, between the margin of the spadix and the ovum. This is filled by a perfectly transparent fluid.

In the next stage observed (Plate 42, SP′, OV′) the spadix is still further complicated at its margin by subdivision of its lobes, which form a sort of network over one-half of the surface of the ovum, terminating in a fringe of numerous tentacular-like lobes. The ovum is a large ovoid mass, composed of fine rounded particles densely packed together.

In the next stage observed, the developing ovum has already assumed the elongate-cylindrical form of a planula (Plate 42, P1). The stages by which the planula breaks its connexion with the spadix were not traced. The earliest planulæ observed appeared to be composed entirely of a uniform mass of fine rounded particles, like those constituting the substance of the latest stage seen in relation with the spadix. The formation of the ectoderm appears to take place by delamination. As the planula developes it becomes much elongated, and an outer layer becomes gradually more and more plainly observable on its surface as distinct from a general mass beneath it (Plate 42, P 2). The early-formed ectoderm layer (Plate 44, fig. 5) is composed of closely set, very fine rounded particles; whilst the inner mass, or endoderm, is made up of larger transparent oil globules. As development proceeds, the ectoderm layer thickens and becomes highly transparent, and being colourless contrasts with the more opaque red-pigmented endoderm within.

The mature planula measures nearly a quarter of an inch in actual length, and is so long that it has to be doubled up in order to allow of its accommodation within the gonophore sac. The planula has a thick, highly transparent, gelatinous-looking ectoderm, and a darkly pigmented endoderm. It is long and worm-like in form (Plate 42, P3). The surface of the ectoderm is marked out into polygonal areas, which are defined on the surface of the planula at an early period of development (Plate 44, fig. 6).

A vertical section of the ectoderm of the mature planula (Plate 44, fig. 7) shows that this thick layer is composed of a transparent gelatinous-looking mass, which is traversed by tracts of small rounded non-transparent elements, which stretch vertically to the surface of the planula, from the surface of the endoderm to that of the ectoderm. These tracts are continued outwards from a layer of similar elements, which rests at the base of the ectoderm, directly upon the surface of the endoderm. The opaque tracts are disposed at roughly regular intervals, and form vertical layers which, rising to the surface of the ectoderm and meeting one another, enclose the polygonal areas already described. In these tracts, apparently out of the opaque elements composing them, numerous thread cells of the larger kind are developed, and are more abundant and thickly set towards the surface of the planula; hence, when the lines enclosing the polygonal areas are viewed from the surface of the planula, they appear mainly composed of rows of nematocysts viewed end on, but partly also of the already described opaque rounded bodies (Plate 44, fig. 8). Some of the nematocysts contained in the ectoderm of the mature planula were observed to have their contained threads fully developed.

Planulæ were not examined in the fresh condition, hence the ciliation of their surface doubtless occurring was not observed. In the most mature planula investigated the endoderm consisted of pigmented cells, like those of the endoderm of the mature coral, but evidently in a condition of rapid increase, and of oil globules of various sizes and fine granular matter. The endoderm mass did not show any trace of a central cavity, but appeared homogeneous and solid. The gonophore sacs seem to be permanent in *Cryptohelia pudica*, and the production of planulæ within them to be carried on as a continuous process.

Growth by Budding.—In ordinary growth of the coral by budding, every part of the coral surface would appear capable of producing complete cyclo-systems, for in one specimen procured a new cyclo-system has been abnormally produced as a bud from the upper surface of the lid of an older cyclo-system.

General Remarks.

Summaries of the characteristics of the Hydrocorallinæ and their subdivisons will be given in the sequel under the heading "Classification." A few further points are required to be noted here. The Hydroid affinities of the Stylasteridæ need no discussion; they are borne out by every item of structure.

As in almost all Hydroids, the sexes are on distinct stocks, and these stocks, like those of Sertularins, have a tendency to grow in a flabellate form with alternate gemmation. In having the number of the tentacles borne by the gastrozooids regular in number in each species, possibly in each genus, the Stylasteridæ differ from the Milleporidæ, in which the number is variable. The connexion of an absence of the styles in the gastropores with a flask-shaped form of gastrozooid devoid of tentacles is remarkable. It occurs in apparently otherwise widely separated genera, *Astylus* and *Pliobothrus*. It is possible that the tentacles of the gastrozooids in all the genera would show traces at least of having knob-like or club-shaped ends were they able to be examined in the fresh condition.

The gonophore sacs within the ampullæ, as containing several distinct gonophores, in several genera at least, seem entitled to the term "gonangia," according to Allman's terminology. It seems uncertain whether the central mass in *Astylus*, from which the sperm-developing lobules are budded off, is to be considered as a blastostyle or not; no definite spadices were observed within these lobules.

The radiate arrangement of the cœnosarcal canals around the sacs of the zooids, which is so remarkably developed in *Sporadopora* and *Allopora*, and traces of which appear in nearly all the genera, is very remarkable. It gives the soft structures of *Allopora* at first sight, a still closer resemblance in arrangement to that occurring in *Anthozoans*, than does the very curious simulation which exists in its corallum. The resemblance is, however, in both instances merely superficial, and of no genetic significance.

The branched and fringed processes of endoderm described as embracing the embryos in *Errina* and *Stylaster* appear to correspond with the similarly branched structures in *Cordylophora lacastris*, described and figured by ALLMAN and F. E. SCHULTZE.* I have described them as outgrowths of the spadix, but possibly the cup-shaped endodermal structure supporting the ova should not be so designated.

The endoderm of the Stylasteridæ is always coloured, and seems most frequently to assume various shades of red or violet colouration, but in *Astylus subviridis* it is green. The corallum itself is in some species coloured, especially, it would appear, in *Distichopora*, but no doubt in many instances the colouration ascribed to the calcareous structures is in reality due to endoderm dried up within the interstices of the corallum.

In a former paper I conjectured that possibly shallow water Stylasteridæ might bear free gonophores, and perhaps medusiform ones, and that the occlusion of the gonophores within calcareous structures, and their adelocodonic condition, was due to the fact that the forms examined lived in the deep sea. This suggestion was in accordance with the observations of ALLMAN, who has found fixed sporosacs in all deep-sea Hydroids examined by him.† I find, however, from specimens sent me by Count de POURTALES, that ampullæ are especially well developed on the shallow water *Stylaster roseus*; those in the female stocks being very large and prominent. There can, therefore, be little doubt that these structures occur throughout the family.

In all the Stylasteridæ in which the gastropores have styles, the gastrozooids must be protusible in the expanded condition to a very slight extent. And the fact that in some genera the gastrozooids lose their tentacles seems to bear out this supposition. No doubt in active life the dactylozooids extend like long and filiform tentacles and catch and convey food to the gastrozooid, which nourishes them in return by means of its basal canals and the general circulation. It is to be noted that in those genera in which the gastrozooids have no tentacles, tentacles are wanting in the entire stock.

The nariform and tubular projections of *Errina* are no doubt contrivances for extending the reach of the dactylozooids, whilst at the same time protecting them. In *Acanthopora* the bases of the dactylozooids are pushed out to a remarkable distance from the gastropore mouths, and subsidiary dactylozooids of a smaller kind seem to be necessary to ensure the conveyance of the food to the gastrozooid.

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* F. E. SCHULTZE, Ueber den Bau und die Entwicklung von *Cordylophora lacastris*. Leipzig: W. ENGELMANN, 1877, p. 34, plates 3, 4.

† ALLMAN, "Gymnoblastic or Tubularian Hydroids," Vol. II., p. 155. *Nature*, Oct. 28, 1875, p. 556.

G. O. Sars, who is the only naturalist who has observed a Stylasterid alive, never saw the zooids raise themselves above the levels of the mouths of their cyclo-systems.

In the building up of the corallum, which must be deposited, as in *Millepora*, by the ectodermal covering of the cœnosarcal canals, absorption of already formed hard structures must take place during the gradual increase in size of the ampullæ and the widening of canals, which, as shown in the figures, are larger in bore in the deeper than in the younger superficial regions of the corallum. A re-deposit must also take place constantly, for old ampullæ, in the deeper parts of the coralla, are to be found in all stages of obliteration. Sometimes in some genera a rejuvenescence of parts of the corallum takes place; a previously dead area becoming overgrown from its margins by a living lamina, which spreads over and covers it.

Parasites.

The coralla of nearly all Stylasteridæ are liable to become much distorted in growth by the presence upon them of parasites of various kinds, each of which appears by the special kind of irritation which it offers to produce a particular form of abnormal growth in the part of the corallum it infests, producing thus, as it were, an animal gall. The commonest distortion is the reduction of the stem of a coral or branch, or of one side of these, into a hollow canal or deep furrow, more or less roofed over by a thin wall. This condition is produced by the adherence to the growing stem of an *Aphroditacean Annelid.* It has been noticed and described by Count de Pourtales * and Verrill, in *Stylaster erubecsens* and *Allopora Californica.* I have seen it in *Cryptohelia*, *Stylaster*, *Allopora*, and *Errina.* On *Errina labiata*, a parasitic filiform Nemertean also occurs which twines itself round the tips of the branches in many coils. The branches thus irritated grow out into a burr-like mass of projecting points which are evidently hypertrophied dactylopore prominences, and sometimes assume almost the appearance of the normal spines of *Spinipora.*

The most interesting parasite observed was a form found in the gastric cavities of the gastrozooids of *Pliobothrus symmetricus* contained in small capsules. These were badly preserved, but there seemed little doubt that they contained the remains of larvæ of a Pycnogonid, so that the deep-sea Pycnogonids, which are so abundant, very possibly pass their early stages in deep-sea Stylasteridæ. The formation of a calcareous corallum has not vitiated the capabilities of the Stylasterid Hydroids as hosts for Pycnogonid larvæ. The gastrozooids containing the larvæ were partly aborted.

Classification.

I place the Stylasteridæ with the Milleporidæ in a separate sub-order of the Hydroids, which I term Hydrocorallinæ, in accordance with a suggestion which I

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* Bull. Mus. Comp. Zool., Harvard, VI., p. 136.

made in a paper "On the Structure of *Millepora*," in the Phil. Trans., Vol. 167, Pt. I., 1877, p. 132. The placing of the two families together seems justified in the present stage of knowledge concerning them; but the Milleporidæ, in the general form of their zooids, seem allied to the gymnoblastic Hydroids, whereas the presence of distinct gonangia in the Stylasteridæ seems to ally these latter to the calyptoblastic group. Ampullæ seem certainly to be absent in Milleporidæ, and their gonophores are, therefore, probably developed free of the corallum. Further research may lead to the separation of the two families. The characters of the sub-order Hydrocorallinæ and of the families Milleporidæ and Stylasteridæ are given in the sequel in a concise tabular form, and also in a series of more extended and comprehensive statements in which no known detail of importance is omitted.

The components of the family Stylasteridæ have hitherto been classified from a knowledge of the structure of the corallum alone, and even this has been but imperfectly investigated in most instances; further, the descriptions given of the genera and species have been distorted by the violent efforts made by naturalists to discover septa and interseptal chambers in the so-called calicles of these supposed anthozoan corals.

The descriptions of the genera at least thus required to be rewritten, and modified according to the present knowledge of the structure of the members of the family. This has been attempted in the sequel where the characters of the genera given embrace those derived from the structures of the soft tissues as well as of the hard. Unfortunately, the soft structures are known in only one species in almost all the genera, and in almost all in but one sex. Hence the classification here given will doubtless need subsequent modification. It merely professes to be an attempt to define the genera in the best manner now possible.

In the case of two genera, *Distichopora* and *Labiopora*, nothing is known of the soft structures. Four new genera—*Sporadopora*, *Labiopora*, *Spinipora*, and *Astylus*—are described. Count de POURTALES' genus *Lepidopora* is merged in *Errina*, from which, in the absence of information concerning the soft structures, it can hardly be considered distinct. The lid-like coverings of the gastropores, by the presence of which the genus *Lepidopora* is distinguished, are most frequently composed of fused dactylopore projections, and do not consist of special elevations of the margins of the gastropore mouths themselves. Although this latter is sometimes the case, *Errina labiata*, a new species of which the structure is described in the present paper, seems to form a gradation between the species described as of the genus *Lepidopora* and *Errina aspera*. Count de POURTALES originally placed his *Lepidoporas* under the genus *Errina*.

Cyclopora (VERRILL) and *Stenohelia* (KENT) are further omitted from the list. The latter was formed to include *Allopora Madeirensis*, which seems to come very near to *Astylus* and *Cryptohelia* in that it has the cyclo-systems all directed towards one face of the flabellum; but the presence of a style in the gastropores is decisive in excluding it from this association, and probably points to the existence in it of a gastrozooid bearing tentacles.

The separation of the genera *Allopora* and *Stylaster* is difficult. The different forms of the gastrozooids, and the presence in that of one genus of six, and in that of the other of twelve tentacles, may prove characteristic of the genera. Count de POURTALES sent me specimens of *Stylaster roseus* and *Allopora miniata* in spirit, both species of their genera different from those of which I had determined the anatomy. The soft parts were unfortunately badly preserved in the specimens, but the gastrozooids, although their tentacles could not be counted, appeared in form to correspond with those before observed in the other species of the same two genera. A tendency to alternate budding can be made out in all *Alloporas*. It seems probable that the strong tendency to the development of the cyclo-systems on the sides of the branches only in the flabellum will prove a good characteristic for the separation of the *Stylasters* from the *Alloporas*, which would then include all those species in which the faces of the stem and branches were covered with cyclo-systems. The genus *Endohelia* of M. EDWARDS and HAIME, as already remarked by Count de POURTALES[*] and myself,[†] does not seem in any way separable from *Cryptohelia*. Short characters of the several genera of the Stylasteridæ are given in the table immediately following; more extended descriptions follow.

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[*] "Deep Sea Corals," *l.c.*, p. 34.

[†] H. N. MOSELEY. "On the True Corals dredged by H.M.S. 'Challenger,'" Proc. Roy. Soc., No. 170, 1876, p. 557.

TABULAR Synopsis of the Characters of the Sub-order Hydrocorallinæ, its Families and Genera.

					GENUS.
HYDROIDS forming a corallum, with two kinds of zooids, gastrozooids and dactylozooids. *Sub-order:* HYDROCORALLINÆ.	Dactylozooids with numerous tentacles. Ampullæ absent. *Family:* MILLEPORIDÆ.				*Millepora.*
	Dactylozooids devoid of tentacles. Gonangia contained in ampullæ. *Family:* STYLASTERIDÆ.	Pores sporadic, or not in cyclo-systems. Gastropores with styles. Dactylopores without them.	Dactylopores of one kind only.	Pores of both kinds simple. Gastrozooids with four tentacles.	*Sporadopora.*
				Dactylopores at the tips of tubular projections. Gastrozooids without tentacles.	*Pliobothrus.*
				Gastropores sometimes covered with a projecting scale. Dactylopores within nariform projections. Gastrozooids with four tentacles	*Errina.*
				Pores simple in a triple linear row at the lateral edges of the branches of the flabellum, rarely on its faces	*Distichopora.*
			Dactylopores of two kinds, larger and smaller.	Larger dactylopores within nariform projections arranged in regular rows. Smaller dactylopores at the sides of these	*Labiopora.*
				Larger dactylopores within long spine-like projections. Smaller dactylopores in simple cavities at their bases. Gastrozooids with six tentacles	*Spinipora.*
		Pores occurring in regular cyclo-systems only. Styles present in both kinds of pores, or absent altogether.	Both kinds of pores with styles. Gastrozooids with tentacles.	Cyclo-systems budding from one another somewhat irregularly. Gastrozooids with twelve tentacles	*Allopora.*
				Corallum increasing by regular alternate gemmation of the cyclo-systems from one another. Gastrozooids with eight tentacles	*Stylaster.*
			Styles absent. Gastrozooids without tentacles. Gastropores with two chambers.	Summits of cyclo-systems covered by a lid	*Cryptohelia.*
				Cyclo-systems without a lid	*Astylus.*

CHARACTERS OF THE SUB-ORDER HYDROCORALLINÆ, AND OF THE FAMILIES AND GENERA CONTAINED IN IT, MODIFIED TO INCLUDE THE RESULTS ATTAINED BY THE PRESENT INVESTIGATIONS.

SUB-ORDER *HYDROCORALLINÆ* (H. N. M.).*

Compound Hydroid stocks, growing by gemmation. Hydrophyton consisting of a meshwork of ramified cœnosarcal canals, composed of an ectoderm and pigmented endoderm, lodged within channels permeating a hard calcareous support, "corallum," which is deposited by the ectodermal investment of the canals, and forms masses of very various shape. Surface of the Hydrophyton covered with a continuous layer of ectoderm. Zooids of two forms, the one provided with a mouth and gastric cavity, "gastrozooid," the other mouthless and simply tentacular in function, "dactylozooid." Tentacles, when present, mostly with knobbed extremities. A well-defined muscular layer present in the zooids. Zooids lodged within chambers excavated in the substance of the Hydrophyton, "gastropores" and "dactylopores," lined by reflections of the surface layer of the ectoderm, forming the "sacs" of the zooids. Zooids of the two forms either scattered irregularly over the surface of the stock, or gathered into groups more or less regular, in each of which a centrally-placed gastrozooid is surrounded by a ring of dactylozooids. Cavities of zooids communicating with cœnosarcal meshwork by large canal offsets.

I. *Family Milleporidæ* (L. AGASSIZ).

Corallum irregular in growth, arborescent or encrusting, composed of a thin superficial living layer, supported by a dead mass made up of successive preceding dead layers. Pores devoid of styles, divided into a series of vertically succeeding chambers by transverse calcareous partitions, "tabulæ." Usually scattered irregularly, but in some species grouped with tolerable regularity into systems, in which a centrally-placed gastropore is surrounded by a ring of dactylopores. Nematocysts of two kinds present—the one, the three-spined form, occurring only in Hydroids; the other ovoid in shape, with a thread beset with a spiral of spines. Gastrozooids short, cylindrical, with from four to six tentacles with knob-like tips, set in a single whorl. Dactylozooids long, filiform, and tapering, with an irregular number of short knob-bearing tentacles set on at irregular intervals. Gonophores unknown, but not contained within special cavities in the substance of the corallum "ampullæ."

Genus 1.† *Millepora*, LINNÆUS (Syst. Nat., ed. 10, t. i., p. 790, 1858).—Genus with the characters of the family.

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* Phil. Trans. Roy. Soc., Vol. 167, pt. 1, 1877, p. 132.

† This seems to be the sole genus which can be now included in the family. *Heliopora* I have shown to be an Alcyonarian (Phil. Trans. Roy. Soc., Vol. 166, pt. 1), and I have confirmed Prof. VERRILL'S

II. *Family Stylasteridæ* (GRAY).

Corallum arborescent, with a strong tendency to assume a flabellar form, and to the development of the zooid pores on one face only of the flabellum, or on the lateral margins only of the branches composing it. In some genera a superficial layer only of the coral is living; in others, nearly the entire mass retains its vitality. Pores with tabulæ in two genera also. The gastropores usually provided with a conical calcareous projection, "style," at their bases. In some genera, a rudimentary style present only in the dactylopores. Pores scattered irregularly, or grouped into more or less symmetrical systems, composed of a centrally-placed gastropore surrounded by a circlet of dactylopores. In some genera the mouths of the dactylopores appear as elongate chambers, disposed radially towards the centre of the gastropore into which they open, and the chambers being separated from one another only by thin partitions, "pseudosepta;" the systems, "cyclo-systems," simulate closely calicles of Hexactinian corals. Nematocysts of two kinds, large and small, and of uniform shape in all the genera. Three-spined nematocysts absent. Gastrozooids cylindrical or flask-shaped in form always entirely retracted within the gastropores when at rest; those of the former shape with from four to twelve tentacles, set in one whorl, and regular in number in all the gastrozooids in each species; those of the latter devoid of tentacles. Dactylozooids simple elongate-conical bodies, devoid of tentacles, sometimes capable of entire retraction within the pores, sometimes not. Stocks of distinct sexes. Gonophores adelocodonic, developed within sacs, "gonangia," which are contained within special cavities in the substance of the corallum, "ampullæ." Stocks of the two sexes alike in form as far as known, except in the size of the ampullæ, which are larger and more prominent in the females. Ampullæ containing in male stocks several gonophores; in female, in some genera, a single gonophore, in others several. Spadix, in the female gonophores, cup-shaped, embracing a single ovum only, which becomes developed into a planula within the gonangium.

Genus 1. *Sporadopora*, gen. nov., H. N. M. =*Polypora*,[*] H. N. M. (Proc. Roy. Soc. No. 172, 1876, pp. 94, 95).—Corallum pure white, composed of finely reticular but compact cœnenchym, forming stout vertical stems, usually compressed from before backwards, so as to be oval in transverse section. Stem giving off a limited number of irregularly dichotomous branches, which are flattened like it, and tend to coalesce by their lateral margins and assume a flabellate form, which is sometimes somewhat

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results as to the nature of *Pocillopora*, which is a Hexactinian. There can be no doubt that *Seriatopora* is closely allied to *Pocillopora*, and it is apparently by an oversight that Prof. HUXLEY has retained *Pocillopora* amongst the Millepores in his 'Manual of the Anatomy of Invertebrated Animals,' London, 1877, p. 168. There seem to be no fossil genera, other than *Millepora* itself, which can with certainty be referred to the family.

[*] I was kindly informed that *Polypora* was inadmissible, as being already in use by Mr. ETHERIDGE, junr., F.G.S., &c.

curved. Surface of the corallum smooth and nearly even. Pores of both kinds with simple circular mouths, irregularly scattered. Gastropores larger, less numerous, with a deeply seated brush-like style, and very thin and delicate tabulæ placed at irregular intervals. Dactylopores devoid of a style. Ampullæ, in male stocks, ovoid, entirely immersed beneath the surface of the corallum. Pores and ampullæ more abundant on one face of the flabellum than on the other. Gastrozooids cylindrical, with four club-shaped tentacles, dividing at their bases into four main canals. Dactylozooids of various sizes, retracted entirely within the pores when at rest. Gonophores, in male stocks, ovoid, with a club-shaped spadix; one, two, or three present in each gonangium, attached directly to offsets of the cœnosarcal canals. Female stocks unknown.

Genus 2. *Pliobothrus*, Pourtales.—Corallum branching, with a tendency to form a flabellum. Surface smooth, marked with small linear openings arranged in rows, which in the recent state contain branches of the superficial cœnosarcal meshwork. Inner parts of the corallum very coarsely porous. Pores irregularly scattered. Gastropores circular-mouthed, their cavity tubular above, but expanding below into a basin-shaped chamber, without a style, often with one or two tabulæ. Dactylopores showing as minute openings at the tips of small tubular projections, devoid of styles. Ampullæ rounded cavities; in the female very large, in the male smaller; placed deeply, often in the axis of the corallum. Gastrozooids flask-shaped, devoid of tentacles, communicating with the cœnosarcal meshwork by numerous offsets arising all around their bases. Dactylozooids entirely retracted when at rest. In the female a single gonophore in each gonangium; in the male, a group of gonophores (?) in each ampulla.

Genus 3. *Errina*, Gray.—Corrallum branching with a tendency to form an irregular flabellate expansion. Pores most abundant at the tips of the branches; irregularly scattered. Dactylopores with delicate nariform or scale-like projections, which vary much in form, being sometimes drawn out into tubes opening on one side by a slit as the pore mouth, but often coalescing, so that two or three projections have a common base and form large scales perforated by the pores; devoid of styles; scales all with a tendency to incline towards the tips of the branches. Gastropores with irregularly circular mouths, often seated in depressions; with a deeply seated style. The mouths of the gastropores frequently covered by the dactylopore projections inclined more or less over them. Sometimes the margin of the gastropore itself is raised up on one side into a scale inclined over the pore mouth, but this is usually fused with neighbouring dactylopore projections. Ampullæ on both sides of the flabellum; prominent in the female. Gastrozooids cylindrical, with four club-shaped tentacles and four basal canals. Dactylozooids entirely retracted. Gonophores in the female solitary in the gonangia. The free margin of the cup-shaped spadix becomes converted into a ramified fringe, embracing the embryo as development proceeds. Planula as in *Pliobothrus*. Structure of male stocks unknown.

Genus 4. *Labiopora*, gen. nov., H. N. M. (Type specimen in British Museum, mistaken by Gray for a Bryozoon, and described by him as *Porella Antarctica.*) (Proc. Zool. Soc., 1872, p. 746, plate lxiv., fig. 4).—Corallum minutely reticulate in texture, composed of a few rounded branches with tapering extremities. The entire surface covered with nariform projections, with elongate cavities, which are arranged in rows along the lengths of the branches, often disposed with great regularity for long stretches. The projections of very uniform shape and rising from the branches to a uniform height. All inclined in the directions of the tips of the branches. The elongate cavities, which are extended in the direction of the lengths of the branches, have a defined rounded margin at their ends, situated towards the tips of the branches, but gradually merge at their opposite extremities in the deep and complex hollows by which the surface of the coral is excavated, and which are made up of the confluences of cavities of adjacent nariform projections with the other irregularities of the surface. Dactylopores devoid of styles; two kinds present, larger and smaller. The nariform projections are the outgrown margins of the larger dactylopores, which are continued into the substance of the corallum from the cavities of the projections as tubular pits. The smaller dactylopores have mouths of the same general form as those of the larger ones, but with their longer diameters directed at right angles to these latter. They have their walls fused with those of the nariform projections, or often appear as if excavated in the sides of these. They are of one-third or one-fourth the dimensions of the larger pores. Mouths of the gastropores deeply seated in depressions at the bases of the nariform projections. Circular in outline. Gastropores provided with deeply seated styles with brush-like tips. No ampullæ in the unique specimen. Soft structures unknown.

Genus 5. *Spinipora*, gen. nov., H. N. M.—Corallum branching. Branches rounded. Entire surface thickly beset with long spinous projections inclined towards the tips of the branches. Spines conical, grooved deeply on their sides turned towards the tips of the branches so as to present spout-like openings, which are the mouths of the larger dactylopores. Dactylopores of a smaller kind also present; their mouths appear as minute oval apertures scattered over the bases and sides of the spines. Styles absent in the dactylopores. Gastropores deeply seated in hollows between the bases of the spines; with deeply placed styles. Ampullæ absent in the unique specimen. Dactylozooids of two kinds, the larger attached by elongate bases within the spout-like cavities of the larger dactylopores, incapable of retraction within the pores; the smaller minute; entirely retracted when at rest. Gastrozooids cylindrical, with six tentacles and four basal canals. Gonophores unknown.

Genus 6. *Allopora.*—Corallum branching, but frequently not so as to form a flabellum. Pores in regular cyclo-systems only, excepting in *A. Nobilis*, where some of the systems are not perfected. Tendency to alternate gemmation present, but weak, and usually obscured by an abundant growth of cœnenchym. Cyclo-systems always scattered over the faces of the branches, as well as situate at their lateral margins; often entirely sporadic in disposition. Dactylopores with a more or less rudimentary style affixed to those parts of their walls which are outermost in the systems. Gastropores, simple tubular, with a brush-like style. Ampullæ sometimes prominent, sometimes scarcely showing at the surface. Dactylozooids attached by elongate bases to the sides of their pores occupied by the styles; partly retracted within the pores, partly bent upwards when at rest within the wide pore mouths. Gastrozooids dome-like in shape, with twelve tentacles and numerous basal canals. Gonangia in male stocks containing two or three ovoid gonophores with club-shaped spadices. Structure of gonophores of female stocks unknown.

Genus 7. *Stylaster*, GRAY.—Corallum arborescent usually flabelliform. Pores in regular cyclo-systems only. A strong tendency to the development of these cyclo-systems on the lateral margins of the branches only. Cyclo-systems arising from one another by alternate gemmation. Dactylopores and zooids as in *Allopora*. Ampullæ usually prominent on both faces of the flabellum. Gastrozooids cylindrical, with numerous basal canals and eight tentacles. Gonophores of male stocks as in *Allopora*. Structure of females unknown.

Genus 8. *Astylus*, gen. nov., H. N. M.—Corallum forming a small and delicate flabellum. Pores in regular cyclo-systems, all placed on one face of the flabellum. Cyclo-systems forming cylindrical masses prominent from the branches, and with their axes directed at right angles to the plane of the flabellum. Style absent in both kinds of pores. Gastropores divided into two chambers, an upper and lower, by a constriction of their walls. Opening between the chambers rendered horseshoe-shaped by the projection across it, in the direction of the tips of the branches, from that side of its margin placed nearest the bases of the branches, of a tongue-like excrescence. Ampullæ in the male stocks in a ring around the cyclo-system masses; none scattered on the branches. Dactylozooids, when at rest, doubled down within the upper chambers of the gastropores. Gastrozooids flask-shaped, devoid of tentacles, with numerous basal canals. Gonangia in the male stocks containing a central mass of cells, from the surface of which are developed as buds numerous pedicellate lobular sacs, in which the spermatozoa are produced. Female stocks unknown.

Genus 9. *Cryptohelia*, M. EDWARDS and HAIME.—Corallum closely resembling that of *Astylus* in all respects, excepting that the cyclo-system masses are not so prominent, that the opening between the upper and lower chambers of the gastropores is circular in outline, and that a lid-like lamina of calcareous matter is directed horizontally across the mouths of all the cyclo-systems. The lids are supported on stout columns arising from the margins of the cyclo-systems and inclined over them. They spring from the sides of the systems nearest the bases of the branches, and are directed towards the tips of the branches. In female stocks only a single ampulla and gonangium developed in relation with each cyclo-system. No ampullæ on the connecting branches. In the males several ampullæ in the walls of each cyclo-system. Soft structures as in *Astylus*. In female stocks numerous gonophores present in each gonangium in all stages of development. Spadix cup-shaped, developing, as in *Errina*, into a fringed network at the margin. A solitary ovum developed in relation with each spadix. Planula very long and worm-like.

Genus *Distichopora*, LAMARCK (Plate 35, figs. 6 and 16).—Corallum branching flabelliform with branches usually flattened in the plane of the flabellum; composed of very compact cœnenchym. Pores confined to narrow lines or rows running along the exact centres or edges of the sides of the branches, generally absent on their faces, except as occasional abnormalities or rudimentary branchlets budding in a direction out of the plane of the flabellum. The lines of pores composed of three rows, a central row of larger gastropores with circular or oval mouths; and a row on each side of this of smaller dactylopores, sometimes very minute, often slit-like in aperture, the length of the slit being directed at right angles to the line of the row. Pores very deep, prolonged in curved lines side by side in the plane of the flabellum, inwards and downwards towards the bases of the branches; forming thus throughout the flabellum a thin continuous tract of fragile tubulate tissue, in which the successively-developed curved pore-tubes stand out fanwise, separating from one another the compact masses of cœnenchym forming the opposite faces of the branches. The branches may therefore be readily split into two halves along this tubular tract. Older gastropores with immensely long filiform styles. Styles much shorter in the younger gastropores. Dactylopores devoid of styles. Ampullæ sometimes on one, sometimes on both faces of the flabellum, prominent and often forming confluent masses. Soft structures unknown.*

SPECIES OF STYLASTERIDÆ AT PRESENT KNOWN.

A list of all the species of Stylasteridæ, at present described, here follows. The list is not to be considered as constituting a revision of the species. Access has been had to only a limited number of specimens, and as in the case of many of the species good figures, or indeed any figures at all, are wanting, and the descriptions founded on a false theory as to the nature of the organisms described are necessarily imperfect, a revision has not been found possible. Indeed, such can only be carried out when the soft structures of more species shall have been examined. The list, such as it is, represents an attempt to draw attention to all the species of which an account has been published. Few references are given, those only in each case being selected which are the latest and will supply all further references required when consulted, or those which indicate figures of the species described. The localities are appended in most instances in order to show the distribution of the members of the family.

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* See Postscript, p. 502.

LIST OF THE SPECIES OF STYLASTERIDÆ AT PRESENT KNOWN.*

Family Stylasteridæ, Gray, Ann. and Mag. Nat. Hist., vol. xix., 1847.

(I.) Genus *Sporadopora,* gen. nov., H. N. M. *Polypora,* H. N. M., Proc Roy. Soc., No. 172, 1876, pp. 94, 95.

Species 1. *Sporadopora dichotoma,* sp. n., H. N. M., Plate 34, figs. 3 and 4. *Polypora dichotoma,* H. N. M., Proc. Roy. Soc., No. 172, 1876, pp. 94, 95. Dredged by H.M.S. 'Challenger,' Feb. 14, 1876, in lat. 37° 17′ S., long. 53° 52′ W., off the mouth of the Rio de la Plata, from 600 fathoms.

(II.) Genus *Pliobothrus,* Pourtales, Bull. Mus. Comp. Zool. Harvard, No. 7 ; 'Deep Sea Corals,' p. 57. See Ill. Cat. Mus. Comp. Zool. Harvard, No. 10.

Species 1. *Pliobothrus symmetricus,* Pourtales, Bull. Mus. Comp. Zool., No. 7 ; 'Deep Sea Corals,' p. 57, pl. iv., figs. 7 and 8. Dredged off Florida and Key West, in from 98 to 154 fathoms.

Cold area of North Atlantic, in 500 to 600 fathoms, P. M. Duncan, "Madreporaria of the 'Porcupine' Expedition," Trans. Zool. Soc.. vol. viii., pt. v., p. 336.

Off Sombrero Island, Danish West Indies, in 460 fathoms, Moseley, "Corals dredged by H.M.S. 'Challenger,'" Proc. Roy. Soc., No. 170, 1876, p. 548.

Species 2. *Pliobothrus tubulatus,* Pourtales, 'Deep Sea Corals,' p. 58, pl. iv., fig. 9. Off Havana, in 270 fathoms.

(III.) Genus *Errina,* Gray, Proc. Zool. Soc., 1835, p. 35 ; Savile Kent, Proc. Zool. Soc., 1871, p. 282.

Species 1. *Errina aspera,* Gray, *l.c.* *Millepora aspera,* Esper, Supp. 1, t. 18 ; Lam., ii., p. 202.

Species 2. *Errina carinata,* Pourtales, 'Deep Sea Corals,' p. 39, pl. vi., fig. 5. Off Havana, in 270 fathoms.

Species 3 (?). *Errina fissurata,* Gray, Proc. Zool. Soc., 1872, p. 745. Described from a drawing of the living coral. Specimen lost. Affinities doubtful. Dredged in the Antarctic Ocean.

Species 4. *Errina glabra.* *Lepidopora glabra,* Pourtales, 'Deep Sea Corals,' p. 40, pl. vii., figs. 8 and 9. Off Havana, in 270 fathoms.

Species 5. *Errina cochleata.* *Lepidopora cochleata,* Pourtales, 'Deep Sea Corals,' p. 40, pl. iii., figs. 17, 18, and 19. Off Havana, in 270 fathoms.

Species 6. *Errina Dabneyi.* *Lepidopora Dabneyi,* Pourtales, 'Deep Sea Corals,' pp. 40, 41, pl. vii., figs. 10 and 11. Fayal, Azores. Depth (?).

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* In the preparation of the present list, I was kindly assisted by Dr. F. Brüggemann, of the British Museum.

Species 7. *Errina labiata*, sp. n., H. N. M., Plate 34, fig. 6. Off the mouth of the Rio de la Plata, 600 fathoms.

(IV.) Genus *Labiopora*, gen. nov., H. N. M.

Species 1. *Labiopora Antarctica*, Plate 2, fig. 5. *Porella Antarctica*, GRAY, Proc. Zool. Soc., 1872, p. 746, pl. lxiv., fig. 4. Antarctic Ocean. Mistaken by GRAY for a Bryozoon allied to *Porella Cervicornis*.

(V.) Genus *Spinipora*, gen. nov., H. N. M.

Species 1. *Spinipora echinata*, sp. n., H. N. M., Plate 34, fig. 2; Plate 35, fig. 4. Off the mouth of the Rio de la Plata, 600 fathoms.

(VI.) Genus *Allopora*, EHRENBERG, Corall. des roth. Meer., p. 147, 1834.

Species 1. *Allopora oculina*, EHRENBERG, *l.c.* Coast of Norway, 50 to 100 fathoms, G. O. SARS, Fork. Selsk. Chr., 1872, p. 115.

Species 2. *Allopora miniata*, POURTALES, 'Deep Sea Corals,' p. 37, pl. iii., figs. 14, 15, 16. Off Florida, 100 to 324 fathoms.

Species 3. *Allopora Californica*, VERRILL, Proc. Essex Institute, iii., p. 37.

Species 4. *Allopora venusta*, VERRILL, Trans. Connect. Acad., i., p. 517.

Species 5. *Allopora nobilis*, KENT, Proc. Zool. Soc., 1871, p. 279, pl. xxv.; = *explanata*, KENT, *l.c.*, p. 250, pl. xxv.; = *Allopora venusta*, VERRILL (?).

Species 6. *Allopora subviolacea*, KENT, Proc. Zool. Soc., 1871, p. 280, pl. xxv.

Species 7. *Allopora profunda*, sp. n., H. N. M., Plate 34, fig. 7. Off the mouth of the Rio de la Plata, 600 fathoms.

Species 8. *Allopora pygmæa*. *Dendracis pygmæa*, RÖMER, Beschreibung der Norddeutschen tertiaren Polyparien; MEYERS, 'Palæontographica,' bd. iv., p. 243, taf. xxxix.; bd. ix., fig. 13, *a*, *b*, *c*. Described as a Bryozoon Fossil in 'Oligocœne' of LATTORF.

Allopora tuberculosa. *Dendracis tuberculosa*, RÖMER, *l.c.*; fossil in 'Oligocœne' of LATTORF.

(VII.) Genus *Stylaster*, GRAY, Zool. Miscell., p. 36, 1831.

Species 1. *Stylaster flabelliformis*, M. EDWARDS and HAIME, Hist. Nat. des Cor., t. ii., p. 129. Isle of Bourbon, 160 fathoms.

Species 2. *Stylaster gracilis*, M. ED. and H., Hist. Nat. des Cor., t. ii., p. 129. Australia.

Species 3. *Stylaster roseus*, M. ED. and H., Hist. Nat. des Cor., t. ii., p. 130. American Ocean; littoral on the Florida reefs, two feet below low water mark; POURTALES, 'Deep Sea Corals,' p. 83.

Species 4. *Stylaster sanguineus*, M. ED. and H., Hist. Nat. des Cor., t. ii., p. 130; Ann. des Sci. Nat., t. xiii., p. 96, pl. 3, fig. 2, 1850. Coast of Australia; New Zealand; Florida, shallow water; POURTALES, 'Deep Sea Corals,' p. 83.

Species 5. *Stylaster gemmascens*, M. ED. and H., Hist. Nat. des Cor., t. ii., p. 131.

Indian Ocean; North Atlantic, 530 fathoms, 'Porcupine,' P. M. DUNCAN, Trans. Zool. Soc., vol. viii., pt. v., p. 332; at great depths in the Foldenfjord, Norway, G. O. SARS, *l.c.*, p. 115.

Species 6. *Stylaster granulosus*, M. ED. and H., Hist. Nat. des Cor., t. ii., p. 131; Ann. des Sci. Nat., 3 ser., t. xiii., p. 97, pl. 3, fig. 3, 1850.

Species 7. *Stylaster bella*, DANA, Zooph., p. 696, pl. 60, fig. 6. *Cyclopora bella*, VERRILL, Proc. Essex Inst., v., p. 38, 1866. Paumotu Archipelago. (Without style?)

Species 8. *Stylaster amphiheloides*, KENT, P. Z. S., 1871, p. 277, with figure.

Species 9. *Stylaster punctatus*, POURTALES, 'Deep Sea Corals,' p. 36; Ill. Cat. Mus. Comp. Zool. Harvard, No. 8, pl. vii., figs. 8, 9. Off Florida, 9 to 315 fathoms.

Species 10. *Stylaster Duchassaingii*, POURTALES, 'Deep Sea Corals,' p. 35; = *Eximius*, KENT, Proc. Zool. Soc., 1871; = *Elegans*, DUCH. and MICH.? Near Tortugas, 43 fathoms; off Point Calvo, Brazil, 400 fathoms, 'Challenger.'

Species 11. *Stylaster filogranus*, POURTALES, 'Deep Sea Corals,' p. 35, pl. v., figs. 13, 14. West of Tortugas.

Species 12. *Stylaster asper*, KENT, Proc. Zool. Soc., 1871, p. 278, with figure.

Species 13. *Stylaster erubescens*, POURTALES, 'Deep Sea Corals,' p. 34, pl. iv., figs. 10 and 11. Off Florida reef, 120 to 324 fathoms.

Species 14. *Stylaster rosso-americanus*, BRANDT, Z. wiss Zool., xxii., p. 292. Name given only; no figure or description.

Species 15. *Stylaster tenuis*, VERRILL, Bull. Mus. Comp. Zool. Cambridge, i., p. 45. No figure.

Species 16. *Stylaster elegans*, VERRILL, Bull. Mus. Comp. Zool. Cambridge, p. 45. No figure.

Species 17. *Stylaster densicaulis*, sp. n., H. N. M., Plate 34, fig. 5. Off the mouth of the Rio de la Plata, 600 fathoms.

Species 18. *Stylaster complanatus*, POURTALES, 'Deep Sea Corals,' p. 36, pl. ii., figs. 16, 17. *Stenohelia complanatea*, KENT, Ann. and Mag. Nat. Hist., Feb. 1870. Off Havana, 270 fathoms; off Sombrero Island, Danish West Indies, 460 fathoms, 'Challenger.'

Species 19. *Stylaster Madeirensis*, JOHNSTON, Proc. Zool. Soc., 1862. *Stenohelia Madeirensis*, KENT, Ann. and Mag. Nat. Hist., 1870, vol. v., p. 120, with figure. Madeira, on long fishing line; off Cape Verde Islands.*

(VIII.) Genus *Astylus*, gen. nov., H. N. M.

Species 1. *Astylus subviridis*, sp. n., H. N. M. Off the Meangis Islands, 500 fathoms.

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* See Postscript, p. 503.

(IX.) Genus *Cryptohelia*, M. Ed. and H., *l.c.*, p. 127.

Species 1. *Cryptohelia pudica*, M. Ed. and H., Ann. des Sci. Nat. 3 ser., t. xiii., fig. 3, pl. 3, fig. 1, 1850; = *Endohelia Japonica* (?), M. Ed. and H., Hist. Nat. des Cor., t. ii., p. 128, of which no figure is extant. New Guinea, Japan, Atlantic, West Indies, S. Pacific, Japan, 390 to 1,530 fathoms, 'Challenger;' Moseley, Proc. Roy. Soc., No. 170, 1876, p. 548.

Species 2. *Cryptohelia Peircei*, Pourtales, 'Deep Sea Corals,' *l.c.*, p. 37, pl. ii., figs. 18 and 19. Off Havana, &c., 270 to 600 fathoms. = *C. pudica* (?).

(X.) Genus *Distichopora*, Lamarck, Hist. des Anim. sans Vert., t. ii., p. 198.

Species 1. *Distichopora violacea*, M. Ed. and H., Hist. Nat. des Cor., t. iii. p. 451; M. Edwards, 'Atlas Regne Animal, Zoophytes,' pl. 85, fig. 4, 4*a*, 4*b*, 4*c*. Island of Timor; Fiji.

Species 2. *Distichopora coccinea*, Gray, Proc. Zool. Soc., 1860. Abundant at the Marshall Group, Pacific Ocean.

Species 3. *Distichopora nitida*, Verrill, Bull. Mus. Comp. Zool. Cambridge, i., p. 46.

Species 4. *Distichopora cervina*, Pourtales, 'Deep Sea Corals,' p. 39, *note*; Ill. Cat. Mus. Comp. Zool. Harvard, No. 8, pl. vii., fig. 11. St. Thomas, Danish West Indies.

Distichopora foliacea, Pourtales, 'Deep Sea Corals,' p. 38, pl. iv., figs. 12, 13. Off Florida and Key West, 100 to 262 fathoms.

Species 6. *Distichopora sulcata*, Pourtales, 'Deep Sea Corals,' p. 38, pl. iv., fig. 14, pl. vii., fig. 7. Off Havana, 270 fathoms; off Cuba.

Species 7. *Distichopora barbadensis*, Pourtales, Ill. Cat. Mus. Comp. Zool. Harvard, No. 8, p. 43, pl. vii., fig. 10.

Species 8. *Distichopora rosea*, Kent, Proc. Zool. Soc., 1871. East coast of Australia.

Species 9. *Distichopora antiqua*, Defrance; M. Ed. and H., Hist. Nat. des Cor., t. iii., pp. 451, 452. Tertiary fossil at Chaumont and Valmondois.

Pedigree of the Hydrocorallinæ.

The line of descent of the various genera of the Stylasteridæ from a parent form seems to be traceable with especial clearness. All gradations are present by which simple circular mouthed pores sporadically scattered over the corallum become grouped and modified into cyclo-systems of the most symmetrical and complex character. Since styles appear in some genera in the dactylopores as well as in the gastropores, it seems probable that in the ancestral form or Archistylaster styles were present in both forms of pores. If the Milleporidæ prove closely related to the Stylasteridæ when their gonophores have been investigated, it will follow that the two families have had a common ancestor, and that Hydroids have developed a calcareous support only once in their history and not in two separate instances. This common ancestor may be presumed to have had a hydrosoma composed, as throughout the sub-order Hydrocorallinæ; with its pores sporadic, with tabulæ and without styles, and with two kinds of zooids, with knob-bearing tentacles; with a tendency also of the dactylozooids to form ring-like groups around a gastrozooid.

From this form Archistylaster was developed with a branching corallum; with a strong tendency to assume a flabellate form, and to develop its pores only on one face of the flabellum, or at the sides only of the branches; with its pores sporadic and tabulate, and styles in both forms of them. The dactylozooids of Archistylaster were devoid of the knobbed tentacles, these were, however, retained by the gastrozooid. The gonangia were included in hollows in the corallum.

In *Sporadopora*, the most ancestral *Stylasterid* at present known, the styles of the dactylopores have disappeared, and they only reappear apparently by reversion in *Allopora* and *Stylaster*. Rudimentary tabula are present in *Sporadopora* and *Pliobothrus*, but disappear in succeeding genera. In *Pliobothrus* the margins of the dactylopore mouths are raised up and prolonged into small tubuli, and the genus would thus lead to *Errina*, where the tubuli become nariform, were it not that in *Pliobothrus* the style of the gastrozooid is lost, and that the gastrozooid is devoid of tentacles and flask-shaped: a condition occurring again only in the most highly specialized members of the family *Astylus* and *Cryptohelia.*

Two separate modifications of the nariform projections of *Errina* are presented by *Porella* and *Spinipora*, in both which genera further complication ensues by the differentiation of two kinds of dactylozooids.

The process of the formation of cyclo-systems is seen in all stages in different parts of the surface of the single species *Allopora subviolacea*, as will be seen by reference to SAVILE KENT's figures,[*] or to the diagrams given on Plate 44 of the present Memoir, figs. 10, 11, and 12. In this coral five or six dactylopores are grouped in a circle around a single centrally placed gastropore. In some groups all the pores are simply circular (fig. 10). In others, shallow grooves, often only just indicated, lead radially from the dactylopores towards the gastropore. In others, these grooves are well marked and deep, and a complete cyclo-system is formed. It appears probable that this condition has been brought about by the continual bending inwards of the dactylopores to convey food to the gastropore. The grooves have been the result of the attempts of these zooids to reach the gastrozooid when further and further retracted. Thus, in most *Alloporas* and all *Stylasters*, all the pores have come to form regular cyclo-systems, in which the mouths of the dactylopores are drawn out into elongate chambers, and their tubular prolongations reduced to mere rudiments in many cases. At a very short distance below the surface in *Allopora subviolacea*

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[*] Proc. Zool. Soc., 1871, Plate 25, fig. 2A.

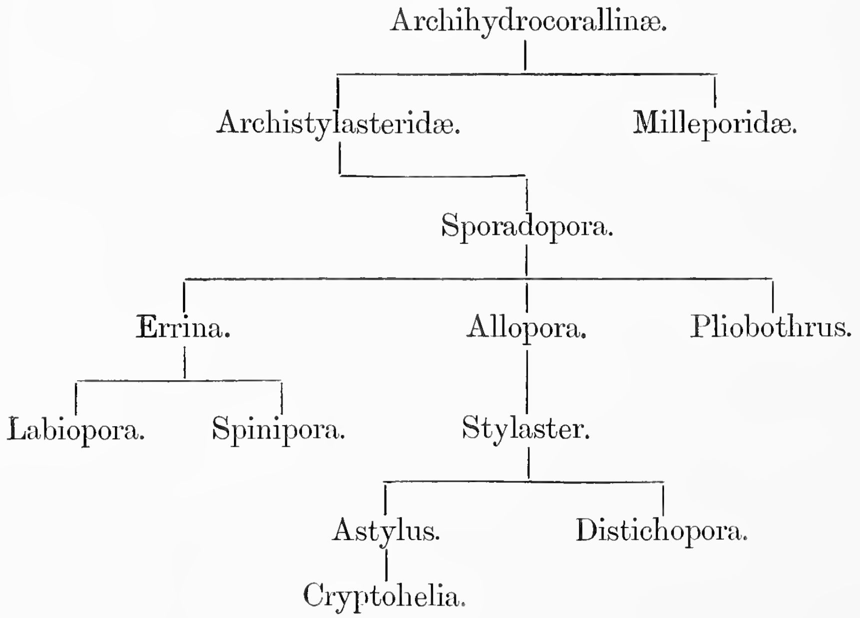
the pores are found to be in all the systems still entirely independent (fig. 12), and this condition is maintained at greater depths in all *Stylasters*.

In *Astylus* and *Cryptohelia* the tentacular zooids have come to place themselves out of harms way, not by retraction within their pores, but by being doubled down within the gastropore, which is divided in two chambers. They are thus enabled to maintain a greater length than they could were they obliged to be retracted within their own pores, and they thus obtain a longer reach.

Allopora Madeirensis (JOHNSTON) forms a gradation towards *Astylus* and *Cryptohelia* in that it has its cyclo-systems all turned towards one face of the flabellum, but differs in having styles in the gastropores. The lid of *Cryptohelia* may be a further modification of the tongue-like process in the gastropore of *Astylus*, or the reverse may be the case, the structure in *Astylus* being a reduction of that in *Cryptohelia*. If the former view be correct, then the tongue-like process represents the scale often present in *Errina* as a covering of the gastropore, and the lid of *Cryptohelia* is a further modification of this which is increased in dimensions and altered so as to cover an entire cyclo-system. It is to be noted that in becoming so remarkably modified into elongate slit-like cavities, the dactylopores of Stylasteridæ, with cyclo-systems, follow an ancestral tendency to modification, for these elongate pores are, taken separately, closely similar in form to the nariform dactylopores of *Errina* and its allies. They only have the lips of all the projections directed radially outwards.

From the appearance of rudimentary cyclo-systems in the pore rows of *Distichopora*, it seems probable that this genus represents an extreme modification of those *Stylasters* in which the systems are confined to the sides of the branches of the flabellum, closely opposed cyclo-systems having degenerated into a triple row of pores, in which the gastropores occupy the central row of the three (Plate 35, fig. 16).

The phylum of the Hydrocorallinæ may, therefore, be represented as follows:—



Distribution in Space and Time of the Stylasteridæ.

The Stylasteridæ range all over the world, and exist at all depths from shallow water on the coasts to great depths in the open oceans. Two species occur close at home off the coast of Norway, viz., *Allopora oculina*, obtained by G. O. Sars in from 50 to 100 fathoms, and *Stylaster gemmascens*, which occurs at great depths in the Foldenfjord. The same species, originally described from the Indian Ocean, occurs in the North Atlantic in 530 fathoms. *Stylaster roseus* is abundant in a depth of two feet below low water mark on the coast of Cuba,[*] and *Stylaster punctatus* occurs in nine fathoms off Florida.[†] *Stylaster sanguineus* occurs at Florida and New Zealand, and I dredged a closely allied, if not the same, species in two fathoms on the Philippine coast. *Cryptohelia* came originally from New Guinea. It was dredged by the 'Challenger' in all parts of the world, and up to a depth of 1,530 fathoms. Some genera, as *Sporadopora* and *Spinipora*, are as yet known only from one locality, but no doubt their range will be extended by further dredging.

No Stylasteridæ are known from geological deposits older than the tertiary; indeed, until now a single species only of one genus, *Distichopora*, has been described as occurring in the fossil condition, viz., *Distichopora antiqua* from tertiary beds at Chaumont, in France. Fossil Stylasteridæ have, however, been confounded with Bryozoa, just as Gray confounded the recent *Labiopora* with *Porella*. Two species of a genus termed *Dendracis*, figured by Fr. A. Römer,[‡] which occur in the Oligocœne of Lattorf, are evidently *Stylasterids*, and probably members of the genus *Allopora*, in which they have been introduced in the present paper in the list of species, as *Allopora tuberculosa* and *pygmæa*. Some calcareous structures from the *Cenoman* (=middle chalk) figured by the Ritter von Reuss, in the same publication as that containing Römer's paper,[§] and placed with Heteroporella as Bryozoa, may very possibly prove allied to *Pliobothrus* on further examination. *Thalamipora*,[||] figured by the same author in the same paper, seems to be a *Stylasterid* bearing large female ampullæ, present in abundance and agglomerated, the pore systems being all at the ends of the branches, whilst a deep central gastropore in each system is surrounded by a circlet of from five to seven dactylopores. Von Reuss is in great doubt as to the affinities of this form, but concludes that it is a chambered foraminifer. It is probable that now that their importance and structure is more fully known, abundance of fossil Stylasterida will be made out. The structure of the Stylasteridæ appears to throw no light upon that of the Graptolites.

Exeter College, Oxford, *January 21st*, 1878.

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[*] Pourtales' 'Deep Sea Corals,' p. 83.

[†] Pourtales, *l.c.*, p. 36.

[‡] Fr. A. Römer. 'Beschreibung der Norddeutschen tertiaren Polyparien.' Meyer's 'Palæontographica,' bd. ix., p. 243, taf. xxxix., fig. 15, *a*, *b*, *c*.

[§] Ritter von Reuss. Die Bryozoen des unteren Pläner's 'Palæontographica,' bd. xx., taf. xxxiii.

[||] *Ibid.*, p. 138.

DESCRIPTION OF PLATES.

NOTE.—The whole of the figures appended to the present paper relating to soft structures have been prepared so as to represent by means of ideal sections the conclusions as to structure arrived at by prolonged study of long series of preparations. It would have been impossible to give *facsimile* drawings of all the preparations from the study of which facts of importance were arrived at, and thus to lay the evidence before the reader in a pictorial form. The practice of illustrating scientific papers treating of minute anatomy by figures which profess to be *facsimiles* of preparations, and in which often all defects due to breakage of sections or obliquity of the line of cutting are reproduced, seems to me to be much to be deplored, and only tends to create confusion and needlessly increase the number of figures without in any way enhancing the credit which will be given to the results. The drawings can never be so accurate as to stand in the place of preparations. They will always represent to some extent the author's views as to what is to be observed in the preparations. It seems far better that in the modern stage of the science of finer anatomy, drawings should represent the results attained, in as complete and concise a form as can be devised, so as to convey these results to others almost at a glance, if possible.

In the present figures all the histological details, as well as the major features of the structures represented, *have been drawn accurately to scale by means of a series of micrometic measurements.* The amount of magnification in diameters is given at the bottom of each plate or figure. Since in the majority of the plates ideal sections through complex canal meshworks are represented, the canals composing the meshworks are necessarily shown as cut open in all directions.

PLATE 34.

Drawings of the Coralla of several species of Stylasteridæ of which the corresponding soft tissues are described in the present paper.

Fig. 1. *Sporadopora dichotoma.* Young vigorous specimen which was obtained in the living condition. Natural size.

Fig. 2. *Sporadopora dichotoma.* Older specimen reduced in size to one-half of its dimensions to show the method of branching in the more fully grown specimens.

Fig. 3. *Spinipora echinata.* Enlarged to twice the natural size.

Fig. 4. *Astylus subviridis.* Several of the branches of the specimen are broken off. Natural size.

Fig. 5. *Stylaster densicaulis.* Portions of a corallum of the natural size.

Fig. 5A. Portion of a tip of a branch enlarged.

Fig. 6. *Allopora profunda.* Natural size.

Fig. 6A. Portion of same enlarged.

Fig. 7. *Errina labiata.* Natural size. The form of the stem of the corallum is much distorted in places by parasitic annelids.

Fig. 7A. Portion of a branch enlarged.

PLATE 35.

Fig. 1. Section vertical to the surface of the corallum of *Sporadopora dichotoma* showing the structure of the hard parts. The general mass is seen to be excessively porous in appearance, being traversed in all directions by canals which, in the recent condition of the coral, contain the elements of the cœnosarcal meshwork. The perforations and canals are smaller towards the surface of the corallum, and coarser in the deeper regions. The cavities in the mass occupied by the zooids and gonophores are excavated within it, and have their walls freely perforated, like the remainder of the corallum.

G Z. Mouth of a gastropore.

S. Style terminating above in a delicate brush of spicules.

T. Thin calcareous tabula.

D Z, D Z. Pores of large and small dactylozooids.

G. Cavity or ampulla occupied by a male gonophore, which is in this genus entirely sunken beneath the surface of the corallum.

Fig. 2. View of the surface of the corallum of *Sporadopora dichotoma* as seen by reflected light.

G Z, G Z. Mouths of gastropores.

D Z, D Z. Mouths of dactylopores.

G, G, G. Shallow depressions in which the ampullæ open to the surface.

Fig. 3. Portion of the corallum forming a single calicular system of *Stylaster densicaulis* laid open by a vertical incision in order to show the arrangement of the hard parts, and enlarged.

G Z. Gastropore.

S. Style of the gastrozooid.

A. Circlet of small rough projections of the corallum, which stand out from the wall of the gastropore just above the top of the style.

D Z, D Z. Dactylopores of the cyclo-system.

P. Walls of the corallum separating the adjacent dactylopores from one another, the pseudosepta of the cyclo-system.

S′. Style of a dactylozooid. This is seen adhering to the outer wall of one of the dactylopores, which is laid completely open in order to show it *in situ*.

Fig. 4. Portion of the corallum of *Spinipora echinata* enlarged to show its outward form. The corallum is covered with long grooved spines, which carry the larger dactylozooids. On the sides of these spines, and about their bases, are numerous simple or slightly lipped smaller pores, occupied by a smaller form of dactylozooid. Deeper in between the bases of the spines lie the pores of the gastrozooids, provided each with a style.

D Z, D Z. Pores of the larger dactylozooids, appearing as grooves in the long projecting spines.
D Z′, D Z′. Pores of the smaller dactylozooids.
G Z, G Z. Pores of the gastrozooids.

Fig. 5. Portion of the corallum of *Labiopora Antarctica* much enlarged. From a drawing by Mr. CHARLES STEWART, F.L.S.

G Z, G Z. Pores of gastrozooids with their styles just visible in their depths.
D Z, D Z. Pores of larger dactylozooids.
D Z′, D Z′. Pores of dactylozooids of the smaller kind.

Fig. 6. View of one of the inner surfaces of a fragment of the corallum of *Distichopora coccinea*, which has been split in half through the line formed by the pores of the gastrozooids; showing the arrangement of these pores, and their very long styles.

G Z, G Z. Pores of gastrozooids.
G Z′. Young similar pore which has as yet little depth.
S, S, S. Styles; that on the extreme left remarkably long.

Fig. 7. Somewhat diagrammatic view of a zooid system of *Cryptohelia pudica*, divided vertically in half by a section passing through the axis and in the direction of the length of the branch on which the system is situate. The dotted areas indicate cut surfaces of calcareous substance, the structure of which is not filled in in the drawing. The gastropore consists of two portions, an upper and lower, separated from one another by a circular constricted aperture. The wall of the upper portion ends below in a thin incurved border bounding the circular aperture, and from the border behind this wall all round a narrowed prolongation of the lower chamber of the gastropore passes up, and leads above by a series of offsets to the lower terminations of the tubular portions of the dactylopores, conveying, in the recent condition of the coral, the main canals springing from the gastropore. The support of the lid of the pore system sends a stout prolongation downwards to fuse with the wall of the upper chamber of the gastropore.

G P. Upper chamber of the gastropore.
G P′. Lower chamber of the gastropore.
D P, D P. Dactylopores.

C. Space behind the wall of the upper gastropore chamber leading to the dactylopore tubules.

L. Lid covering the cyclo-system.

Fig. 8. Pore system of *Astylus subviridis* laid open by a vertical incision through the axis and in the direction of the length of the supporting branch.

G P. Upper chamber of the gastropore.

G P′. Lower chamber of the gastropore.

B. Tongue-like process of the lower border of the wall of the upper chamber of the gastropore, which projecting horizontally in the direction, in each system, of the tips of the branches, converts the aperture leading between the two chambers into the form of a horseshoe.

A. Base of the tongue-like process and part of the wall of the upper chamber cut through.

C C. Space behind the wall of the upper gastropore chamber, leading, as in the last figure, to the dactylopores.

G, G. Ampullæ.

Figs. 9–16. Diagrammatic representations of the arrangements of the gastropores and dactylopores in the several genera of Stylasteridæ, to show the manner in which cyclo-systems and their pseudosepta have become developed in this family.

The following letters apply similarly throughout the series.

G Z. Gastropore. S. Its style.

D Z. Dactylopore.

Fig. 9. *Sporadopora dichotoma.* The pores of both kinds are irregularly scattered over the surface of the corallum.

Fig. 10. *Allopora nobilis.* A number of dactylopores are grouped in a circle around a single centrally-placed gastropore.

Fig. 11. A system of pores from another part of the same specimen of *Allopora nobilis.* Shallow grooves run from the central gastropore to the encircling dactylopores, a cyclo-system being thus commenced.

Fig. 12. Horizontal section through the foregoing group at a slight depth from the surface to show the existence of styles in the pores of the dactylozooids.

Fig. 13. *Allopora profunda.* The connecting grooves between the pores of the cyclo-system are deeper. The system is regular, and the interval between the dactylopores have all the appearance of septa.

Fig. 14. *Allopora mineacea.* (Copied from POURTALES 'Deep Sea Corals,' plate 3, fig. 16.) Here the styles of the dactylozooids are brush-like in form, just like those of the gastrozooids.

Fig. 15. *Astylus subviridis.* There are no styles present in either kind of pore. The pseudoseptal system is complete. The open mouths of the tubular con-

tinuations of the dactylopores appear as a circlet of circular openings at the bottoms of the wide pseudo-interseptal spaces. The gastropore has two mouths, an upper circular and wider one, and a deeper constricted opening, which is rendered horseshoe-shaped by the projecting tongue of calcareous matter B.

Fig. 16. *Distichopora coccinea.* The pores are entirely confined to the central lines of the sides of the branches of the flabelliform coral. The pores here occur in regular straight rows. The gastropores form a median row, and on each side of this is a single row of dactylopores, the mouths of which are elongate in form with their longer axes directed towards the gastropores.

PLATE 36.

Section vertical to the external surface of the decalcified living lamina of *Sporadopora dichotoma.*

The main mass is seen to be composed of a network of ramifying and freely anastomosing canals. The canals are of larger diameter towards the base of the section, where they are continuous with the body cavities of the zooids; but in the most inferior region they are again smaller, being here somewhat atrophied and effete. Towards the outer surface of the coral the canals are of smaller diameter and enclose smaller interspaces than the larger deeper canals. The interspaces throughout the meshwork are, in the recent condition of the coral, filled by the calcareous corallum.

Lying in special cavities of the meshwork are seen a gastrozooid and two dactylozooids in the retracted condition, together with two sets of male gonophores and three nematophores. The calcareous style of the gastrozooid is introduced in order to show the position which it occupies in the retracted condition of the zooid.

G Z. External opening of the sac of a retracted gastrozooid.
O. Mouth of the gastrozooid.
S. Gastric cavity lined in its upper part by large elongate ovoid gastric cells; in its lower, by ordinary endoderm cells.
T. One of the tentacles of the zooid, of which a pair are seen in section.
M. Longitudinal muscular layer of the zooid. The muscles are continued down on to the four main cœnosarcal canals leading from the base of the zooid.
E. Ectoderm layer of the zooid.
C. Cavity of one of the four large canals into which the zooid cavity divides at its base in order to become continuous with the canal system of the cœnosarc. This canal is here shown as cut open, and is seen to be lined with endoderm cells, the layer of which is continuous with that lining the zooid cavity.
S′. The calcareous style, here introduced to show the position which it occupies

within the cavity of the zooid in the retracted condition of the latter. It is covered by a layer of ectoderm, and the endoderm lining layer of the zooid cavity is reflected over it.

F, F. Walls of the sacs of the zooids.

D Z, D Z. External openings of the sacs of two retracted dactylozooids, one of which is very small, the other of the largest size occurring.

B. Body cavity of the larger dactylozooid. In this zooid the ectoderm, E, is thrown into a series of folds in the retracted condition of the zooid. It presents on its outer surface a continuous layer of nematocysts. The zooid cavity is lined by a thick layer of endoderm. The zooid is attached to the side of the base of its containing sac, and is thus bent upon itself somewhat at its lower region.

R. Retractor muscles, continuations on to the main basal canal of the zooid of the longitudinal muscular layer, which is seen in section in the upper portion of the zooid.

N, N. Nematophores.

D. Surface layer of the ectoderm.

G, G′. Male gonophores. Those on the right seen in complete section, that on the left with its sac only opened, the generative masses being left intact. In G a ripe male sac filled with mature spermatozoa is seen situate nearest the surface of the coral, and beneath is an immature gonophore with its centrally placed spadix of endoderm. The axes of the two generative masses not lying in the same plane, the spadix is not seen in the upper riper sac, which is not divided by the section through the axis.

X, X. Spaces between the branches of the cœnosarcal network in the region immediately adjoining the sac of the zooid, where these branches have a peculiarly radiate arrangement, called here " inter-radial spaces."

A, A. Slips of fine membrane attached to the radial offsets.

PLATE 37.

Section vertical to the surface of a portion of a decalcified female stock of *Errina labiata,* showing the form and dispositions of the zooids and gonophores. The meshes of the cœnosarcal network are, as in *Sporadopora,* closer and smaller in the more superficial than in the deeper regions of the coral. The zooids are all inclined towards the tip of the branch of the coral.

D Z, D Z, D Z. Dactylozooids. In two of these the sac or lining membrane of the pore is shown as cut open, in order to exhibit its relations to the contained and partly retracted zooid.

D. A dactylozooid in process of development as a bud.

G Z. Mouth of the sac of a gastrozooid, which sac is cut open in order to show the contained zooid.

T. Tentacle of the zooid. O. Its mouth.

C. Gastric cavity. St. Style.

X, X. Spaces in the cœnosarcal network, homologous with the inter-radial spaces in *Sporadopora*.

Three gonophores are represented in the figure, showing three successive stages of development, the central one of the three being most advanced.

O V. Ovum. S. Spadix shown in section.

In the central gonophore the planula which is shown in section is doubled up somewhat, being fully developed, and ready to escape.

E C. Ectoderm of the planula.

E. Endoderm.

B. Membrane immediately covering the planula where not in contact with the spadix.

A. Surface layer of endoderm reflected over the wall of the ampulla.

In the remaining gonophore the developing planula and its spadix are shown entire, the surface membrane only covering the wall of the ampulla being seen in section.

P. Planula.

S′. Spadix showing the manner in which it forms a network and becomes digitate or fringed towards its outer margin.

N, N, N. Nematophores.

PLATE 38.

Longitudinal section through the axis of a branch of a stock of *Spinipora echinata*, decalcified, showing the cœnosarcal network and the surface membrane, with the various zooids in their mutual relations.

D Z, D Z, D Z. Indicate some of the larger dactylozooids, of which many are shown. These larger dactylozooids are situate near the extremities of spine-like processes of the corallum, represented here by the corresponding laminæ of soft tissue, extensions of the surface layer of ectoderm. The processes are grooved on the side lying towards the tips of the coral branches, for the reception of the zooids, which are all shown here as much contracted.

A, A. Examples of the fusing of two adjacent processes.

B, B. Processes represented as cut open in order to show the disposition of the bases of the dactylozooids within, and their connexion with the cœnosarcal network.

D, D. Small dactylozooids protruded from simple pores near the bases of the spine-like processes.

G Z, G Z, G Z. Gastrozooids seen ; some retracted within their sacs, others partially expanded. They have each a well-developed hypostome and six tentacles, and join the cœnosarcal network at their lower extremities by means of four main canals.

PLATE 39.

Vertical section, through one of the cyclo-systems of zooids of a male stock of *Allopora profunda* decalcified. Right and left of the centrally placed system seen in section, are represented parts of two other similar systems, which are shown as seen from their outer aspects by transmitted light, and not in section.

D Z, D Z. Dactylozooids retracted. Arranged in a circlet around the mouth of the pore of the gastrozooid.

P, P. Sacs, or soft tissue walls of the pores, of dactylozooids, separated from one another by pseudosepta. Two of these sacs are shown cut open to display the attachment of the bases of the zooids within them.

G Z. Sac of the gastrozooid, between which and the place occupied in the recent condition by the wall of the pore of the zooid is a wide space, traversed by radially disposed offsets of the cœnosarcal network, R R.

X, X. Interradial spaces between these effects.

Z. Gastrozooid with twelve tentacles, disposed in a single whorl, and an almost hemispherical hypostome. Large canals spring from the base of the zooid, and form communications with the basal canals of adjacent zooid cyclo-systems.

D Z′, D Z′. Dactylozooids in their sheaths seen from behind within the adjacent zooid cyclo-systems.

G, G, G. Gonophores. The sac of one of these is cut open to show the ovoid male generative masses and their spadices lying within.

PLATE 40.

Shows the structure of *Stylaster densicaulis* as seen in the decalcified condition. The cylindrical masses formed by three cyclo-systems of zooids are shown cut open, so as to demonstrate the arrangements of the zooids and cœnosarcal ramifications within them. The positions of the two additional cyclo-systems are indicated by outline. In the centrally placed cyclo-system the entire ramifications of the cœnosarcal canals are shown.

In the system displayed on the left side of the figure, the wall of the sac of the gastrozooid, and the superficial network beneath it, are removed, in order to show the connexions of the bases of the dactylozooids with the deeper canals.

In the system shown on the right in the figure, the details of the connexion of the base of the gastrozooid with the cœnosarcal canals only are given.

The gonophores are seen through the superficial networks existing in the walls of the ampulla.

Z, Z. Gastrozooids situate one at the base of each cyclo-system.

D Z, D Z. Dactylozooids. G G. Male gonophores.

N, N. Nematophores.

A. Reflection of the surface layer of the ectoderm forming the sac of the gastrozooid and lining the gastropore. The sac-wall is very thin and transparent. Curved lines crossing the dactylozooids transversely about their middles, in the central cyclo-system shown in the figure, mark the lower and innermost margins of the dactylozooid sacs, where these become continuous with the sac of the gastrozooid. In the cyclo-system shown on the left in the figure, the sac of the gastrozooid and the portions of the sacs of the dactylozooids fronting the gastropore are represented as removed, together with the superficial network of the cœnosarc immediately beneath them, in order to display the connexions of the deeper systems of large main canals which connects the zooids of the cyclo-system directly with one another.

B, B. Spaces in this deeper system of main canals.

PLATE 41.

Fig. 1. Shows the structure of the soft parts of *Astylus subviridis*. A single cyclo-system, divided in half, together with portions of the branch on which it rests, is represented in the figure. At the lower part of the figure the large cœnosarcal canals occupying the axis of the branch are seen passing right and left. These place the cyclo-system in connexion with the other adjacent cyclo-systems on the branches of the coral. The gastrozooid, which is devoid of tentacles, is seen resting retracted at the bottom of its sac (A).

The base of the gastrozooid is rounded and basin-shaped. Large canals spring from the margin of the basin to join the cœnosarcal meshwork, and carry into the general circulation the products of digestion; but none such arise from the direct under surface of the zooid.

G Z. Cavity of the upper chamber of the sac of the gastrozooid.

G Z′. Cavity of the lower chamber of the sac of the gastrozooid.

Z. The gastrozooid.

O. The mouth appearing as a crucial slit with symmetrically-arranged elongate gastric cells.

B. Tongue-like process of the wall of the gastropore which projects forwards horizontally over the summit of the retracted gastrozooid at a point where there is a sudden constriction of the pore. The projection of the tongue forms the opening of the constriction into a horseshoe-shaped aperture.

D Z, D Z. Dactylozooids retracted into their pores, and doubled down into the mouth of the sac of the gastrozooid.

P, P. Mouths of the sacs of the dactylozooids, occupying in the recent condition the dactylopores. These mouths are in this species elongate in outline, and simulate the interseptal spaces of anthozoan corals.

G, G. Male gonophores in special sacs, and springing from branches of the cœnosarcal network.

C. Deep axial cœnosarcal canals, of the branches of the coral on which the cyclo-systems rest.

S, S. Superficial networks of finer canals lying immediately beneath the superficial external layer of the ectoderm.

R, R. Radially disposed offsets of the cœnosarcal network springing from the sac of the gastrozooid.

X, X. Inter-radial spaces (*cf.* Plate 39, R R, X X).

Fig. 2. Shows the structure of the soft parts of a female stock of *Pliobothrus symmetricus.* The structure is exposed by means of a section vertical to the surface of the coral. The mass of the coral, the hard skeleton being removed, is composed of the usual cœnosarcal meshwork which is bounded externally by a continuous surface layer of ectoderm containing large nematocysts. Embedded in the meshwork are two kinds of zooids and the gonophores. The sac or sheath of the single gastrozooid shown in the figure is opened in order to display the zooid within.

Z. Gastrozooid. S placed in the neck of the sac of the gastrozooid.

X, X. Spaces in the meshwork corresponding to the inter-radial spaces in *Sporadopora.* Here the radial arrangement is hardly to be discerned.

D Z, D Z. Dactylozooids. The transverse lines drawn incircling the bodies of these zooids indicate folds into which the bodies of the zooids are thrown in extreme retraction.

O. Rests on a cup-shaped spadix, bearing a mature unimpregnated ovum, containing a germinal vesicle.

G. Impregnated ovum in an early stage of development.

P. Planula nearly mature contained within its sac.

Fig. 3. Male gonophore of *Pliobothrus symmetricus.*

PLATE 42.

Shows the structure of the soft parts of the *Cryptohelia pudica* displayed by decalcification.

The figure represents two cyclo-systems of zooids, together with the short branch of the coral connecting them. The cyclo-system on the left hand in the figure is represented as laid open by a vertical cut passing through the axis of the gastrozooid, and the disposition of the several parts is here shown in detail. The breeding sac of this system is in an early stage of development. The dactylozooids are shown as protruded.

In the cyclo-system on the right hand the superficial membrane is mostly left entire, and the cyclo-system is not opened, but a view is obtained into the open mouth of the gastropore showing the dactylozooids doubled down into it. The breeding sac is here shown in its fullest activity, and containing a planula ready for emergence. The sac is represented as cut open in order to exhibit the contained structures.

G Z. Cavity of the sac of the gastrozooid.
O. Mouth of the gastrozooid, which is devoid of tentacles.
S. Digestive cavity of the zooid lined with elongate gastric cells.
M. Longitudinal muscular layer of the zooid. As in *Astylus subviridis*, numerous canals pass off from the margin of the rounded base of the zooid, but none from the under surface of the base itself.
D Z, D Z. Dactylozooids.
P, P. Mouths of the sacs of the dactylozooids.
L, L. Peculiar protective laminæ, or lids, folded in this genus in front of the openings of the cyclo-systems. In the case of the left hand system of the figure the ramifications of the superficial network of the cœnosarc, prolonged into the body of the lid, are shown, and also the nematophores with which the lid is provided. In the other system represented, the superficial layer of the cœnosarc is shown intact. The downward prolongation of the base of the lid, which should pass between the sac of the gastrozooid and its main upward canals, is omitted for clearness sake.
N, N. Nematophores. One of these is situate at the outer margin of the cyclo-system in each interval between the sacs of the dactylozooids.
G. Female gonophore contained, with two others in earlier stages, in a special brood-sac as yet not far advanced in development.
G′. Brood-sac in full development containing ova and embryos in all stages of development.
Ov, Ov′. Fertilized ova in advancing stages of development and cell multiplication.
P 1. Early stage of planula embryo. The thin ectoderm appears as just differentiated.
P 2. Planula fully advanced with more fully developed and thickened ectoderm.
P 3. Fully developed planula with highly differentiated ectoderm containing nematocysts, &c.

PLATE 43.

Shows certain details of structure of the soft parts of *Sporadopora dichotoma* and of *Astylus subviridis*.

Fig. 1. Section transverse to the longitudinal axis of a gastrozooid of *Sporadopora*

dichotoma, taken at a point above the junction of the tentacles with the body of the zooid. Hence the tentacles appear in this section as isolated structures.

S. Digestive cavity.

G. Large gastric cells with their nuclei.

M. Muscular layer and basement membrane at this region of the zooid comparatively little developed. Shows longitudinal fibres cut across.

E. Ectoderm here containing very few nematocysts but numerous nuclei.

T, T. Tentacles in section.

Fig. 2. Section transverse to the longitudinal axis of a large dactylozooid of *Sporadopora dichotoma.*

B. Body cavity of the zooid.

L. Endoderm consisting of a double layer of large transparent nucleated cells with smaller cells on its inner aspect.

M. Muscular layer and basement membrane showing a series of stout isolated fibres.

E. Ectoderm. This forms a thick layer which contains numerous nuclei and developing nematocysts, and at its outer surface is protected by a continuous layer, one cell deep, of nematocysts of the smaller form.

Fig. 3. Section parallel to the surface of the soft tissues of *Sporadopora dichotoma* taken at a very slight depth just beneath the superficial layer of the ectoderm. The soft structures about the mouths of the sacs of a gastrozooid and a contiguous dactylozooid are shown, and the radiate arrangement of the cœnosarcal tubes around the sac openings.

G Z. Lumen of the upper part of the sac of a gastrozooid.

R M. Wall of the sac containing radially disposed fusiform muscular elements which are prolonged outwards on to the radial offsets of the cœnosarc. The wall of the sac is lined internally by small rounded cells.

R, R. Radially disposed offsets of the cœnosarcal meshwork which pass inwards to join the outer surface of the sac all round.

A, A. Slips of fine transparent membrane containing nuclei attached to the radial offsets.

X, X. Inter-radial spaces.

D Z. Lumen of the sac of a dactylozooid with somewhat similar arrangement to that in the case of the gastrozooid.

N, N. Nematophores. Closely packed batteries of the larger form of nematocysts, which are here seen in transverse section.

Fig. 4. Tentacle of a gastrozooid of *Sporadopora dichotoma,* from a sketch made of the animal in the fresh condition.

K. Elongate knoblike tip of the tentacle thickly beset with nematocysts of the smaller form.
E. Ectoderm.
Tr. "Apparent septa."
C. Axial cavity of the tentacle.

Fig. 5. Section transverse to the axis of the zooid of a segment of the body wall of a gastrozooid of *Sporadopora dichotoma,* taken near the lower region of the zooid.

S. Wall of the sac of the zooid seen in section.
E. Ectoderm.
M. Muscular and basement layer showing a series of stout longitudinal slips in section.
L. Layer of transparent endodermal cells, the representatives in this region of the larger elongate gastric cells which exist higher up in the body cavity.
E N. Pigmented endoderm cells, such as line the canals of the cœnosarcal meshwork.

Fig. 6. Portion of the muscular layer of the body wall of a gastrozooid of *Sporadopora dichotoma* viewed from its inner surface. The layer is seen to be composed of a closely set series of longitudinal narrow muscular slips. The layer is crossed by fine transverse striations, the nature of which was not determined, no definite circular muscular fibres having been detected in the zooids. The striations probably are caused by wrinkles in the basement membrane.

Fig. 7. One of the longitudinal muscular slips of the last figure, much enlarged, to show that it is composed of fusiform nucleate closely-packed elements.*

Fig. 8. These fusiform muscular elements still more magnified.

Fig. 9. Nematocysts of *Sporadopora dichotoma.*

a. Elongate form of nematocyst occurring only in the nematophores and surface layer of the ectoderm, and that investing the more superficial cœnosarcal canals.
a′. The same, with the thread protruded.
b. Smaller form of nematocyst, abundant in the tentacles of the gastrozooids, ectoderm of the dactylozooids, and other regions.
b′. The same, with the thread protruded.
c. d. e. Successive stages in the development of the smaller form of nematocyst.

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* N.B.—By error in plate, marked + 100 instead of + 600.

Fig. 10. Section through a portion of a male gonophore sac of *Astylus subviridis*, showing various stages in the development of spermatozoa (*cf.* Plate 41, G.)

S. Wall of the gonophore sac, a reflection of the ectoderm.

S′. Reflection of the same over one of the lobules of the generative mass.

S″. Thin membrane enclosing the spermatozoa within the lobule.

E N. Endoderm cells. These apparently continuous with those forming the lining of the canals of the cœnosarcal meshwork, X X, expand into a centrally-placed mass from which the lobules spring. The lobules as they approach maturity become attached by narrow pedicles.

a. One of the earliest stages in the development of a lobule.

b. More advanced stage showing a multiplication of the contained cells.

c. Further stage with more numerous and smaller spermatogenous cells.

d. Further stage. The character of the contained cells is changed.

e. More advanced stage.

f. g. h. Further stages. *h. h.* Containing mature spermatozoa.

Fig. 11. Successive stages in the development of the spermatozoa of *Astylus subviridis.*

a. Ordinary endoderm cell.

d. The opaque constituents of the cell are entirely condensed into the nucleus.

e. Smaller similar cell produced by division of the above.

e′. f. g. Successive stages showing the development of the spermatozoon from the nucleus.

Fig. 12. Ripe spermatozoa of *Sporadopora dichotoma* as seen in the living condition. A vesicle is present situate between the head and commencement of the filament of the spermatozoon. In the case of the lower spermatozoon of the three figured, the head, which is flattened, is viewed edge on.

PLATE 44.

Fig. 1. Gastrozooid of *Cryptohelia pudica,* removed from its sac, and viewed directly from above so as to be much foreshortened in the drawing.

A. The cruciform mouth, with a lining of elongate gastric cells.

B. Main canals given off from the outer margin of the base of the zooid all round. These soon branch, and join by their offsets the general cœnosarcal meshwork.

The zooid is seen to be devoid of tentacles.

Fig. 2. Part of a section, cut at right angles to the axis, of a cyclo-system of *Astylus subviridis,* showing the structures surrounding the pores of the dactylozooids at the margin of the system.

D. Transverse section of a dactylozooid, showing ectoderm, endoderm, and intermediate muscular layer.

P. Cavity of the sac of the zooid, occupying, in the recent condition, the wide upper chamber of the dactylopore.

D′. Another dactylozooid, seen in section. The zooid, being doubled back into the outer part of the dactylopore where cut in section, its cavity appears partly as a lumen at T′, partly as an elongate hollowed area, in which are seen the strong longitudinal retractor muscles of the zooid.

C, C. Tortuous canals, offsets of the general cœnosarcal meshwork, which pass radially outwards in the substance of the pseudosepta, between the pores of the dactylozooids. The canals ramify as they reach the outer margin of the calicular system, and join by their branches the superficial outer network of the cœnosarc.

N. Large ovoid nematophore, full of closely-packed nematocysts. One such nematophore is present in each interval between the outer margins of the mouths of the pores of the dactylozooids.

Fig. 3. Earliest stage in the development of the ovum in *Cryptohelia pudica.*

A bud-like mass of endoderm cells is gathered together within an offset of a branch of the cœnosarcal meshwork.

Fig. 4. The same, in a further stage of development.

O. Ovum, with germinal vesicle and spot.

S. Spadix, composed of endoderm cells.

E. Thin layer of the ectoderm, continuous with that covering the spadix, and investing the free surface of the ovum.

Fig. 5. Section through a planula of *Cryptohelia pudica,* in a very early stage.

E. Ectoderm.

En. Endoderm.

Fig. 6. Portion of a planula of *Cryptohelia pudica,* in a more advanced stage than the foregoing, viewed from the outer surface.

E. Ectoderm, transparent, and showing a demarcation into the polygonal areas.

En. Endoderm cells, seen through the transparent ectoderm.

Fig. 7. Section vertical to the surface of a planula of *Cryptohelia pudica* when fully developed and ready for exit from the broad sac.

En. Endoderm, composed mainly of oily globules.

E. Ectoderm, which is extremely thick, and for the most part transparent and gelatinous in appearance.

N, N. Nematocysts.

A. Tracts composed of small, rounded, non-transparent ectodermal elements, which run from the endoderm region at intervals vertically, towards the surface of the planula. In these tracts the nematocysts are developed.

B. A layer of the same elements lying next to the outer surface of the endoderm.

Fig. 8. A portion of the same planula, viewed from the surface, to show the manner in which the surface is marked out into polygonal areas, by the special arrangement within the substance of the ectoderm of the nematocysts and tracts in which they are developed.

N, N. Nematocysts, seen in optical section.

A, A. Elements amongst which they are developed.

Fig. 9. Section, vertical to the surface, of a nearly-mature planula of *Errina labiata*.

E. Ectoderm, composed of alternate transparent and more opaque tracts, disposed vertically to the surface.

A. The more opaque tracts, containing numerous nuclei and young thread cells.

B. Basement membrane.

En. Endoderm, composed of fatty bodies and granules, and containing—

N, N. Developing nematocysts;

O. Large oil globules.

Fig 10. Portion of the surface layer of the ectoderm of *Errina labiata*, viewed from the outer surface.

S, S. Polygonal nucleated cells composing the layer. These in places overlap.

N, N. Nematocysts seen *in situ* within these transparent superficial cells.

Fig. 11. Section, vertical to the surface, of the ectoderm of a gastrozooid of *Errina labiata*.

E. Superficial layer, composed of inflated transparent nucleated cells.

E′. Inferior layer of the ectoderm containing numerous nuclei.

Fig. 12. Section, transverse to the axis, of a cyclo-system of *Allopora profunda*, taken at some little distance below the level of the mouths of the pores of the zooids, showing the sac only of the retracted gastrozooid in section, but both sacs and zooids of the dactylozooids. The whole is represented as decalcified, with the exception of the styles of the dactylozooids, which are introduced to show the position which they occupy in the recent state of the coral (cf. Plate 39).

R, R. Radially disposed offsets of the cœnosarc.

G Z. Cavity of the sac of the gastrozooid.

X, X. Interradial spaces between these.

D Z. Dactylozooid, showing in section its three composing layers, ectoderm, endoderm, and intermediate muscular and membranous layer.

C. Style of one of the dactylozooids, seen in section.
B. Large canals of the cœnosarc, occurring in the pseudosepta or intervals between the dactylopores.
S. Surface layer of the ectoderm. The main mass of tissue is composed of the finer ramifications of the cœnosarcal meshwork.

Fig. 13. Small portion of the cœnosarcal meshwork of *Sporadopora dichotoma*, greatly magnified in order to show the histological structure; as seen in osmic acid preparations.

C. Channel of the canal.
En, En. Endoderm layer.
M M. Membranous layer.
E E. Ectoderm.
T. Nematocyst in process of development.

Fig. 14. Two pigmented cells of the endoderm of the same, highly magnified.

POSTSCRIPT.

(Added September 24, 1878.)

Since the above paper was written, I have been able to examine the structure of the soft parts of a species of *Distichopora* (*D. Violacea*) of which I obtained specimens preserved in spirits from the Museum Godeffroy, in Hamburg.

The structure of the soft parts is essentially similar to that in *Errina.* In the dactylozooids, however, which are stoutly formed, there is present an excessively long muscular slip which runs down very far into the long tubular pores, and must be attached at the bottoms of these. The gastrozooids are short and cylindrical, with four small clavate tentacles. The gonophores appear identical in structure to those of *Errina* and *Pliobothrus.* Both male and female specimens were examined. The latter are distinguished by the prominence of the ampullæ. In the males, the ampullæ are invisible from the anterior, being sunk beneath the surface. All stages of development of the ova were seen, and they appear identical with those of *Errina* and *Pliobothrus.* In the male no spadix was made out within the gonangia, but these contained four or five ovoid masses full of spermatozoa and their parent cells.

Amongst the corals dredged by the 'Challenger' is a *Distichopora* which I had not seen until after the present paper was written, which I have named *D. irregularis,* in which the lines of pores are unusually irregular in their disposition, and traverse the faces of the flabellum all over as well as the edges of the branches as usual.

Further, specimens in spirit, of *Lepidopora cochleata,* and *Pliobothrus tubulatus,* were most kindly sent to me by Count de POURTALES, and I was able to determine some points in the structure of the soft parts of these, though they were not completely hardened and in best condition, not having been specially prepared with a view to microscopic examination.

Lepidopora cochleata is essentially similar in the structure of the soft parts to *Errina labiata*. The dactylozooids are especially numerous and closely packed. The gastrozooids have four tentacles. The nematophores are like those of *Errina*, as are also the female gonophores. A female specimen only was available for examination. The specimen of *Pliobothrus tubulatus* was not well preserved, but in the gastrozooids there was seen an appearance as of very short tentacles more than four in number. I am uncertain whether tentacles are really present; it seems improbable that such can be the case.

From a study of the specimens of *Stylaster Madeirensis* (JOHNSTON), *Stenohelia Madeirensis* of KENT, in conjunction with some specimens obtained by H.M.S. 'Challenger,' but only lately available for examination, I conclude that the genus *Stenohelia* should be preserved with the following characters:—

Genus *Stenohelia*, SAVILE KENT (Ann. and Mag. Nat. Hist., 1870, vol. v. p. 120).—Corallum delicate; branching flabelliform; pores in regular cyclo-systems only. Cyclo-systems all turned towards one face of the flabellum. Dactylopores without a columella or with a very rudimentary one. Gastropores very deep and curved, so as to tubulate in all but the older branches the entire lengths of the axes of the branches; with small styles, seated at the bottoms of these tubes and directed parallel to the axes of the branches at right angles to those of the mouths of the cyclo-systems.

Species *Stenohelia Madeirensis*, KENT.

Stenohelia profunda, sp. n., H. N. M.—Off St. Thomas, Danish West Indies, 450 fathoms.

A further specimen, also dredged by H.M.S. 'Challenger,' seems to require the formation of an additional genus which I call *Conopora*, of which the following are the characters:—

Genus *Conopora*, gen. nov., H. N. M.—Corallum delicate; with pores in regular cyclo-systems; branching irregularly, the cyclo-systems having their mouths turned in all directions. Cyclo-system masses conical in form. Both kinds of pores devoid of a style. Gastropore with two chambers, the upper opening into the lower by a circular aperture. Differs from *Cryptohelia* and *Astylus* in having no lid or tongue-like process and in not forming a regular flabellum.

Species *Conopora tenuis*, sp. n., H. N. M.—Dredged off the Kermadec Islands, in 650 fathoms.

Possibly this species should be placed in the same genus as VERRILL'S *Cyclopora bella*, = *Stylaster bella* (DANA), see p. 481; but the descriptions in the old terminology are inadequate to determine the point. If so, *Cyclopora* may be revived as a genus with the above characters.

XV. *On the Development of the Parasitic Isopoda.*

By J. F. BULLAR, *B.A., Trin. Coll., Cambridge.*

Communicated by Dr. MICHAEL FOSTER, *F.R.S., Prelector of Physiology in Trinity College, Cambridge.*

Received March 14,—Read April 4, 1878.

[PLATES 45–47.]

THE following paper contains an account of observations on the development of the species *Cymothoa œstroides* and *C. parallela* of MILNE EDWARDS; but the forms of the young seem to show that several species are really included under these two names. In the early stages of development the only observable difference that exists between the embryos is one of size, but in the later stages they differ very markedly from each other in their external characters. From adult individuals answering the description of *C. œstroides* I have obtained four varieties of embryos: two with long antennæ and two with short.[*]

In the two former the first pair of antennæ are but slightly longer than the head, while the second pair are longer than the body; the eyes are small. In one of the varieties thus characterised the abdominal appendages are fringed with long hairs (fig. 20), and in the other they are smooth.

In the two latter both pairs of antennæ are short, and of about the same length, and the eyes are large. This group also contains one variety with smooth, and one with hairy abdominal appendages. From *C. parallela* I have obtained but a single form of embryo (fig. 21), which differs from the embryo of *C. œstroides* with short antennæ and smooth abdominal appendages only in being smaller, and not having so many pigment spots on the dorsal surface.

My work has been mainly carried on in the zoological station at Naples, and I take this opportunity of thanking Dr. DOHRN and Dr. EISIG for the kind way in which they have forwarded my researches. I have prepared the eggs in the way described by BOBRETZKY (Zeit. für Wiss. Zool., bd. xxiv.) in his paper "On the

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[*] *C. œstroides*	I. Embryos with second pair of antennæ long. Eyes small	1. Abdominal appendages hairy. 2. Abdominal appendages smooth.
	II. Embryos with second pair of antennæ short. Eyes large	3. Abdominal appendages hairy. 4. Abdominal appendages smooth.

Development of *Oniscus Murarius*," namely, by heating them in water and then hardening them, first in bichromate of potash and then in alcohol, beginning with 70 per cent., and gradually increasing the strength to absolute. Some of the specimens were embedded in a mixture of solid paraffin and ordinary paraffin oil,* others in spermaceti and castor oil. In all cases the sections were stained with KLEINENBERG'S hæmatoxylin and mounted in Canada balsam. I found that the most convenient way to heat the eggs was to put them in a test tube with a little sea water, and hold this in a vessel of boiling water for a few minutes.

The eggs are carried by the parent in a large brood-pouch until the young are able to swim as among the non-parasitic Isopoda. I have found both the adults† and young in all stages of development from the beginning of January to the end of June, so breeding probably continues all the year round.

The eggs when first laid are surrounded by a single somewhat tough structureless membrane (fig. 1, O.M.).

I have not been able to observe the first stages of segmentation; the earliest which I have to describe is that in which a circular patch of cells has appeared on the egg (fig. 1). The cells have already become rather numerous, so that a diameter of the blastoderm passes through about twenty. The cells (figs. 1 and 2) are of considerable size, and contain very large granular nuclei. Seen from the surface, they appear of much the same size in all parts of the blastoderm; but a section shows that in the central part they are polygonal in shape and more than one layer thick, while towards the circumference they form only a single layer, and at the extreme edge become flattened. Extending from the edges of the blastoderm over the yolk is an exceedingly thin granular layer.

The cells increase in number and decrease in size, the blastoderm spreads more and more over the yolk, and the central thickening, or *Keimstreif*, losing its circular outline, begins to shape itself into the form of the future embryo. In fig. 2A the head end of the embryo is already distinct, and the two frontal lobes (Fr.) have appeared, though as yet there are no traces of appendages. The posterior part of the embryo is still quite indistinct. Whether the blastoderm spreads by the division of the cells of the circular patch described above as in *Oniscus*‡ and *Mysis*, or by the separation of fresh protoplasm from the food-yolk as in *Asellus*,§ I have not been able to determine. The *Keimstreif* becomes more and more elongated, and soon attains the stage shown in fig. 3. In addition to the original membrane, a second very thin one has now appeared, which is for the most part closely applied to the surface of the yolk, but at certain points is slightly separated from it and is easily seen. It is not cellular in structure.

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* The kind used for lamps.

† "Gen. organs of Parasitic Isopoda," 'Jour. Anat. and Physiol.,' vol. xi., p. 118.

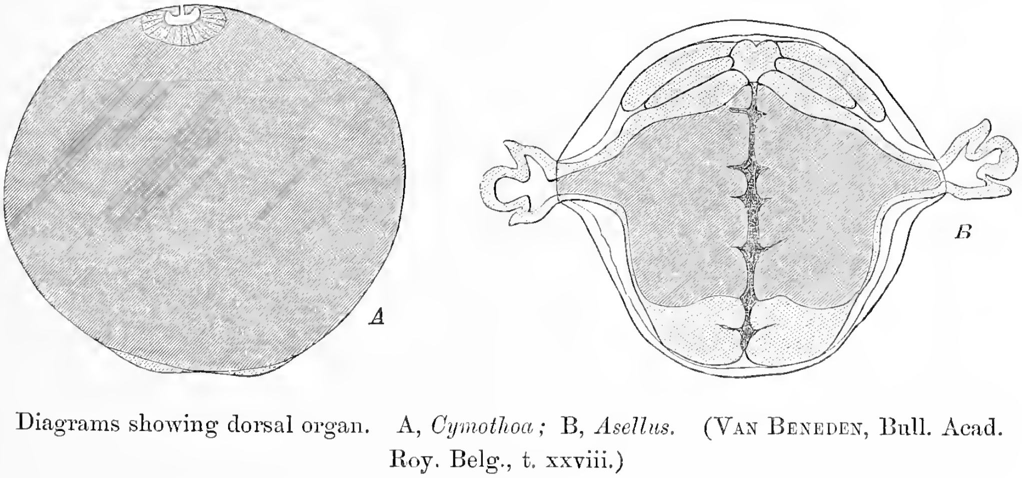
‡ BOBRETZKY, *loc. cit.*

§ VAN BENEDEN, Bull. Acad. Roy. Belg., t. xxviii.

Fig. 3 shows the ventral side of the egg at this stage. The segments of the head and thorax are quite distinct; but in the abdominal region they have not yet become distinguishable. A depression (M) at the anterior end of the embryo is the involution to form the fore-gut; it is very slight, and appears as a dark mark when seen from the surface. Extending from the mouth towards the hinder end of the body in the median line, is a dark mark caused by the surface of this portion of the embryo being lower than the rest. It is along this line that the nervous system will be developed. The ventral half of the egg is covered by a layer of flattened cells extending in all directions from the edges of the *Keimstreif*, and in the hardened egg appears to the naked eye much lighter in colour than the dorsal surface. With the microscope it may be seen that there are cells also on the dorsal surface, but they are very much thinner and less easily seen than those on the ventral surface. On the dorsal surface of the egg at the anterior end is a transversely elongated band of cells (fig. 4, D) connected with the embryo only by means of the flattened dorsal cells. This is the rudiment of a peculiar organ which at a latter period occupies the dorsal part of the first thoracic segment, and which I shall speak of in future as the *dorsal organ*.

It may be well at this point to say a few words about this organ, and the various structures which have been regarded as identical with it in other crustacea. In *Cymothoa* it does not long retain the structure described above, but soon becomes circular, and the cells composing it more columnar. As development proceeds it increases in size, and causes a considerable depression in the yolk; at the same time, a cavity is developed in its interior, of the shape shown in the diagram A (fig. 1, *woodcut*). It is now seen to be in connexion with the inner egg-membrane, though whether this has been the case all along I am unable to say. Having attained this form, it undergoes no further modification during the period in which I have been able to observe it. Considerably after the embryo had escaped from the egg-

Fig. 1.



Diagrams showing dorsal organ. A, *Cymothoa;* B, *Asellus.* (VAN BENEDEN, Bull. Acad. Roy. Belg., t. xxviii.)

membranes it was still present, and it then showed no signs of atrophy. In the adult, however, I could find no trace of it.

Organs more or less resembling this have been described in many of the Crustacea, and in some other groups of Arthropods. In the Amphipods* it appears to be constantly present, and tolerably uniform in structure. It consists of a circular mass of cells attached to the inner egg-membrane, in which usually a cavity becomes developed. It occupies the same position as in *Cymothoa*, and disappears probably about the same time.

Among the Isopoda it is much more variable. In *Oniscus*† it appears first as a patch of rounded cells on the dorsal surface of the embryo. At a later stage this disappears, and is replaced by a short cellular stalk, from the upper end of which a cellular membrane spreads, like a broad saddle, over the back and sides of the embryo.

In the embryo of *Asellus*‡ the well-known paired leaf-like appendages probably represent the dorsal organ. They arise on each side of the body behind the head as simple oval masses of cells, which soon increase in size and become three-lobed. A cavity is formed in each lobe, which becomes filled with fluid. The three cavities eventually unite, and form a single trilobed cavity open to the yolk, in which free cells and drops of food-yolk are sometimes found. At a certain period these organs attain so great a development that they break through the egg-membranes, and protrude freely on each side of the embryo. Diagram B (fig. 1, *woodcut*), taken from one of VAN BENEDEN's figures, represents these organs in this condition. They occupy nearly the same position as the two ends of the elongated cellular band found in the early stages of *Cymothoa*.

In *Praniza*§ an unpaired dorsal organ is formed in the usual position. It consists of a rounded mass of cells, and is in connexion with the inner egg-membrane, which at an early stage is evidently cellular, especially in front.

Among the other groups of Crustacea, *Cuma*‖ possesses a similar organ.

In *Mysis*¶ a mass of cells appears on each side of the body behind the head, in the same position as the leaf-like appendages of *Asellus*. A hollow appears in each of these masses, which becomes filled with fluid, and apparently communicates with the yolk. They disappear very early. It is very doubtful whether these organs occur in other Podophthalmata.

In the Spiders,** a mass of cells, "cumulus primitif" of CLAPARÈDE, appears very

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* MEISSNER, Zeit. für Wiss. Zool., bd. vi.; LA VALETTE, Abhand. der Naturforsch-Gesell. zu Halle, bd. v.; FRITZ MÜLLER, 'Facts for Darwin'; EMIL BESSELS, Jenaische Zeit., bd. v.

† BOBRETZKY, Zeit. für Wiss. Zool., bd. xxiv.

‡ DOHRN, Zeit. für Wiss. Zool., bd. xvii.; E. VAN BENEDEN, Bull. Acad. Roy. Belg., t. xxviii., 1869.

§ DOHRN, Zeit. für Wiss. Zool., bd. xx., 1870.

‖ DOHRN, Jenaische Zeit., bd. v., 1870.

¶ E. VAN BENEDEN, Bull. Acad. Roy. Belg., t. xxviii., 1869.

** CLAPARÈDE, 'Recherche sur l'Evolution des Araignées.'

early on the dorsal surface of the embryo, and was supposed by its discoverer to represent the dorsal organ of the Crustacea.

LEUCKART is stated to have described a similar structure in the Pentastomidæ, but, unfortunately, I have been unable to refer to his paper. Various speculations have been put forward with reference to this organ, the most important of which will be found in two papers by Dr. DOHRN—one in the 'Journal of Anatomy and Physiology,' 2nd series, vol. i., 1868, and the other in the 'Jenaische Zeitschrift für Naturwissenschaft,' bd. v., 1870—and in a paper by M. EMIL BESSELS, 'Jenaische Zeit.,' bd. v., 1870. In the two last papers full references are given to the extensive, literature of this organ. A lengthy discussion of these theories would be out of place here, but I may say that they all agree in regarding the dorsal organ, including the leaf-like appendages of *Asellus*, as being rudiments of some ancestral structure, and no longer of any use to their possessors. There is, no doubt, a good deal to be said for this view; and in cases such as *Cuma*, *Praniza*, and more especially the Spiders, where the organ is never large and soon disappears, it may well be regarded as a rudiment. But in the case of *Asellus*, *Oniscus*, and *Cymothoa*, the organ attains a considerable size, and its production must be a great tax on the resources of the egg—so great, indeed, that it is difficult to believe it would have retained such dimensions unless it performed some important function. It seems possible that the great variations in structure which it presents in different animals may be explained by supposing that it is really the remains of some lost structure, and that, in some cases, it is useless and has nearly disappeared, while in others it has been taken advantage of to perform various special functions, and has thus become modified in various ways.

I will now return to the development of *Cymothoa*.

Sections of an embryo at stage II. show that the ventral wall is formed of a single layer of columnar cells, beneath which are a few scattered cells more or less easily distinguishable as a distinct layer. From the columnar epiblast cells will be developed the epidermis, the central nervous system, the eye, and the fore- and hind-guts, together making up practically the whole gut of the adult. From the lower layer of cells the muscles, connective tissue &c., the liver, and a temporary mid-gut (Dottersack) surrounding the yolk, will be developed.

Fig. 5 is a tranverse section through the thorax; in the centre a slight hollow is seen, formed by the lateral parts being somewhat raised above the general level of the surface of the egg. The hollow is the cause of the dark mark running down the ventral line (fig. 3); from the raised lateral portions the limbs will be developed. At the edges of the *Keimstreif* the columnar epiblast cells (Eb.) alter in character rather rapidly, and pass into the flattened cells spreading over the yolk.

At the anterior end of the embryo the epiblast is thickened on each side of the median line, and consists of several layers of cells (fig. 6). It is from these two lateral thickenings that the cerebral ganglia and eyes will be developed. In longitudinal sections (fig. 7) the segmentation of the body is indicated by the epiblast being thrown into a series of more or less distinct undulations. The lower layer of cells is thicker beneath the centre of each segment than between the segments, but it cannot be said to be distinctly segmented. At the posterior end of the body the cells are less numerous than at the anterior end. Some of these cells are slightly sunk in the yolk and appear to be isolated from the rest, but they may very possibly have been connected with them by means of other cells which lay outside the plane of the section, and which have thus been removed. They are not nuclei, but distinct nucleated *cells*.

On the dorsal surface of the egg, towards the anterior end, the dorsal organ is seen in transverse section (fig. 7D); it consists of a single layer of small cells, thickened in the middle, and causing a slight depression on the surface of the yolk. A series of flattened cells may be seen extending over the surface of the yolk for some distance from both the head and tail ends of the *Keimstreif*, and elongated nuclei occur at various points beyond them.

Though these cells do not appear to form a continuous layer in any one section, yet, after the examination of a number of sections and from the appearance of the entire egg, I have no doubt that they do so in reality.

Their non-appearance as a continuous layer in a single section is probably due to their extreme thinness and to the fact that, like the cells of the dorsal organ, they do not stain nearly so readily as the other cells of the embryo.

In the next stage the inner membrane has become thicker and is less closely applied to the yolk. All the segments are now quite distinct except the seventh thoracic, which does not make its appearance till a later period.

The direction of the telson (fig. 9, T) should be noticed. It is generally stated that one of the differences between the Amphipoda and Isopoda consists in the direction of the tail of the embyro, which is bent downwards in the Amphipoda and upwards in the Isopoda. Thus FRITZ MÜLLER ('Facts for Darwin,' p. 71) says: "The curvature of the embryo upwards instead of downwards was met with by me, as well as by RATHKE, in *Idothea*, and likewise in *Cassidina*, *Philoscia*, *Tanais*, and the Bopyridæ; indeed, I failed to find it in none of the Isopoda examined by me." And at p. 74: "The Amphipoda are distinguishable at an early period in the egg by the different position of the embryo, the hinder extremity of which is bent downwards." If my figure (fig. 2, B, *woodcut*) is compared with VAN BENEDEN'S[*] figure of *Mysis*, and with BOBRETZKY'S[†] figures of *Oniscus*, we shall, I think, be able to see the way in which this difference of bending has come about. In *Mysis* (A), the posterior end of the body is formed very early, and is bent sharply forwards. In *Cymothoa* (B), development proceeds gradually from before backwards, and the posterior end is the last part to appear; it is bent forwards in precisely the same way as in *Mysis*, but naturally forms a smaller protuberance as it here represents but a single segment (the telson), while in *Mysis* it represents several. If the embyro *Mysis* were drawn out as

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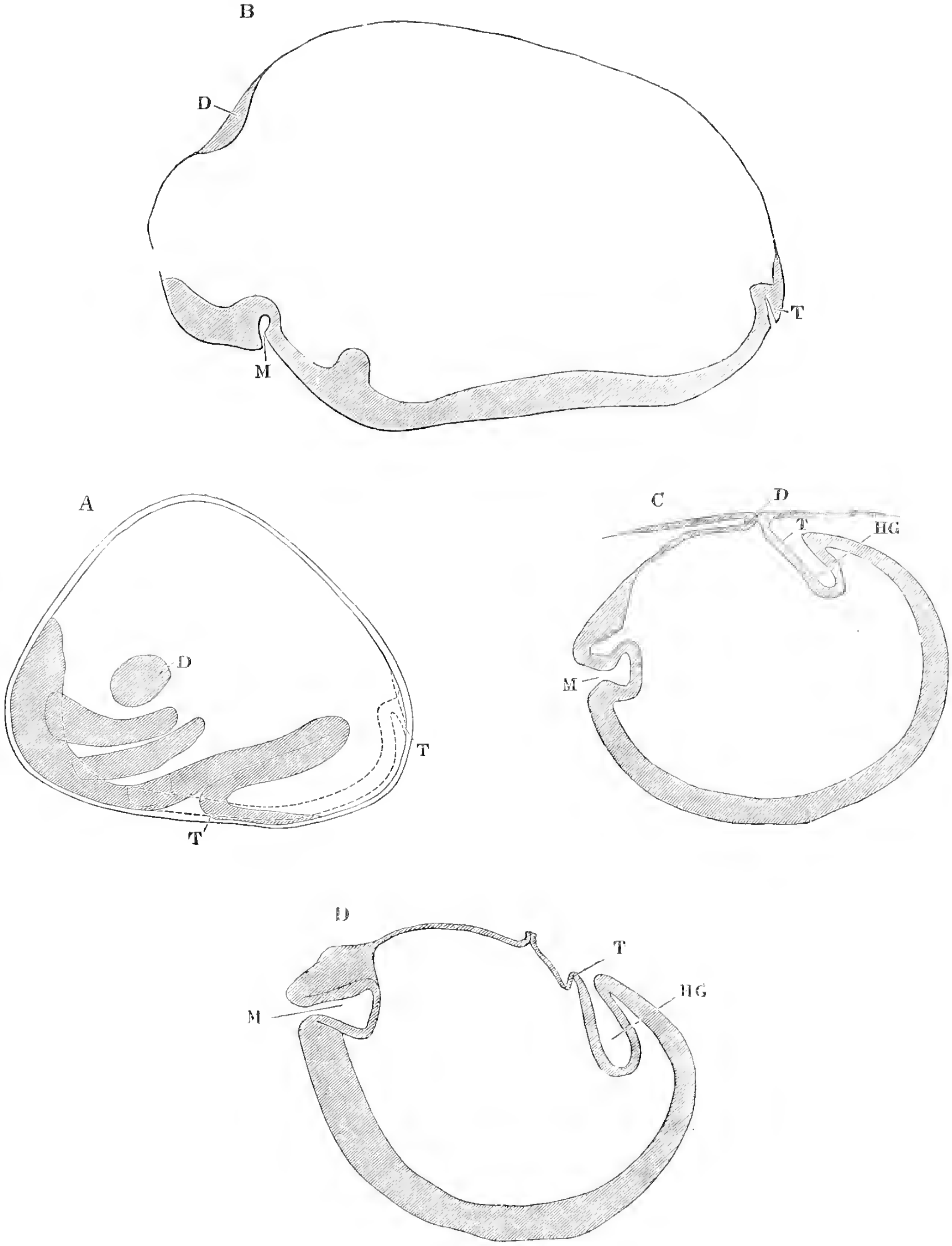
[*] "Dévelopment des Mysis," Bull. Acad. Roy. Belg., t. xxviii., 1869.

[†] Zeit. für Wiss. Zool., bd. xxiv., pl. xxii., figs. 15 and 17.

Fig. 2.



Diagrams illustrating the curvature of the embryo.
A. *Mysis* (Van Beneden, Bull. Acad. Roy. Belg., t. xxviii., 1869). B. *Cymothoa.*
C and D. *Oniscus* (Bobretzky, Zeit. für Wiss. Zool., bd. xxiv.).

indicated by the dotted line it would almost exactly correspond with *Cymothoa*, and this drawing out is what would naturally happen if the development proceeded more gradually from before backwards as in *Cymothoa*. In *Oniscus* (C) the drawing out is completed, the embryo grows almost quite round the egg, and the hind-gut (HG) appears before the telson is formed. When it does appear (D) it is directed dorsally, not on account of any change in its real direction of growth, but because its development has been delayed so long that the embryo has had time to grow nearly round the egg, causing the whole of the posterior ventral surface to point dorsally.

Cymothoa may thus be regarded as a connecting link between forms like *Mysis*, in which there is a strong forward bending, and forms like *Oniscus*, in which there is apparently an equally strong backward bending.

The rudiments of the limbs have appeared as small rounded projections, whose free ends point toward the middle line: they are formed of a fold of epiblast, and their central part is filled with a number of cells from the lower layer. Fig. 8 is a transverse section through the thorax; it is slightly oblique, and so does not pass through the point of junction of both the limbs with the body. In the median line between the limbs a thickened mass of epiblast has appeared from which will be developed the ventral nervous system. At this stage it still consists of undifferentiated epiblast cells, and has not become separated from the external or epidermic layer of the epiblast.

The only other point to be noticed in this section is the increase in the number of the cells of the lower layer. Fig. 9 is a longitudinal section slightly to one side of the middle line. At the anterior end the cerebral ganglion is very well marked, and some fibrous tissue has been formed in it. Immediately behind this is the involution to form the mouth and fore-gut; it has increased considerably in size since the last stage. The most interesting feature of this section is the solid rounded mass of lower layer cells (fig. 9, L) situated in the third post-oral segment from which the liver will be developed.

The involution to form the hind-gut has begun, but is less marked than the oral involution. Behind it the telson (fig. 9, T) is very distinct.

The dorsal organ now consists of several layers of cells, and has become circular in shape. In its interior a peculiarly-shaped cavity has made its appearance, and it rather closely resembles one of the "blattformige Anhänge" of *Asellus*.

In the fourth stage the egg has altered somewhat in shape; in the last stage it was a nearly regular oval, but now the head has become more distinct. The positions of the eyes are marked by slight prominences, and the abdominal region has become narrower. The limbs can be distinctly seen through the egg-membranes; they have increased a good deal in size, and are now marked by slight constrictions indicating the positions of the future joints.

Fig. 10 is a transverse section through the anterior part of the thorax. In the middle line between the appendages (fig. 10, Ap.) the ventral nerve cord (N) is seen. The external layer of epiblast cells has become separated from it everywhere, except at the most ventral point. In the dorsal part of the nervous system two fibrous cords have been formed; these are continuous with the fibrous tissue in the cerebral ganglion. Longitudinal sections show that the ventral nervous system is no longer a continuous mass of cells, but that segmentation has begun at the anterior end and is extending backwards. The bodies marked Ap. are portions of the appendages, which owing to their oblique position have been cut nearly transversely. Their central part consists, as it did in the last stage, of undifferentiated cells. The cells of the lower layer are especially abundant in the neighbourhood of the liver, two of whose cœca are seen on the right and one on the left hand side of the figure. These tubes have been developed from the solid mass of cells seen in the last stage (fig. 9, L).

Their relations to the other parts of the embryo are best seen in a horizontal section, such as that represented in fig. 11. This section passes through the liver on both sides of the animal. Each half of the liver consists of three very short cœcal tubes, opening by a common aperture to the yolk, and, at any rate as far as can be made out from sections, quite unconnected with each other, and with the fore and hind-guts. Between, and a little in front of the liver cœca, the section passes through the top of the fore-gut (FG.) Both the fore- and hind-guts have increased a good deal, and the fore-gut has become dilated at its posterior end to form the stomach.

At the sides of the body the epiblast is folded at each segment, and beneath it are a quantity of still undifferentiated cells, from which the lateral muscles, &c., will be developed. There are also some scattered cells round the liver cœca, and a large number near the fore-gut and cerebral ganglion. The large masses of cells at the anterior end of the section are parts of the cerebral ganglion. The two of these which are in apposition in the middle line, are continuous, on the one hand with the circum-œsophageal cords, and on the other, at a point dorsal to that through which the section passes, with the lateral masses of the cerebral ganglion.

The epiblast covering the sides of the head has become split off from the ganglion, and already at this stage is considerably thickened in the position of the eyes.

Before going on to the next stage it will be best to notice shortly the fate of these two thickenings. I will first describe the structure of the eye in the adult.* Fig. 12 represents somewhat diagrammatically a section passing through the eye transversely to the long axis of the head. Only two divisions of the eye are shown, but as these are all alike they will be enough for the purpose. Beginning with the external surface of the eye, each division is covered by a thickening of the cuticle (C); beneath this comes a transparent, refractive, nearly spherical body (*kristal kegel*), somewhat flattened in front, surrounded by seven large cylindrical pigmented cells (Z). At a short distance beneath the *kristal kegel* these cells are marked by

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* For an account of the structure of the eye in the different groups of the Arthropoda, see "Untersuchungen über das Arthropoden-Auge," Dr. H. Grenacher, 'Klinische Monatsblätter für Augenheilkunde.' Mai-heft xv. Jahrgang. Rostock, 1877.

a constriction, and appear to pass through the pigmented membrane (Mbn.) which is parallel to the external surface of the eye. Extending from this membrane to the cuticle, and separating the different divisions of the eye from each other, are a number of pigmented connective tissue partitions (C.T.). These parts are shown in transverse section—that is, in a section made parallel to the surface of the eye—in fig. 13, which represents a single complete division and parts of the partitions (C.T.) dividing it from the six adjacent divisions. The circle (X) marks the position of the *kristal kegel* as it would be seen if the focus were adjusted to a point slightly above the level of the paper. Below this are the seven cells (Z) arranged so as to radiate like the spokes of a wheel from the centre of the *kristal kegel;* each cell at its inner edge contains a small transparent unpigmented body (*Sehstäbchen*).

Following these cells (fig. 14, Z) towards the brain they pass through the membrane (Mbn.), run on separately for a certain distance, and then all coming into close contact, become again constricted and pass into a fine bundle of nerves (figs. 12 and 14, Z'') which soon becomes lost in the fibrous tract marked Op. T. (fig. 14).

All the parts of the eye external to the membrane (Mbn.) are developed from the epiblastic thickenings. Whether the parts internal to this, marked Z' and Z'', are developments from the epiblast, from the cerebral ganglion, or from both, I have not been able to determine.

In the fifth stage the external appearance is much the same as before; the head and abdomen are a little more distinct, and the limbs have increased in size, the antennæ now covering the mouth appendages. A little pigment has appeared at the posterior border of the eyes.

The segmentation of the ventral nervous system is now complete, and there are sixteen distinct ganglia; these are well shown in fig. 15: a longitudinal section. The two fibrous cords extend to the posterior end of the nervous system, and the skin is now entirely separated from the ganglia.

The mouth and œsophagus are very narrow; the stomach has become larger, and two projections have appeared in its floor.

The hind-gut extends forwards as far as the anterior edge of the fourteenth ventral ganglion. Its anterior end is still closed.

The liver cœca are a great deal larger than in the last stage, and have come into contact with the fore-gut.

Above the hind-gut, and extending beyond it in front as far as the tenth post oral ganglion, and therefore occupying the same position as in the adult, is the heart. Its walls consist of elongated nucleated cells. From its anterior end a band of cells passes forwards to the dorsal surface of the embryo, and from about the middle of its ventral surface another band passes to the anterior end of the hind-gut. Posteriorly it is attached to the dorsum and to the anterior ventral wall of the telson.

The dorsal organ has enlarged a little, but otherwise remains the same as in the third stage.

The formation of the muscles has begun in the abdomen and limbs, the cells being arranged in rows.

The sixth stage I have taken is that in which the egg-membranes burst, so that some of the embryos are still in the egg, while others are free.

In the living embryo (fig. 16) most of the organs can be more or less distinctly seen. In the abdominal region the heart (fig. 16, H) is very conspicuous and beats vigorously.

The blood enters it by four valvular openings (V) arranged in a spiral, and leaves it by five anterior aortæ, each of which is provided with valves.

The blood corpuscles are not very numerous, but there are enough to show the course of the circulation, and in those individuals which have escaped from the egg, they can be seen passing through the branchiæ.

Round the edge of each branchial plate, between the two chitinous laminæ of which it is composed, there is a clear space answering the purpose of the afferent and efferent vessels of the organ. The central part of the lamella is occupied by a number of cells, leaving a meshwork of spaces between them.

The blood flows up the inner edge of the lamella, and passes across to the outer edge to return to the heart.

The passage of the corpuscles to and from the gill is very rapid, but they often get entangled in going from one side to the other and remain in the gill for some time.

Occupying the greater part of each side of the thorax are the liver cœca (fig. 16, L). They are of a different colour from the yolk and are filled with a fluid in which are suspended numerous oil drops. These are kept in constant motion by the contractions of the cœca, and sometimes pass from one cœcum to another; the cœca lie entirely above the yolk, and by gently moving the cover-glass can be made to glide about over it without in any way injuring the embryo.

In the second segment behind the head the liver cœca are attached to the fore-gut. The arrangement of these parts cannot be made out in the living state, but by cutting the embryo in half so as to allow the yolk to escape without injuring the liver cœca, it is possible to see the duct, and by gently pressing the cover-glass one can make the oil drops pass from the liver into the fore-gut. Fig. 17 is an optical section of these parts. Two projections bearing hairs may be seen in the front part of the stomach.

The hind-gut can be traced from beneath the heart, in the abdomen, to its junction with the fore-gut, although from its transparency it is not at first very easily seen. Like the liver, it lies entirely above the yolk. For the greater part of its course it is very narrow, but at the anterior end it appears to dilate a little before joining the fore-gut; its walls at this point are extremely thin. At a rather later stage, when the yolk has decreased to about half its present size, it is quite easy to tease out the embryo, so as to see the junction of the fore- and hind-guts and liver cœca.

The yolk at this stage is surrounded by a membrane (Dottersack) continuous with the point of junction of the fore- and hind-guts. Fig. 18 shows the yolk, liver cœca, and fore-gut teased out in the fresh state; the hind-gut, owing to its extreme tenderness, has been broken off. The yolk-membrane is very thin, and bursts so as to let the yolk rush out on very slight pressure. Its presence is most easily demonstrated by cutting off the abdomen close to its junction with the thorax, and then allowing the cover-glass to press very gently on the embryo. This causes the yolk to protrude from the cut end, and its outline may be seen to be quite smooth and sharp; the least extra pressure at once causes the membrane to burst, and the yolk then immediately streams out. With care, the whole yolk-sac may be dissected out still attached to the fore-gut, as in fig. 18.

In the process of hardening, the yolk-membrane always breaks, and thus allows the yolk to escape into the body cavity, and to occupy positions in which it is not found in the living animal. For this reason sections are not entirely trustworthy, the hind-gut and liver cœca often appearing embedded in the yolk, whereas in the living embryo they are quite free from it. The yolk-membrane cannot in any one section be traced the whole way round the yolk; but in those places where it is supported, such as on each side of the nerve-cord and between the liver cœca, the membrane can be seen in almost every section. There seem to be elongated nuclei here and there in it.

I think there can be no doubt that the yolk-membrane and the cells forming the liver are to be regarded as the hypoblast or lining of the mid-gut, for the liver is only a diverticulum from that organ.

It appeared in an earlier stage (stage III.) that the liver arose from a mass of cells on each side of the embryo, and apparently had no connexion with the rest of the digestive system.

This mode of development is perfectly explained if the yolk-membrane is regarded as the mid-gut. Though its presence cannot be demonstrated in the earlier stages, it no doubt really exists in them; and it would probably not be wrong to trace it back to some of the lower layer cells in the second stage, from which the liver (stage III.) arose, or even perhaps to the lower cells in the first stage. It is thus probable that the liver does not arise, as at first sight might appear, from two isolated masses of cells, but in the normal way by a prolongation of the cavity of the mid-gut.

The arrangement of the parts becomes clear on remembering the large quantity of food-yolk in the egg.* It may be supposed that, owing to the growth of the hind-gut being more rapid than the absorption of the food-yolk, the point of junction of the hind- and mid-guts gets carried more and more forward (see diagrams A, B, C, fig. 3, *woodcut*) along the dorsal surface of the yolk until at last it reaches the fore-gut, into which the hind-gut opens, the mid-gut remaining as a sac enclosing the yolk, and opening at the junction of the fore- and hind-guts.

As the yolk disappears the mid-gut disappears with it, and the alimentary canal

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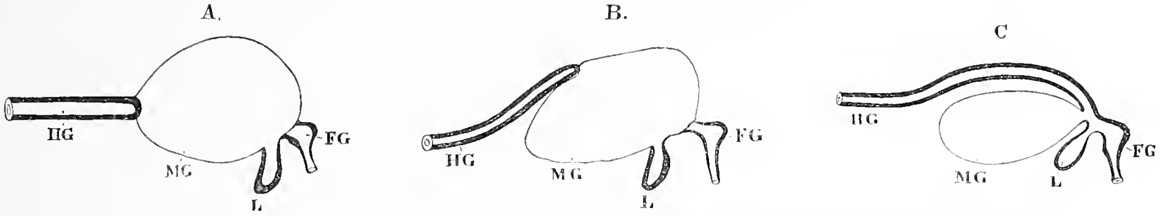
* More than in any other Crustacean except *Astacus* (?).

of the adult consists practically of the fore- and hind-guts alone, the liver being the only permanent representative of the hypoblast. The mid-gut in these animals is therefore only a temporary structure. That this interpretation of the facts is the true one, is rendered more probable by the fact that among certain other Crustacea the mid-gut is formed from the yolk-membrane.

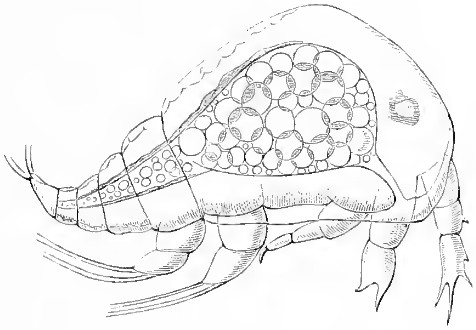
Fig. 3.



Diagrams illustrating successive stages in the development of the alimentary canal.
HG. Hind-gut. MG. Mid-gut. FG. Fore-gut. L. Liver.

Fig. 4 (*woodcut*), copied from Van Beneden's paper on *Hessia colorata*,* shows this very clearly, and corresponds almost exactly with the woodcut, fig. 3, A. Van Beneden does not give a detailed account of the mode of development of these parts, but mentions incidentally that the mouth and œsophagus are formed by involution. He thus describes the figure: "L'œsophage relie la base de la trompe buccale à l'élargissement considérable du tube digestif (yolk-membrane) qui renferme les restes du vitellus; il s'est considérablement allongé, et l'on distingue facilement ses parois. On peut suivre le tube digestif dans toute sa longueur: fortement renflé en avant, il se rétrécit progressivement en arrier (hind-gut) pour s'ouvrir à l'extrémité postérieure de l'abdomen entre les deux appendices caudaux. Il renferme le reste de la masse deutoplasmatique qui a singulièrement diminué dans le cours du développement embryonnaire, ayant été employée à former successivement les différents organes de l'embryon."

Fig. 4.



Hessia colorata (Van Beneden, Bull. Acad. Roy. Belg., t. xxix., 1870, pl. xxix., fig. 15).

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* Bull. Acad. Roy. Belg., t. xxix., 1870, pl. xxix., fig. 15.

The same mode of development occurs probably in *Cuma, Praniza, Portunus,*[*] &c. In his account of the development of *Asellus aquaticus,* Dr. DOHRN[†] goes rather fully into the question of the meaning of the yolk-membrane (Dottersack). After describing his observations on the development of the digestive organs, which appear to agree in all essential points with what I have found in *Cymothoa,* he says:—"Ueber das Verhältniss der Entstehung der Leberschläuche—denn durch Auswachsen des hinteren Endes nehmen diese Organe bald die Schlauchform an—zum Dottersack habe ich nur Vermuthungen. Möglich ist es, dass der Dottersack schon vor der Bildung der Leberwandungen besteht, dass letztere also Ausstülpulgen desselben würden. Wahrscheinlich ist mir indess dass Gegentheil, und dass der Dottersack sich um die innere Oeffnung des Leberschlauches herumlegt. Diese Bildung allein lässt uns aber schon die Möglichkeit einsehen, dass die Abscheidung des Dottersacks kein so allgemeiner Vorgang ist, als es haüfig aufgefasst wird. Gewiss stehen keine theoretischen Bedenken der gleichzeitigen und allgemeinen Umhüllung des Dotters durch den Dottersack in den Fällen entgegen, wo seine Wandung sich zum Mitteldarm umwandelt und wo alle accessorischen Gebilde des Darmcanals durch Ausstülpungen entstehen. Und wenn man die directe Beobachtung für diesen Vorgang ins Feld führen kann, so muss ja jegliches auf abweichenden Untersuchungen in analogen Fällen basirtes Urtheil seine Beweiskraft verlieren. Aber ich glaube, es würde irrthümlich sein, den Analogieschluss von derartig festgestellten Entwicklungs- und Bildungsweisen nun auf die übrigen noch zweifelhaften Fälle anzuwenden, und meine bisherigen Untersuchungen lassen mich durchaus bezweifeln, dass bei *Asellus aquaticus* ein Dottersack ensteht, der an allen Puncten gleichzeitig den Dotter umhüllt. Leider bin ich trotz tagelanger und häufig wiederholter Beobachtungen nicht im Stande, ein sicheres Resultat über die, wie es scheint, sehr verwickelten Verhältnisse zu bieten; ich kann nur angeben, was ich gesehen und was ich daraus schliesse, ohne mehr für meine angaben in Anspruch zu nehmen, als dass sie wahrscheinlich seien."

I think there can be no doubt that fig. 18 proves that the yolk is completely surrounded at one time by the yolk-membrane, and it may therefore be fairly assumed that the same is the case in *Asellus,* although Dr. DOHRN did not absolutely succeed in demonstrating it.

In *Oniscus,*[‡] according to BOBRETZKY'S very clear description, the mode of development is somewhat different, although it results in the formation of a digestive tract having the same homologies as that of *Cymothoa.*

The first step in the development of the egg is the separation of the formative yolk from the food-yolk. The former then segments and a single layer of cells is formed which spreads gradually over the surface of the egg. Before the yolk has

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[*] DOHRN, "Untersuchungen Ueber Bau und Entwick. der Crustacean," Jenaische Zeit., bd. v., 1870, and Zeit. für Wiss. Zool., bd. xx., 1870.

[†] DOHRN, "Asellus," Zeit. für Wiss. Zool., bd. xvii., 1867.

[‡] BOBRETZKY, Zeit. für Wiss. Zool., bd. xxiv., 1874.

been completely surrounded a second layer of cells has made its appearance under the central part of the blastoderm. These cells rapidly increase, and at the same time sink into, and become filled with the yolk. Meanwhile the blastoderm has surrounded the egg, the *Keimstreif* has been formed, and the involutions for the fore- and hind-guts have appeared. The hypoblast cells now begin to separate themselves again from the yolk, and gradually become arranged so as to form the wall of the liver, which, from the first, surrounds a considerable portion of the yolk. There is no yolk-membrane. The hind-gut grows forwards and becomes connected with the liver, in which the whole of the yolk is now contained, and eventually unites with the fore-gut.

The following seems to me to be the most satisfactory explanation of the differences in the development of the hypoblastic organs in *Cymothoa* and *Oniscus*. On comparing the yolk-membrane of *Cymothoa* with the mid-gut of *Hessia* (fig. 4, *woodcut*), it appears that a degeneration has taken place. In *Hessia* there is a distinctly cellular and functional mid-gut, while in *Cymothoa* it is extremely thin and disappears entirely in the adult. In *Oniscus* it has completely disappeared, and at the same time the mode of development of the liver has become altered to enable it to perform to a certain extent the function of the yolk-membrane in enclosing the yolk. Instead of arising as in *Cymothoa* (fig. 9) from a solid mass of hypoblast cells which penetrates into the yolk only to a small extent, and then gradually becomes hollowed, the liver is formed from hypoblast cells, which, at an earlier period, penetrate much more deeply into the yolk, and arrange themselves in such a way that, from the first, they surround a large portion of the yolk, and very soon completely enclose it. We must therefore regard the penetration of the hypoblast cells into the yolk in *Oniscus* as in no way connected with a primitive invagination or gastrula, but as a modification of the mode of formation of the liver and mid-gut found in *Cymothoa*.

In *Astacus*[*] the case appears different, and until we have a more complete history of its development, probably no comparison of any real value can be made between it and *Cymothoa*.

Fig. 19 is a transverse section through the thorax of an embryo, of the same stage as fig. 16. The hind-gut (HG) is a very narrow tube, with scarcely any lumen. On each side of it are the three liver cœca (L); their walls are very thin, the cells forming them scarcely seeming to be in contact. The muscles in the sides of the body and limbs are now distinctly striated.

The dorsal organ (fig. 16) is still of the same form, and contains the same peculiarly shaped hollow.

After their escape from the egg, the embryos remain in the brood-pouch of the mother until the whole of the yolk is absorbed. When they leave her they are very active, and swim well. Fig. 21 represents one of these young. It was one of a brood which I succeeded in bringing up by keeping the fish, in whose mouth the mother

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[*] REICHENBACH, Zeit. für Wiss. Zool., bd. xxix., 1877; BOBRETZKY, *Astacus*.

lived, in a glass covered with a piece of gauze to prevent the escape of the young. They all, however, succeeded in escaping except three. When these first came out their livers were filled with oil-drops. I kept them and examined them every day with the microscope. There was less oil each day, and they got more and more transparent. For several days I could not get any proper fish to put them on, and the oil had so much diminished that they had only one or two small drops left in their livers, and must have been very hungry, for directly I put them on the fish they stuck in their claws and held on tight. It is worthy of notice that, although embryos of a species which lives in the mouth, they began by fixing themselves on the outside of the fish, where they seemed quite happy. I was unfortunately unable to trace their development further, as the fish on which they were placed died in the night, and in the morning when I found them the embryos were dead. Although before they fixed themselves to the fish they were so transparent that all the organs could be easily seen, I was unable to make out any trace of either the generative organs or of the so-called renal tubules.

It seems probable that these organs may be developed at the time that the limbs of the seventh thoracic segment appear, since they are both connected with it.

Description of the Figures.

The same letters are used in all the figures.

Ab. Abdomen.
An. Antenna.
Ap. Appendages.
C. Thickened cuticle forming cornea.
C.G. Cerebral ganglion.
C.T. Connective tissue.
D. Dorsal organ.
E. Eye.
Eb. Epiblast.
FG. Fore-gut.
FG′. Gullet.
FG″. Stomach.
Fr. Frontal lobes.
H. Heart.
HG. Hind-gut.
I.M. Inner Membrane.
L. Liver.
L.D. Duct of liver.
M. Mouth.
Mb. Cells beneath the epiblast.
Mbn. Pigmented membrane.
Ms′. Muscles.
N. Nervous system.
O.M. Outer membrane.
Op.T. Optic tract.
T. Telson.
Th. Thorax.
V. Valves of heart.
X. *Kristal kegel.*
Y. Yolk.
Z.Z′.Z″. Sensory parts of eye.

Fig. 1. Whole egg, Stage I. From a hardened example.
Fig. 2. Section through centre of blastoderm, Stage I.
Fig. 2A. A little later than the last. From a hardened egg.
Fig. 3. Whole egg, Stage II. Ventral side. From a hardened egg.
Fig. 4. The same ; dorsal surface.
Fig. 5. Transverse section through thorax of the same.
Fig. 6. Transverse section through frontal lobes of the same.
Fig. 7. Longitudinal section of the same not quite in the middle line.
Fig. 8. Transverse section, Stage III.
Fig. 9. Longitudinal section of the same, a little to one side of the middle line.
Fig. 10. Transverse section, Stage IV.
Fig. 11. Horizontal section, same stage.
Fig. 12. Longitudinal section of the eye (transverse of head): adult.
Fig. 13. Horizontal section of the eye (longitudinal of head): adult.
Fig. 14. Transvere section of head: adult.
Fig. 15. Longitudinal section in median line, Stage V.
Fig. 16. Whole embryo alive, Stage VI.
Fig. 17. Optical section of junction of liver cœca and fore-gut, fresh state, same stage.
Fig. 18. Yolk enclosed in its membrane and liver cœca teased out in the fresh state, same stage.
Fig. 19. Transverse section through thorax, same stage.
Fig. 20 and 21. Embryos as they leave their parent.

XVI. *On the Placentation of the Apes, with a Comparison of the Structure of their Placenta with that of the Human Female.*

By WM. TURNER, *M.B. (Lond.), F.R.SS. L. & E., Professor of Anatomy in the University of Edinburgh.*

Received March 11,—Read March 28, 1878.

[PLATES 48 AND 49.]

CONTENTS.

THE placenta in the Quadrumana has from time to time engaged the attention of anatomists.

JOHN HUNTER[*] seems to have been the first to describe and figure the placenta of a monkey, which had been shed in the ordinary course of parturition, after the birth of a single fœtus. The placenta was divided into two oblong contiguous lobes, and each lobe was made up of smaller lobes closely united together. Fissures were seen on the uterine surface of the placenta, in which were situated veins or sinuses that received the blood laterally from the lobes, and that passed through the decidua to enter the substance of the uterus. The substance of the placenta seemed to be "cellular" as in the human subject: an arrangement which allowed a communication to be kept up between different parts of each lobe, as well as between different lobes. HUNTER recognised the chorion and amnion. The decidua was thicker than in the human subject. The allantois was absent. HUNTER does not give the generic name of this monkey, but Professor OWEN calls it *Macacus rhesus*, and in the "Catalogue

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[*] 'Animal Œconomy, 1786.' Also in collected works by PALMER, vol. iv., p. 71, plates xxxv., xxxvi., and fig. 2, xxxiv. The uterine and fœtal surfaces of the same placenta are figured by Sir E. HOME, 'Comparative Anatomy,' vol. iv., plates 167, 168. The description of the placenta of an Ape given in 'Les Œuvres de VICQ-D'AZYR,' vol. v., p. 318, 1805, is an almost verbatim translation of HUNTER's description, though HUNTER's name is not given as the authority.

of the Museum of the Royal College of Surgeons"[*] he points out that the filamentary fœtal villi include the capillary loops of the umbilical vessels; but instead of lying free in the alveolar cavities of the maternal placenta, they are connected or entangled with the fine cellular structure which receives the blood from the uterine arteries; the uterine veins have stronger and more definite coats than in the human placenta.

RUDOLPHI gave, in 1828, a short account, with figures, of the gravid uterus of an *Hapale jacchus*, in which were twin fœtuses.[†] The fœtal membranes consisted of a chorion, with an oval placenta and an amnion for each fœtus, but with no trace of an allantois. Each fœtus had an umbilical vesicle of about the same size as in the human embryo of the third month. The vesicle possessed a long peduncle containing delicate blood vessels, but no trace of an omphalo-mesenteric duct. The placenta was divided into a fœtal and a maternal part. RUDOLPHI also figured the placenta of a howling monkey (probably *Mycetes ursinus*), which consisted of a single oval lobe of greater thickness than the placenta of *Hapale*. In the fœtus not only of *Hapale* and *Mycetes*, but also of a Capucin monkey, two umbilical veins extended from the placenta up to the liver.

In 1845 M. BRESCHET published an elaborate memoir on the gestation of the Quadrumana,[‡] in which he described and figured the fœtus and placenta in a species of *Hylobates*, in *Cercopithecus sabæus*, *Cynocephalus sphinx*, *Semnopithecus mitratus* and *nasicus*, *Mycetes seniculus*, and *Chrysothrix sciurea* (Saimiri). His description comprises an account of the form of the placenta, the presence of a decidua, the arrangement of the umbilical vessels, and the general disposition of the chorion and amnion in the several specimens. Some observations are also made on the appearance of the gravid uterus of *C. sabæus*. No allantois was present in any of the specimens, but in *sabæus* indications of an umbilical vesicle were seen at the junction of the cord with one of the lobes of the double placenta. No reference is made to the minute structure of the organ.

In the Chimpanzee the placenta is stated both by Professor OWEN[§] and by Professor HUXLEY[||] to be single, and the latter anatomist says that in a specimen he examined, where the fœtus was 11½ inches long, the placenta was simple, rounded, 3½ inches in diameter, and 0·6 inch thick in the centre. The umbilical cord was inserted near one of its edges.

Professor ROLLESTON, in a memoir communicated to the Zoological Society,[¶] gave an account of the examination of an injected placenta of *Macacus nemestrinus*, which had been preserved in spirit in the Oxford Museum for many years. He described

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[*] Physiological Series, vol. v., p. 145, and 'Comp. Anat. of Vertebrates,' vol. iii., p. 746.

[†] Abhand. der Akad. der Wissensch. zu Berlin, 1828.

[‡] Mémoires de l'Institut, Paris.

[§] 'Comparative Anatomy of Vertebrates,' vol. iii., p. 747.

[||] 'Manual of the Anatomy of Vertebrated Animals,' p. 487, 1871.

[¶] Trans. Zool. Soc., vol. v., 1863.

the uterine surface of the placenta as covered by a smooth continuous deciduous membrane, between which and the muscular coat were loose lamellæ, the deeper of which consisted of cells with large nuclei and tapering ends. A number of large blood vessels were seen amidst these lamellæ. The lamellated tissue represented the non-deciduous serotina. From the deciduous layer on the surface of the placenta processes of decidua passed into its substance. ROLLESTON obviously inclines to the view that the maternal vessels within the placenta formed a sinus system as in the human placenta. The fœtal villi were arborescent.

Signor ERCOLANI* has examined a placenta ot *Cercopithecus sabæus* which had been preserved in alcohol. As he could not find any essential difference between its structure and that of the human placenta, he did not think it necessary to describe it, but states that the intra-placental lacunæ for the maternal blood are smaller than in the human placenta, and that manifest traces of decidua serotina are present on the uterine face of the organ, which are continued on to the fœtal villi, for which they form an external membrane.

KONDRATOWICZ has recently described† the uterus of a species of *Macacus*, in the last stage of pregnancy, preserved in the Museum at Warsaw. As it was not practicable to inject the placenta he gives no account of its structure, but he relates some facts bearing on the structure of the decidua vera. He states that the uterine mucous membrane is thickened, and he recognises on it an epithelium, which is blended with the chorionic epithelium. Subjacent to the uterine epithelium is a finely fibrous connective tissue with spindle cells and opaque yellow clumps of pigment; in the deeper layers the cells increase in number, become multipolar, and assume the character of large epithelial-like decidua cells. These cells, he says, seem to form the immediate boundaries of wide blood-containing spaces or canals. Deeper than these cells and next the muscular coat is a layer of delicate connective tissue, the trabeculæ of which bound large spaces lined by an epithelium formed of short hexagonal cylinders, with large nuclei. He regards these spaces as the expanded deeper ends of the utricular glands, the ducts of which have become obliterated. No gland-like spaces were recognised in the tissue between the placenta and the muscular coat of the uterus.

By the researches of the several investigators above referred to, observations on the form of the placenta in the Apes, on the presence of a decidua, on the absence of the sac of the allantois, on the absence or rudimentary condition of the umbilical vesicle, and on the arrangement of the amnion have gradually been accumulated.

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* Sul processo formativo della porzione glandulare o materna della placenta. Mem. dell' Accad. delle Scienze di Bologna, 1870, p. 53.

† His memoir, entitled "Przyczynek do histologii cięzarnej macicy," is in the Pamietnik Towarzystwa Lekarskiego, Warszawa, Zezzyt iii. 1875, p. 259. As this memoir is in Polish, I have been unable to read it in the original, but owe my knowledge of its contents to an abstract by HOYER in HOFMANN and SCHWALBE'S "Jahresberichte," 1876.

To some extent, also, observations have been made on the minute structure of the placenta, so that by the common consent of anatomists the placenta in the Apes is regarded as approximating in many particulars, both of form and structure, to the human placenta. As the recorded observations have for the most part been made on specimens preserved often for many years in spirit of wine, many questions connected with the minute structure of the organ have either been left in doubt or not examined; more especially I may refer to the arrangement of the intra-placental decidua, the presence or absence of the utricular glands in the serotina, the relation of the maternal vascular system to the fœtal villi, and the general disposition of the intra-placental system of maternal blood spaces. As I felt that it was important to inquire into these matters on a fresh specimen, and with the aid of transparent injections and other modern histological appliances, I requested, some time ago, my friend Dr. JAMES MURIE to obtain for me, if possible, a monkey well advanced in pregnancy. This he very kindly succeeded in doing in the year 1876, and I have been able with the aid of this specimen not only to confirm many of the observations of my predecessors in this branch of investigation, but to ascertain a number of new facts in the structure of the monkey's placenta.

Gravid Uterus of Macacus cynomolgus.

The monkey obtained for me by Dr. MURIE was an adult *Macacus cynomolgus*, well advanced in pregnancy. After having been completely anæsthetized with chloroform it was killed by dividing the carotids and jugulars. As soon as respiration had ceased the abdominal cavity was opened into by a crucial incision, when the uterus was seen to occupy a large part of the abdominal cavity. It was broadly ovoid in shape, and ascended from the pelvis to within $2\frac{3}{4}$ inches from the xiphi-sternum. No portion of the small intestine was visible. The transverse colon appeared at the left side of the fundus uteri, and the cœcum projected immediately above the fundus. Both the cœcum and colon were covered by the great omentum, which contained bright yellow lobules of fat. Above the cœcum the pyloric end of the stomach occupied the epigastrium. No other abdominal viscus was visible until the uterus was pushed to one side.

The uterus was of a delicate bluish-grey colour mingled with pink. Slight changes in the contour of its anterior wall took place during about half an hour after the abdominal cavity was opened. Bulgings arose here and there which slowly subsided, to be succeeded by similar bulgings in adjacent parts of the wall. These movements were obviously due to slight movements of the fœtus within the uterine cavity.

From the sides of the uterus the broad ligaments passed to the postero-lateral walls of the abdomen. Large ovarian veins distended with fluid blood lay between their folds, and ramified on the wall of the uterus beneath the serous membrane. From the lower part of each antero-lateral aspect of the uterus a distinct ligamentum teres passed downwards and forwards to the inguinal region. The Fallopian tube arose from the fundus in line with, but $1\frac{1}{4}$ inch above, the uterine attachment of the round ligament. The ovaries were attached by their ligaments to the fundus, posterior to the uterine ends of the Fallopian tubes. The right ovary was purplish coloured, and contained a corpus luteum; the left ovary was cream coloured, and with numerous semi-transparent specks on its surface. No appearance of an organ of ROSENMÜLLER was to be seen in either of the broad ligaments. The uterus diminished in size as it entered the pelvis and blended with the wall of the vagina. The body of the uterus was 5 inches long, the cervix uteri was $\frac{3}{4}$ inch, and the vagina $2\frac{1}{4}$ inches.

As one of the objects I had in view in the investigation of the structure of the placenta was to determine the relations of the fœtal and maternal vessels, I decided upon injecting both systems of vessels. With the object of doing this with as little disturbance of the parts as possible, without removing the uterus, I introduced a pipe into the abdominal aorta, and tied off the external iliac arteries so as to stop the flow of the injection into the lower limbs. I then made a mesial longitudinal incision through the anterior wall of the vagina, cervix uteri, and lower part of the body of the uterus. A colourless viscid mucous plug, which occupied the cervix, was removed, and the chorion opposite the os uteri internum was exposed. The depending part of the fœtal membranes was then cut through, when a gush of liquor amnii took place and the hairy breach of the fœtus was exposed. The finger being introduced into the uterine cavity, the umbilical cord was drawn down and injecting pipes introduced into the umbilical vein and one of the umbilical arteries. A simultaneous injection of the uterine and chorionic vessels was then made. My assistant, Mr. A. B. STIRLING, gently passed a red transparent injection through the aorta into the uterine arteries, whilst I threw at the same time a blue transparent injection into the umbilical vessels. It was unnecessary to inject specially the uterine veins, as the large veins in the broad ligaments became filled from the injection which had passed along the uterine arteries. When the injection had become set the incision was prolonged through the uterine wall to the fundus, and the fœtus was exposed.

The fœtus lay in the uterus with its breech in close relation to the os internum, and the head to the fundus; the back was directed to the left side of the body of the uterus, and the belly to the right (fig. 1). The head was bent forward, the chin being in contact with the front of the chest, and the snout resting between the hands. The thighs were flexed on the belly, and the legs on the back of the thighs. The ankles were flexed on the front of the legs, and the toes on the soles of the feet. The upper arms were close to the sides of the chest, the fore-arms slightly bent on the upper arms, the hands in the position of palmar flexion, and the fingers bent on the palms. The tail was curved forwards on the right side of the fœtus, so that its tip was in relation to the top of the head. The fœtus, a male, was covered with hair, and the nails were well developed. The upper and lower eyelids were firmly united together. From the snout to the root of the tail was $8\frac{1}{4}$ inches: the tail measured $7\frac{1}{4}$ inches in length. The upper limb was 4 inches long, the lower limb 5 inches. The size and state of development of the fœtus confirmed the opinion I had formed during the life of the monkey that she was far advanced in pregnancy.

The umbilical cord was $4\frac{3}{4}$ inches long, and passed from the belly of the fœtus between the back of the left thigh and leg to the more anterior lobe of the placenta. The cord was then cut through and the fœtus removed, when the position and form of the placenta were seen. The placenta was divided into two distinct lobes; the one attached to the anterior, the other to the posterior wall of the uterus. The posterior lobe was almost circular in form; its vertical diameter being $3\frac{1}{2}$ inches, its transverse diameter 3 inches. Its upper border almost reached the summit of the fundus uteri, its lower border was $2\frac{1}{4}$ inches from the os internum. Its fœtal surface was traversed by shallow furrows which indicated a division into six lobelets. The anterior lobe was circular in form, its vertical and transverse diameters being alike 3 inches; the upper border did not approximate so closely to the fundus as that of the anterior lobe; the lower border was 1 inch from the os internum. The division of the anterior lobe into lobelets by superficial furrows was not so distinct as in the posterior lobe. The anterior and posterior lobes were separated from each other by intermediate smooth non-placental parts of the chorion, situated in relation to the lateral aspects of the uterus; the smooth part to the right was 1 inch in transverse diameter at its narrowest, whilst the smooth part to the left had a transverse diameter of 2 inches at its narrowest. The summit of the chorion at the fundus uteri, where it passed between the upper borders of the two placental lobes, was also smooth, and the most depending part of the chorion, in relation to the os internum, was smooth over a considerable area.

The umbilical cord joined the fœtal surface of the anterior placenta two inches from its upper border, and $1\frac{3}{4}$ inch from its left border. The umbilical vessels then branched, some of the branches entered the anterior placenta, but others ran with a tortuous course over its fœtal surface and across the intermediate smooth parts of the chorion, in relation both to the sides of the uterus and its fundus, to reach the posterior placenta. No vessels, however, ramified in the smooth chorion opposite the os internum. The ramifications of the blue injected fœtal vessels were very distinct on the fœtal surface of both placentæ, and the smaller vessels of the placental chorion were extremely tortuous. The substance of the anterior placenta contained a larger proportion of blue injected vessels than did that of the posterior, whilst the posterior placenta had a larger proportion of red injection in it than the anterior. This difference in the relative proportion of the two injections is capable of the following explanation. In injecting the vessels of the cord, the chief pressure was on the anterior placenta, for the cord directly joined it, and its fœtal vessels were therefore more completely filled with the blue fluid. But in injecting the uterine vessels through the aorta, the chief pressure had been on the posterior placenta, and therefore its maternal vascular spaces had been most completely filled with the red-coloured gelatine.

The umbilical cord contained one large vein and two arteries, with a small quantity of the gelatinous tissue of WHARTON, but no trace of an allantois or umbilical vesicle could be seen. The umbilical vein after entering the abdomen ascended as an undivided vessel as far as the under surface of the liver. The pair of umbilical arteries passed as the hypogastric arteries to the posterior wall of the abdomen. Between the hypogastric arteries was a fibrous cord, the obliterated urachus, which extended from the apex of the bladder to the umbilicus. The cord was invested by the amnion and the fœtal surface of both the placental and non-placental parts of the chorion was lined, as in the human female, by this membrane. The surface of the amnion next the amniotic cavity was covered by a continuous layer of polygonal flattened epithelium cells, which were translucent and with faintly granulated protoplasm, but with the nucleus relatively large and distinct. These cells rested on a thin layer of delicate connective tissue.

The uterine ends of the Fallopian tubes opened at the sides of the uterus opposite the non-placental parts of the chorion situated between the two placentæ. The cavity of the uterus communicated with the canal of the cervix at a well-defined os internum, the lip of which formed a projecting ring around the os. The cervical canal was about ½ inch long; its mucous membrane was elevated into folds, some of which were vertical, but others were directed obliquely, so as to form an arrangement somewhat like the arbor vitæ in the human cervix. The os externum was rounded and bounded by a distinct ring-like projecting lip. The vagina was 1¾ inch long; its mucosa was partly smooth and partly thrown into sinuous rugæ, some of which, near the os externum, were longitudinal, but the greater number were transverse.

Structure of the wall of the Uterus.

The wall of the uterus possessed an external serous coat as in the human female. The muscular coat was thin, and, relatively to the size of the uterus, much thinner than in the human uterus. In this respect the muscular coat resembled what is usually found in the mammalia where the impediments to parturition are not so great as in the human subject.

The uterine cavity was lined by a well-marked mucous membrane, which formed a layer homologous with the decidua vera, or uterina of the human gravid uterus. In the region near the internal os, the chorion could be raised from the mucous membrane by gentle traction, without doing much injury to the free surface of the mucosa. This surface was not smooth, but presented numerous shallow ridges and furrows, which for the most part ran transversely to the long axis of the uterus. At the internal os, however, the ridges were vertical and were continuous with, though not so prominent as, the longitudinal columns of the mucosa in the canal of the cervix uteri. In the region of the fundus uteri, the adhesion between the non-placental part of the chorion and the surface of the mucous membrane was much more intimate than near the os internum; so that on raising the chorion it was usual to peel off flakes of the more superficial part of the mucous membrane along with it. If very great care was taken, the chorion could be separated from the mucous surface, which was then seen to present a ridge and furrowed appearance arranged much more irregularly than in the lower third of the uterus. The corresponding surface of the non-placental part of the chorion possessed multitudes of fine ridges which fitted into the furrows of the mucosa so closely as to make it difficult to strip off the chorion without removing a part of the mucous surface along with it. The close co-aptation of the chorion to the mucous membrane, even in the non-placental regions, was well seen in vertical sections through the wall of the uterus and fœtal membranes, in which the chorion followed the windings of the mucous surface and in close contact with its epithelial covering (fig. 2).

The uterine mucosa was covered by an epithelium which varied in character in different parts of the uterus. In the lower third or fourth of the cavity the cells had the characters of columnar epithelium. Their attached ends were either attenuated into a fine process, or truncated or occasionally bifurcated; at their free ends delicate processes, not unlike short cilia, were seen; but as this observation was made some time after death no movement was visible. The nuclei were nearer the attached than the free ends of the cells. A small proportion of these epithelial cells were swollen out, so as to have more the appearance of goblet epithelium. In the region of the fundus, and in the interval between the placental lobes, the epithelium was neither columnar nor ciliated. The cells were polygonal in form, and fitted together like the tiles of a pavement, but they were not such thin scales as one sees in the tessellated epithelium of the mouth. The nucleus was large, rounded or ovoid in form, and contained one or more nucleoli. In their general appearance these cells had a resemblance to the colossal cells of the human decidua.

The sub-epithelial tissue of the mucous membrane contained a considerable proportion of fusiform corpuscles closely crowded together, which were larger than the corpuscles of the sub-epithelial tissue in a specimen of the non-gravid uterus of a *Macacus*, with which they were compared. These corpuscles were not unfrequently elongated at their poles into slender filaments, though at other times the poles were short and stunted. Other cells were not fusiform, but polygonal, or even irregularly stellate. The protoplasm of the cells was dimly granulated, and the nuclei relatively large and elliptical in shape. Occasionally thin flakes of nucleated protoplasm were seen, in which the differentiation into definite cells could not be recognised. The abundance and variety in form of the cells in the sub-epithelial tissue gave one the impression of a texture in which a rapid cell growth had taken place. Bundles of white fibrous connective tissue were sparingly distributed in the sub-epithelial tissue, though a filamentous appearance, due to the filamentous poles of so many of the fusiform cells, was not uncommon. Parallel and next to the deep surface of the sub-epithelial tissue was a band, more or less distinct in different sections, which could sometimes be peeled off in a long stripe. This band contained spindle-shaped cells of larger size than the fusiform corpuscles of the sub-epithelial tissue, and was, I believe, the layer of muscularis mucosæ (fig 2).

A search was then made for utricular glands in the non-placental part of the mucosa. I did not see the mouths of any glands opening on the free surface of the mucosa; but on tearing off thin portions of the membrane and magnifying them 100 diameters I occasionally saw an appearance of elongated tubes, situated in the deeper part of the membrane. The tubes were tortuous, and contained an epithelium, the cells of which were so granular and opaque that their form could not be definitely determined. The sub-epithelial tissue was very vascular, the capillary network having been injected from the aorta and uterine arteries. At the fundus uteri, where the adhesion between the chorion and mucous membrane was so close, the injected sub-epithelial tissue was often torn through and remained attached to the chorion, when that membrane was raised from its position.

In vertical sections through the mucosa numerous spaces were seen in the sub-epithelial tissue. Most of these were blood vessels divided either transversely, obliquely, or longitudinally, and they were either partially or wholly filled with injection. Other spaces, however, were occasionally present which were not vascular; some of these were elongated slits, others were more irregular in outline. KONDRATOWICZ has described spaces in the decidua vera of his specimen, lined by an epithelium of short columnar cells, which he regards as the deeper ends of the utricular glands. The spaces which I have observed had no epithelial lining, their wall being directly formed of the corpuscles of the sub-epithelial tissue. If these spaces were not gaps in the tissue, formed in the act of making the section, they were probably dilated utricular glands, the epithelium lining which had degenerated and disappeared. The sub-epithelial tissue of the mucosa, though not so compact as in a non-gravid uterus of a *Macacus* to be subsequently described, did not possess the loose, spongy character of the decidua serotina in this animal.

The muscularis mucosæ of the decidua vera was attached to the proper muscular coat of the uterus by a thin and lax layer of delicate submucous connective tissue.

No decidua reflexa could be seen as a distinct layer, for as already stated the chorion was closely adapted to the ridges and furrows of the mucous surface. The flakes of uterine tissue which were adherent to the chorion, when that membrane was stripped off, were the more superficial parts of the decidua vera—sometimes its epithelial layer alone, or at other times both epithelium and sub-epithelial tissue—and not a separate layer of decidua reflexa.

A thick layer of modified mucous membrane intervened between the placental lobes and the adjacent part of the wall of the uterus, and formed a decidua placentalis, homologous with the decidua serotina in the human uterus. The description of the decidua placentalis will be more appropriately considered along with the structure of the placenta.

Structure of the Placenta.

Each lobe of the placenta was half an inch thick at its centre, but was much thinner at the circumference of the disc. On making a section into its substance, it was seen to have a spongy appearance similar to the human placenta. Its fœtal surface was limited by the chorion ; its uterine surface by a thick layer of modified uterine mucous membrane homologous with the decidua serotina of the human gravid uterus (fig. 5). The stems of the villi arose at intervals of from $\frac{1}{10}$th to $\frac{2}{10}$ths of an inch from each other. Each stem was, as a rule, so thick that it could be readily followed out with the naked eye or a simple lens, and be traced through the substance of the placenta to be attached to the hillock-like elevations of the decidua to be presently described. The stems were usually oblique in their direction, and it was not uncommon to see one, notwithstanding the branches which arose from it, reaching the decidua without experiencing any material diminution in size. In their course the stems of the villi gave off numerous branches, which divided and subdivided until they ended in the lateral and terminal bud-like offshoots of the villi (fig. 6). These buds varied in shape and size, some being elongated and cylindriform, whilst others were short and stunted, and it was by no means uncommon to find the latter arise in pairs. Branches of the umbilical vessels of considerable size, derived from the vessels in the chorion, were prolonged into the stems of the villi, and from them offshoots proceeded into the branches of the villi. The lateral and terminal buds contained a looped arrangement of capillaries. The loop might be single, or twisted so as to have a double coil, and the capillaries from one bud, more especially when the buds arose in pairs, might pass from one bud into the adjacent one. Sometimes the capillaries formed a network.

The placental chorion consisted of connective tissue distinctly fibrillated, and with fusiform connective tissue corpuscles. In many sections through it I observed rows of cells, arranged parallel to and at no great distance from the surface on which the amnion rested. These *intra-chorionic* cells did not form a continuous layer, but were found in patches (fig. 8). Sometimes they were in a single row, at others in two or more. They were elliptical, ovoid, rounded, or somewhat polygonal in form, with granulated protoplasm, and varied in size from a white blood corpuscle to about twice that size. The cells in each group were closely related to each other. They were quite distinct, both in position and appearance, from the amniotic epithelium. The fibrillated connective tissue was prolonged into the stems of the villi ; but in the finer branches, and in the lateral and terminal buds, the tissue became so delicate that a differentiation into fibrillæ could not be recognised. A partially injected network of capillaries, continuous with the smaller tortuous umbilical vessels, was situated in the connective tissue of the chorion in the intervals between the origins of the stems of the villi, and formed an extra-villous capillary plexus.

The surface of the chorion next the interior of the placenta was covered by a stratified arrangement of cells, which from their position may appropriately be called the *sub-chorionic* cells (figs. 7, 8). They formed a yellowish-white membrane, sufficiently thick to be seen with the naked eye and to be peeled off with a pair of forceps, which was continued over the whole surface of the chorion. I examined microscopically these sub-chorionic cells, both in vertical sections through the chorion and in preparations made by cutting away the stems of the villi and looking at the placental surface of the chorion. The layers of cells were not uniform in number, but varied from four or five to eight or ten, or even more. The cells in the most superficial layers were flattened and irregularly polygonal, and two or three times larger than the intra-chorionic cells just described. The cells in the deeper layers were elliptical, or even elongated into small spindles, not unlike in shape, but larger than, the fusiform corpuscles of the connective tissue of the chorion. In the more superficial cells the nucleus was very distinct, and in the whole the protoplasm was granulated. When seen in mass these cells had a yellowish colour, which contrasted with the white connective tissue of the chorion. They differed also very materially in appearance from the amniotic epithelium in relation to the opposite surface of the chorion. In some of the sections through the chorion the line of demarcation between the sub-chorionic cells and the proper tissue of the chorion seemed to be as definite as that between the cuticle and cutis in a section through the skin. But when thin sections were made and examined with a magnifying power of 350 diameters, the proper tissue of the chorion seemed to pass between the cells of the deeper layers, so as to give one the impression that they were derived from the chorion and were not cells superadded upon its free surface, but not arising from it.

The stems of the villi at their origin from the chorion were invested by a stratified arrangement of cells similar to, and continuous with, the sub-chorionic cells; and at the angle between the place of origin of the villus and the adjacent part of the chorion a great crowd of these cells was collected, for as many layers of cells enveloped the base of the villus as were situated on the immediately-adjacent part of the chorion. The layers diminished in number as the villous stem was traced farther away from the chorion. The cellular covering was also prolonged on to the larger branches of the villi, but with a considerable reduction in the number of layers. On the smaller branches of the villi, and on the terminal and lateral buds, the cellular covering of the villi was reduced to a single layer of somewhat flattened, though not squamous, cells, rectangular in outline and closely applied to each other by their margins.

The decidua placentalis or serotina formed a thick well-defined layer between each lobe of the placenta and the corresponding part of the muscular coat of the uterus. Opposite the intervals between the lobelets into which each lobe was divided, the modified mucous membrane was thicker than opposite the lobelets, and approached more closely to the chorion; and the villi which grew from the chorion, corresponding to these intervals, were much shorter than those entering into the substance of the lobelets. At the margin of the placental lobes the decidua serotina was directly continuous with the decidua vera, and the spongy character of the serotina was prolonged into the vera immediately continuous with it.

When the placenta was carefully stripped off the uterus, the decidua placentalis split into two portions, the one adhered to the uterine surface of the placenta, the other remained on the uterus, and the latter was considerably thicker than the former. The separation into the two layers was effected by tearing through delicate bands and flakes of the decidual tissue, which were continuous with the septa between the loculi to be immediately described, in the uterine layer of the decidua serotina.

The layer of decidua which remained attached to the uterine surface of the placenta was thin, and of a fawn colour. Its placental surface was very irregular owing to numbers of hillock-like prolongations of its substance, which projected into the interior of the placenta in a manner not unlike that in which stalagmites project from the floor of a cavern (fig. 10). These hillocks were usually somewhat conical in form, and were set so closely together that narrow intervals only intervened between the bases of adjacent hillocks. The arrangement of the hillocks was examined both in vertical sections through the placental decidua and in preparations made by detaching the villi from the decidua and examining its placental surface. In both these modes of preparation the terminal parts of the stems of the villi were seen to be intimately attached to the hillocks of decidua; so close, indeed, was the union that some force had to be used to draw them asunder, and in the act of separation it was usual for either a portion of the decidua to be drawn away with the villus, or for a fragment of the villus to remain attached to the decidua. To carry out the comparison which I have already made, if the hillocks of decidua resemble stalagmites projecting from the floor of a cavern, the villi of the chorion are like stalactites depending from its roof, and the attachment of the villi to the decidua is due to the mutual growth and fusion of the two structures.

When examined microscopically this layer of the decidua placentalis was seen to be chiefly composed of irregularly polygonal, somewhat flattened cells, composed of a large and distinct nucleus surrounded by a granulated protoplasm. These cells had a stratified arrangement and a yellow colour when seen in mass. They closely resembled, both in appearance and arrangement, the superficial layers of sub-chorionic cells. Although, as a rule, the cells were compactly arranged in the several strata, they at times, in the deeper strata, were surrounded by a delicate, translucent, faintly fibrillated matrix, apparently an imperfectly differentiated connective tissue. Groups of fusiform cells were also occasionally seen.

The hillocks of the decidua possessed a very similar structure. The greater part of their thickness was composed of a stratified arrangement of the irregularly polygonal, somewhat flattened, granulated and nucleated cells; but the axis of the hillock was not so richly cellular, but consisted mainly of delicately filamentous connective tissue containing fusiform corpuscles. The end of a villus attached to a hillock was completely surrounded by the stratified cells of the decidua, so that the villus was imbedded for some distance in the hillock, and at times even extended close up to its base of origin (fig. 11). The extent to which the villus penetrated into the substance of the hillock was recognised, not only by its structural difference from the decidua, but by the passage of the branches of the umbilical vessels, filled with a blue injection and surrounded by the proper tissue of the villus, for a greater or less distance down the axis of the hillock. As the villus emerged from the apex of a hillock the strata of decidua cells which invested it diminished both in number and thickness, so that at a short distance from the hillock the cellular covering of the villus was reduced to the single layer of cells investing the placental villi.

Slender bands and thin membrane-like flakes were occasionally to be seen passing between and connecting together the free ends of the bud-like offshoots of the villi. These bands and flakes were continuous with the cellular investment of the villi. Sometimes a slender band, terminating in a pointed end, extended for a short distance from a bud, and apparently had been a connecting band which had been torn in two. These bands and flakes were most probably derived from the decidua.

The thick layer of the decidua serotina which remained attached to the wall of the uterus on stripping off the placenta presented a peculiar and very characteristic spongy or honey-combed appearance, owing to the numerous areolæ or loculi which it contained (fig. 5). The cavities of these loculi varied in size, some being so small as only to be capable of containing a small shot, whilst others could hold a common pea. The loculi were imperfectly separated from each other by thin semi-translucent septa, and the cavities of adjacent loculi freely communicated through gaps in the septa, bounded by a well defined and often falciform edge. The free surfaces of the septa were quite smooth.

When examined microscopically the walls of the loculi were seen to consist of a layer of cells on the free surface, and of a subjacent vascular connective tissue. The cells on the free surface had an epithelial-like arrangement, but they varied materially both in shape and size. The largest of the cells were as big as, or even larger than, the more superficial cells of the tessellated epithelial lining of the mouth, whilst others were not more than one-third or one-fourth that size (fig. 9). They were fitted together by their edges so as to form a continuous pavement. They had not, however, the irregular polygonal form of an ordinary squamous epithelium. The larger cells were often rounded along a part of the edge and indented, or with short processes at other parts; their protoplasm was very pellucid or faintly granulated, and the nuclei were of large size, often rounded in form, and so granulated that the nucleolus was usually obscured. Many of the smaller cells were flask-shaped or fusiform, with pellucid protoplasm and granulated elongated nuclei. At and near the falciform openings in the septa the cells were very much smaller than on the plane surface of a septum; in many cases they were not so large as the nuclei of the larger cells. Their shape was polygonal, their nuclei rounded or egg-shaped, and in many cases the nucleus was enveloped by only a thin layer of protoplasm. These surface cells separated very readily from the subjacent tissue, which could be then examined. It was found to consist largely of elongated fusiform nucleated corpuscles, such as one is familiar with in the embryonic connective tissue, for they were bigger than the similar shaped corpuscles seen in the connective tissue of the adult; both in form and size they resembled the elongated cells of a spindle-celled sarcoma. Intermingled with the fusiform cells were spherical corpuscles like those of lymph, together with a small proportion of oval and elliptical corpuscles somewhat larger than those of spherical form. Occasionally thin flakes of protoplasm in which nuclei were imbedded, but where a differentiation into distinct cells had not taken place, were observed.

Blood vessels ramified in the septa between the loculi. They had been filled with the red injection and formed a capillary plexus, which was obviously concerned in the nutrition of the decidual tissue; many of these capillaries were of greater calibre than one usually finds in capillary networks.

But in addition to the vessels connected with the nutrition of the decidua, the serotina was traversed by veins and arteries passing to the placenta. The utero-placental veins passed obliquely through the uterine layer of the decidua serotina, and when they reached the placental layer they ran almost parallel to its outer surface, which was grooved for their lodgment. They could sometimes be followed in close contact with the placental layer for half an inch, or even a greater distance, and they were seen to branch two or more times. They were cylindriform vessels, smaller in size than the digital veins of the human hand; they possessed definite coats, which with a little care could be separated from the surrounding decidual tissue. When these vessels were carefully cut open, their smooth lining membrane was recognised, and they were seen to communicate with the interior of the placenta. The aperture of communication was crescentic in form, and was directed obliquely, so as to act like an imperfect valve. In the posterior lobe of the placenta, where the maternal system of vessels was more fully injected, the utero-placental veins were filled with the red injection, and when the placenta was gently squeezed, after a vein had been opened, the injection could be made to ooze out of the interior of the placenta through the crescentic opening into the vein.

The wall of the uterus did not contain a tortuous arrangement of the branches of the uterine arteries so characteristic of these vessels in the human gravid uterus, neither were curling arteries to be seen in the decidua serotina. This layer was, however, traversed by slender arteries, so fine indeed that it was difficult to isolate them from the surrounding decidua. They could be traced up to the placental layer of the decidua serotina when they tore through abruptly, so that their exact mode of entering the placenta could not be ascertained; but it seemed as if the supply of arterial blood to the maternal placenta was obtained through numbers of fine arteries which pierced the placental layer of the decidua at various points, and quite independently of each other.* The red injection of the maternal vessels had freely passed

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* A more explicit description of the utero-placental arteries is given in the Appendix to this Memoir, pages 561, 562.

into the more posterior placental lobe, and had insinuated itself between the fœtal villi, where it formed a continuous network extending from the placental layer of the decidua to the stratified arrangement of sub-chorionic cells covering the placental surface of the chorion. This network of red injection was not contained in cylindriform tubes, so that it had no definite pattern, but exhibited an irregular anastomosing arrangement. In those parts of the lobe where the injection was most perfect, the stems, branches, lateral and terminal bud-like offshoots of the villi were seen to be surrounded by the red injection which was separated from the blue injection, occupying the umbilical vessels of the villi, by the cellular investment of the villi and by the more peripheral part of their delicate connective tissue. The spaces occupied by this network of red injection are, I believe, those through which the maternal blood flows in the living placenta. That they are not artificial channels produced in the act of injection is I consider established by their free communication with the utero-placental veins, and from the fact that they were filled with but a slight pressure on the piston of the injecting syringe. These spaces were not of uniform diameter; as a rule they were not equal to the transverse diameter of the villi, but sometimes they were of equal width, and occasionally even were wider than the villi. The width of the channels through which the injection, or the maternal blood, flowed would undoubtedly, within certain limits, vary with the pressure on the blood or injection contained in the maternal vessels. I could not satisfy myself that the spaces containing the red injection were enclosed by a definite membrane, continuous with the wall of the uterine arteries or veins, and separating the injection from the cellular investment of the villi. All that I saw was in favour of the view that the injection, and consequently the maternal blood in the living animal, was in direct contact with the cellular investment of the villi, the sub-chorionic cellular covering of the placental surface of the chorion, and the cellular surface of the placental layer of the decidua serotina. It is not unlikely, however, that at the relatively large crescentic openings of communication between the utero-placental veins and the spaces in the interior of the placenta, the lining membrane of the vein is prolonged for some distance along the placental surface of the placental layer of the decidua.

Comparison of the Placenta of the Ape with that of the Human Female.

In the course of my description of the gravid uterus and placenta of *Macacus cynomolgus* I have incidentally referred to some points of correspondence with, or difference from, the human gravid uterus and placenta, but it may not be uninteresting to make a more detailed comparison of the one with the other. More especially is it advisable to compare the minute structure of the organs, for up to this time satisfactory material for doing so has not been in the possession of anatomists. In the course of this comparison I shall refer to a number of original observations on the human gravid uterus and placenta, many of which, though incorporated in one of my lectures on the Comparative Anatomy of the Placenta, delivered at the Royal College of Surgeons of England in June, 1876, have not otherwise been published.

Both in the *Macacus* and the human female the uterus is single and ovoid in form in its gravid condition. The gravid uteri of *Hapale jacchus* and *Cercopithecus sabæus* figured by RUDOLPHI and BRESCHET possessed a similar form, so that there can be little doubt that the pregnant uterus has this shape throughout the Quadrumana. In the Lemurs, on the other hand, the uterus is divided into two cornua, and though in the later stages of gestation the non-fecundated cornu becomes so compressed as to be scarcely visible on external examination, yet, as I have shown in a previous memoir,* when the uterus is opened it can be distinctly seen.

In the gravid uterus of the *Macacus*, when the abdominal cavity was opened, and the organ examined without disturbing its position, not only the round ligaments and Fallopian tubes were seen at the lateral borders of the anterior surface of the uterus, but the ovaries were also distinctly visible. In the human gravid uterus, again, these structures are situated more at the sides of the organ, so that to obtain a complete view of them the uterus must be drawn either forwards or to one side. From this difference in their relative position it would seem as if in the human female the growth and expansion during pregnancy of the anterior wall of the uterus was greater than in the Ape, so as to throw these structures more towards the sides of the organ.

In the *Macacus* the breech and not the head was the part of the fœtus which presented at the os uteri, and in JOHN HUNTER's case also it is stated † that the young one was born with the hind parts first. Neither RUDOLPHI nor BRESCHET, though they describe the characters of the fœtus found in several gravid uteri which they examined, state the nature of the presentation. Observations have been therefore too scanty to enable one to say if the breech is the part which normally presents in the Apes, or if its presentation was exceptional in JOHN HUNTER's and in my specimen.

The arrangement of the fœtal membranes in *Macacus* closely resembles that of the human placenta.

The division of the placenta into two distinct and separate lobes is in accordance with what has been seen by other observers to be the rule in the tailed Apes of the old world. In the tailless *Hylobates*, also, M. BRESCHET has figured a two-lobed placenta, but in the Anthropoid Chimpanzee the placenta is single. In the human placenta, as is well known, the placenta is also single, but cases have occasionally been seen in which the organ has been divided into two separate lobes. Professor HYRTL not only figures‡ two human placentæ where in each the division into two lobes is

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* "On the Placentation of the Lemurs," Phil. Trans., 1876.

† Works, edited by PALMER, vol. iv., p. 72.

‡ 'Die Blutgefässe der Menschlichen Nachgeburt.' (Wien, 1870.) HYRTL also figures a human placenta almost completely divided into three lobes, and refers to cases where the division into a still greater number of lobes has been seen.

almost complete, but a specimen where they are completely separated. The umbilical vessels are directed to the edge of one of the lobes, but just before reaching it they give off branches which pass to the other lobe. HECKER* and J. MATTHEWS DUNCAN† have also referred to cases of bilobed human placentæ. In my specimen of *Macacus*, in the one figured by JOHN HUNTER, and in several of the placentæ figured by BRESCHET, the division into lobelets by furrows on the surface of the organ was also distinct.

The mucous lining of the non-placental part of the uterine cavity of *Macacus* obviously corresponded generally with the decidua vera of the human gravid uterus; but it differed from it in several points of structural detail. From the intimate adhesion which subsisted between the chorion and mucosa in the *Macacus* it was not easy to make a complete separation between the two surfaces, but when this was effected, the fine ridges and furrows described in a previous section of this memoir (p. 530) were observed.

In the human uterus the chorion and the free surface of the decidua are not so intimately adherent to each other as in the *Macacus*, although, as has been pointed out by various observers, at least in the later months of pregnancy adhesions do occur. I have seen in a human uterus at the fifth month the free surface of the decidua vera finely corrugated into numerous delicate convoluted folds, whilst thread-like prolongations of the chorion, apparently atrophied villi, were adherent to this surface. In another specimen, at the seventh month the outer surface of the chorion was closely adherent to the decidua lining the uterine cavity. When the chorion was stripped off, a well marked layer of decidua, the decidua reflexa, was attached to it, and a somewhat thicker layer remained on the uterine wall. To separate these from each other, flakes and threads of decidual tissue had to be torn through; but as the separation was not made precisely in the same plane throughout, it was clear that the decidua vera and reflexa had become blended together. After the separation had been made, the free surface of the vera was seen to be furrowed, and to these furrows ridges of the chorion were closely adapted. In a human uterus in the ninth month, whilst the chorion, with the decidua reflexa, was generally adherent over the whole extent of its outer surface to the vera, a smaller amount of traction was sufficient to draw them asunder. When the chorion and reflexa were removed, the surface of the vera was smooth and not corrugated.

Observations are wanting on the appearance of the decidua vera in the early stages of gestation in the monkey, but in the human female as early as the twelfth or thirteenth days of gestation REICHERT's observations have shown‡ that the free surface of this decidua was divided by furrows into "islands" which had an irregular

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* 'Klinik der Geburtskunde.' Band II.

† 'Mechanism of Natural and Morbid Parturition,' Note, p. 313. Edinburgh, 1875.

‡ 'Beschreibung über frühzeitigen Menschlichen Frucht. Abhand. der König. Akad. der Wissenschaft.' Berlin, 1873.

polyhedral form. The free surface of these island-like areas was not quite smooth, but subdivided by minute furrows. The surface of the decidua was perforated by numerous apertures, much more distinctly seen in and near the furrows between the different islands than on the summit of the islands ; these openings were the mouths of the utricular glands. In the beautiful drawings of the human gravid uterus from the twentieth to the twenty-fifth day, published by M. Coste,[*] the division of the free surface of the decidua vera into islands is well displayed, as well as the orifices of the numerous utricular glands. In two human gravid uteri, at from the third to the fifth week, which I examined in June, 1876, similar appearances were seen. But I have also observed these characters in more advanced specimens. In a uterus at the fourth month the "islands" were very distinct, as well as the corrugated appearance of their surfaces. Numerous gland-mouths opened into and near the furrows between the islands. Along the fundus and sides of the uterine cavity the mouths of the glands were much more closely set in a given area than on the anterior and posterior surfaces, the orifices were transversely elongated, and the gland-tubes were short. In the specimen at the fifth month, already referred to (p. 539), the orifices of the glands were distinct, and also varied in number in different parts of the mucous surface ; the division of the surface into island-like areas was no longer recognisable. In the uterus at the seventh month the islands had disappeared, but rounded orifices leading into shallow tubular depressions were scattered in varying numbers over different parts of the surface of the mucosa. In the specimen at the ninth month, the free surface of the decidua showed neither islands nor furrows, but numerous shallow pits opened on its free surface. The obliteration of the island-like areas and the intermediate furrows in the later months of pregnancy in the human uterus is doubtless due to the great distention of the organ, and the same cause has also converted the ducts of the tubular glands into the shallow pits I have just referred to.

In my *Macacus cynomolgus* at the stage of pregnancy at which the uterus was examined, no island-like areas or mouths of utricular glands were to be seen on the surface of the mucosa.

Few observations have as yet been recorded on the glands in the non-gravid uteri of the Apes. In 1873 I described[†] these structures in the uterus of *Ateles gricescens,* and in my lectures on the 'Comparative Anatomy of the Placenta'[‡] I made some general observations on the utricular glands in some other genera of Apes, but did not give a detailed description of them, an omission which I now supply.

I have specially examined the non-gravid uteri of *Macacus rufescens, Semnopithecus entellus,* and young specimens of a *Hylobates agilis,* and a Chimpanzee, in order to determine the characters of the mucous membrane in the unimpregnated uterus, and the mode of arrangement of the utricular glands. In all these specimens the mucous

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[*] 'Histoire du Développement des Corps Organisés.' 1847.

[†] On the "Placentation of the Sloths," Trans. Roy. Soc. Edinburgh, vol. xxvii.

[‡] First series, p. 28. Edinburgh, 1876.

membrane formed a definite layer lining the uterine cavity. By its deep surface it was closely united to the subjacent muscular tissue. Its free surface was covered by a single layer of columnar epithelial cells; these cells were much shorter than the columnar cells covering the surface of the intestinal or respiratory mucous membrane; their nuclei were relatively large, and the cells themselves were not unlike in appearance the germ-epithelium corpuscles one sees on the surface of a young ovary. I am unable to say if the cells were ciliated at their free ends, as the uteri had been preserved in spirit for some time before they came into my possession;* but there can be little doubt that in the Apes, as in the uteri of other mammals, the cellular covering of the mucosa is a ciliated epithelium.

The sub-epithelial connective tissue of the mucosa was vascular, and contained a great abundance of corpuscles. Although the corpuscles were diffused throughout the whole thickness of the tissue, yet they were in some places relatively more numerous than in others. In *Macacus*, for example, they were more closely crowded immediately subjacent to the epithelium than in the deeper parts of the mucosa, and, in many of the sections through the mucosa of this and the other specimens, rows of corpuscles lay almost vertical to the free surface, as if following the course of the larger blood vessels. In the *Macacus, Hylobates,* and Chimpanzee, the mucous membrane was thrown into broad folds separated by shallow furrows, but in *Semnopithecus* the free surface of the mucosa was almost plane.

The glands were not uniform either in arrangement or relative numbers in these genera of Apes. In the young Chimpanzee they seemed to be absent in the substance of the broad folds of the mucosa, but to be situated at the sides and bottom of the furrows which separated these folds from each other. In these localities vertical sections through the mucous membrane exhibited transverse and oblique sections through tubular glands, but it was only seldom that the gland tube could be followed to its opening on the surface. As the divided gland tubes were always seen to be collected together in clusters, it is probable that the tube was tortuous and perhaps branched, so that a vertical section passed through various parts of its length. The deeper parts of the glands lay in close proximity to the muscular coat and distinct fasciculi, apparently continuous with the muscular bundles, passed between them. In the young *Hylobates* the glands were more numerous, and more equally distributed throughout the mucosa than in the Chimpanzee. In vertical sections through the membrane they were often divided both obliquely and transversely, so that their course was oblique to the surface and probably tortuous, and it was only seldom that the opening of a gland on the surface could be seen. In *Macacus rufescens* tubular glands were arranged as in the Chimpanzee in the neighbourhood of the furrows which separated the longitudinal folds of the mucosa from each other. A tubular appearance was also seen in the folds themselves; but from the absence of an

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* These uteri were obtained from animals that had died in the Zoological Gardens, and were kindly given to me by Professor A. H. Garrod.

epithelial lining (which may perhaps have been shed) I could not say definitely that these were gland tubes.

In *Semnopithecus* the glands were very distinct and uniformly distributed throughout the mucosa. For some distance from the surface the tubes passed almost vertically into the substance of the mucosa, so that they were divided longitudinally in the sections. The gland tube was constricted into a comparatively narrow neck near its mouth, below which it dilated and assumed a flask-shaped form. In the deeper parts of the mucosa the glands were more tortuous and branched, and the sections through the tubes were oblique and transverse.

In all the specimens the glands were lined by a columnar epithelium, resembling in its characters the epithelium covering the free surface of the mucosa. The cells did not occupy the whole transverse diameter of the tube, but left a central lumen. I am unable to say if these cells were ciliated, as has been observed by ALLEN THOMSON* and LEYDIG† in the epithelial lining of the uterine glands in the pig, and by LOTT‡ in the cow, sheep, rabbit, mouse, and bat.

In the non-gravid human uterus the utricular glands are elongated and even slightly tortuous tubes, which extend from the free surface of the mucosa in an oblique direction throughout its substance, and reach by their closed ends the muscularis mucosæ. They branch occasionally in their course, and sometimes two neighbouring glands join to form a short duct, which opens by a single aperture on the free surface of the mucosa. They are lined by a columnar§ epithelium, which FRIEDLÄNDER and JOHN WILLIAMS‖ have observed to be ciliated, and which resembles the epithelium covering the free surface of the mucosa itself.

That great changes take place in the mucous membrane of the human uterus during pregnancy is well known to anatomists. Not only is the surface epithelium, but the subjacent tissue and the utricular glands are modified in an important manner. The changes in the glands and inter-glandular tissue were studied many years ago by E. H. WEBER,¶ DR. SHARPEY,** and the late Professor GOODSIR,†† more especially in the early stages of gestation. Of late years renewed attention has been given to the condition of the glands, particularly in the later stages of gestation, and important observations have been recorded by FRIEDLÄNDER,‡‡ KÖLLIKER,§§ KUNDRAT and ENGELMANN,‖‖ LANGHANS¶¶ and LEOPOLD.***

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* QUAIN'S 'Anatomy,' 8th edition, vol. ii., p. 466.

† MÜLLER'S 'Archiv,' 1852, p. 375.

‡ STRICKER'S 'Handbuch:' Article, "Uterus."

§ 'Physiologisch-Anatomische Untersuchungen über den Uterus,' Leipzig, 1870.

‖ "The Structure of the Mucous Membrane of the Uterus," Obstetrical Journal, 1875.

¶ WEBER'S edition of HILDEBRANDT'S 'Anatomie,' vol. iv.

** BALY'S translation of MÜLLER'S 'Physiology.'

†† 'Anatomical and Pathological Observations,' Edinburgh, 1845, and 'Anatomical Memoirs,' 1868.

‡‡ *Op. cit.*

§§ 'Entwicklungsgeschichte,' 1st edition, 1861; 2nd edition, 1876.

‖‖ STRICKER'S 'Med. Jahrb.,' 1873.

¶¶ 'Archiv. für Anat. und Phys.,' 1877, p. 188.

*** "Studien über die Uterus-schleimhaut," 'Archiv. für Gynäkologie,' 1877.

The decidua vera in the human uterus is thicker than the mucous membrane of the unimpregnated uterus, and this thickening is always considerably greater in the earlier than in the later months of pregnancy. The superficial part of the membrane is comparatively compact, whilst the deeper part possesses a looser and more spongy character. The ciliated columnar cells, which form the epithelial covering of the free surface of the non-gravid membrane, disappear. The glands become expanded, and though they retain for a time their elongated tubular form, yet in the later months of pregnancy, in correlation with the growth of the ovum and the expansion of the uterine wall, they are so altered that the evidence of their presence is so much obscured that they are recognised with difficulty.

The human gravid uterus at the fifth month, with its contained placenta, closely approximates in size to the gravid uterus of the *Macacus cynomolgus*, which forms the special subject of this Memoir; and it may not be without interest to compare the structure of the human decidua vera at this stage of gestation with that of the *Macacus*. The human decidua vera consisted of a superficial compact and of a deeper spongy layer (figs. 3, 4). The compact layer was composed of a laminated arrangement of irregularly polygonal and somewhat flattened cells with distinct nuclei. The cells were closely crowded together, and the protoplasm in many was elongated into angular processes. It is not unlikely that the stratified cells of this compact layer were derived from and represented in the vera, the epithelium of the non-gravid mucosa, though they widely differed from the ciliated and columnar cells of the unimpregnated uterus. In this respect the epithelial covering of the vera in the human uterus differs in a material manner from what I have described in the gravid *Macacus*, in which animal it retains in the lower part of the uterus its columnar and ciliated character, though in the region of the fundus it forms a single layer of polygonal cells arranged in a pavement-like manner.

In vertical sections through this layer in the human decidua short tubes were traced from the orifices opening on its surface; they passed almost vertically through its thickness, but I did not see them communicate with the spaces of the subjacent spongy layer (fig. 3). In horizontal sections through the compact layer the tubes were transversely divided, and they were seen to be circular or elliptical in form. Occasionally two tubes were close together, but more usually they were separated from each other by considerable intervals occupied by the proper cellular constituents of the compact layer. These tubes had no special epithelial lining, but their walls were formed by the cells of the compact layer. Notwithstanding the absence of an epithelial lining, I have no doubt that these tubes were the ducts of the utricular glands which had become much more dilated than in the non-gravid condition.

Subjacent to the compact layer was the spongy layer of the mucosa, the more superficial part of which had the appearance of an imperfectly differentiated connective tissue, in which were numbers of elongated fusiform corpuscles, swollen out somewhat at the body of the cell, where a large elongated nucleus was situated.

These corpuscles were arranged very definitely in rows, with their long axes parallel to the surface of the decidua; sometimes the rows of cells were close together, but at other times not only the different rows, but the cells in the same row were separated by an imperfectly differentiated connective tissue. These cells were not aggregated into bundles, but the outline of each cell was distinct. In their form and appearance they bore a close resemblance to the fusiform corpuscles of involuntary muscular tissue (fig. 4).

The spongy tissue was several times thicker than the compact layer, and owed its loose or spongy character to the numerous spaces in it. Some of these spaces were undoubtedly sections through blood vessels, for they contained a red injection which had been thrown into the vessels from the uterine arteries. Others again, and these the most numerous, were seen on section to be, in some instances, elongated spaces, in others spaces varying in shape and size, which were arranged in several tiers, irregularly superimposed over each other, and were evidently non-vascular. These spaces were separated from each other by trabeculæ, which were sometimes broad bands of tissue, but at other times slender bars. Fusiform corpuscles, similar to those immediately subjacent to the compact layer of the mucosa, entered into the formation of most of the trabeculæ, and in some instances constituted their chief constituent, but at other times irregularly polygonal, and somewhat flattened cells with rounded nuclei were the predominant structures in the trabeculæ. I saw no appearance of an epithelial lining to these spaces.

We may now inquire into the nature of the non-vascular spaces in the spongy tissue. They were obviously natural spaces in the mucosa, and not produced in the act of making the section.

Friedländer, Kölliker, and the other observers above referred to, regard the decidua vera as retaining its glands throughout the whole period of gestation in the human uterus, which by becoming greatly widened and convoluted give rise to the spongy character of the vera I have just described. The epithelium, they say, also degenerates and disappears from many parts of the widened glands, the disappearance taking place in those parts of the glands which lie nearest the surface, but in the deeper spaces of the spongy layer patches of columnar or cubical cells have been seen by Kölliker and Leopold up to the end of pregnancy. I was not so fortunate as to see an epithelial lining, either complete or partial to the gland spaces, in my specimen at the fifth month, or in uteri at later stages of gestation; and there can, I think, be little doubt, though we may regard the spaces as produced by the great dilatation of tubular glands, yet that the degeneration and loss of their epithelial contents must have impaired if not destroyed, at least in the later stages of gestation, their function as secreting organs.

In the decidua vera of the *Macacus* the glands had also become modified from the non-gravid condition. If the spaces referred to in the descriptive part of this memoir (p. 531) were glands, then their epithelium had disappeared, and the tortuous tubes with an opaque epithelium, also referred to, were so sparing in number that they were very seldom seen. As the uterine glands in the non-gravid *Semnopithecus* are much more numerous and more distinct than in the non-gravid *Macacus*, it is possible that in the former animal they may during gestation undergo a smaller amount of change both in form and structure. The spongy character of the decidua vera in the human uterus was much more pronounced than in the uterus of *Macacus cynomolgus*. I am unable to say if the decidua vera is shed in parturition in the *Macacus* or remains on the surface of the muscular coat.

In the absence of any information of the early stages of gestation in the Apes, it is not possible to say with certainty that the impregnated ovum is in the early weeks of pregnancy shut off from the general cavity of the uterus, and confined in a special ovigerous chamber by the development of a periovular decidua, the decidua reflexa, like that of the human female. M. BRESCHET, in his figures of the placenta and membranes within the gravid uterus of the two specimens of *Cercopithecus sabæus* which he described, has represented the decidua as divided in some places into two layers, as if one were the vera and the other the reflexa. Professor OWEN, in his description of the shed placenta of *Macacus rhesus*, states* that portions of the deciduous membrane remain attached to the outer surface of the placenta and chorion. Dr. ROLLESTON† was unable to say, from the condition of his specimen, if the decidua reflexa was as complete as BRESCHET had described and figured.

From the observations of these anatomists it would seem that, in their judgment, the Apes possess a decidua reflexa, although the demonstration of its arrangement does not appear to have been very complete in any of the specimens. Still, from the general similarity, both in form and structure, of the placenta in the Ape and the human female, there is every probability that a reflexa grows around the young ovum of the Ape, as it does in the human uterus. My dissection, however, proves that in the later stages of gestation, the chorion and uterine decidua have become so intimately attached to each other, that the demonstration of the decidua reflexa as a distinct layer is not possible; and during parturition, if the chorion had carried away, on its outer surface, a covering of decidua, it would have been difficult to say if that covering were reflexa or vera. This, however, is no proof that a reflexa did not at one time exist as a definite layer, for, as I have already stated, even in the human uterus, in the later months of pregnancy, the chorion, with its covering of reflexa, becomes closely attached to the corresponding surface of the vera. Moreover, the reflexa cannot, as was at one time supposed, be regarded as an exclusively human structure; for, as I have elsewhere shown,‡ it exists as a definite layer on the surface of the non-placental part of the chorion of the two-toed Sloth, *Cholopus Hoffmanni*.

Both in the human placenta and in that not only of *M. cynomolgus* but of the other monkeys described by previous anatomists, a well-defined and even tolerably thick

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[*] "Catalogue of the Museum of the Royal College of Surgeons," Physiological Series, vol. v., p. 145.

[†] Trans. Zool. Soc., vol. v., 1863.

[‡] Trans. Roy. Soc. Edinburgh, 1873, p. 77.

layer of decidua serotina intervened between the placenta and the muscular coat of the uterus; and in the Apes the serotina was proportionally thicker than in the human female. In both, the serotina can be divided into two strata, the one comparatively thin and immediately related to the placenta, the other considerably thicker and in relation to the muscular coat of the uterus. The thin stratum is, without question, shed along with the placenta during parturition; whilst the thicker stratum, either altogether or in great part, probably remains on the placental area of the uterus. One of the most prominent characters of the thicker stratum in *Macacus cynomolgus* was its loose spongy appearance, due to the numerous areolæ or loculi which it contained; and in the separation of the placenta there can be little doubt that the septa between those loculi which lie next to the thinner and proper placental layer are torn through. M. BRESCHET has figured,* but not described, in a vertical section through the placenta of *Cercopithecus sabæus*, a series of spaces; but the small scale to which the figure is drawn makes it somewhat difficult to say if these spaces are intended to represent spaces within the placenta, or loculi in the serotina similar to those present in *M. cynomolgus.* Dr. ROLLESTON, in his account of the placenta of *Macacus nemestrinus,*† described numerous loose lamellæ as intervening between the placenta and the muscular coat of the uterus, the deeper of which had a horizontal direction, whilst those nearer the placenta were vertical. I have little doubt that these lamellæ were the septa between a system of loculi, similar to those I saw in *M. cynomolgus.*

In human uteri at various stages of gestation, the spongy appearance of the stratum of the serotina next the muscular coat of the uterus has been described by several anatomists. FRIEDLÄNDER, KUNDRAT and ENGELMANN, KÖLLIKER and LEOPOLD, have all recognised it, and the stratum has been regarded as homologous with the spongy layer of the decidua vera. My attention has also been directed to this matter, and I shall now describe the more important characters that I have observed.

In two human gravid uteri, from the third to the fifth week,‡ the decidua serotina was irregularly sinuous on its free surface, as is so well delineated by M. COSTE in his figures of this structure in the early stages of pregnancy. It was nearly two-tenths of an inch in thickness, and when vertical sections were made through it was seen to consist of a more compact superficial, and of a deeper spongy part. The superficial compact portion consisted of two strata. The stratum next the ovigerous chamber had a folded or laminated appearance, but its minute structure was not well defined. It seemed to consist of an indefinite fibrillated material, which did not colour with hæmatoxylin. Scattered through this material, and somewhat more numerous in the more advanced of the two specimens, were a few corpuscles having the size and form

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* Memoir already cited, plates 3 and 4, fig. 5.

† Memoir already cited, p. 300.

‡ For the opportunity of examining one of these specimens, from the Museum of St. George's Hospital, I am indebted to Dr. ROBERT J. LEE; the other, a somewhat more advanced specimen, in the Museum of Guy's Hospital, Drs. PYE-SMITH and HILTON FAGGE kindly gave me permission to examine.

of blood corpuscles; and in the less advanced specimen some irregularly-shaped cells, three or four times larger than blood corpuscles and with distinct nuclei, were occasionally seen. The obscure anatomical characters of this stratum made it difficult to give an opinion on its nature. From its position on the free surface of the serotina, it might be regarded as the coagulated secretion of the utricular glands, but the presence of blood corpuscles in it is against such a supposition, and would lead one to think that it was coagulated blood. But the paucity of the corpuscles was such as to throw doubts upon this interpretation.

Immediately subjacent to, and in part commingled with, this stratum, was the deeper stratum of the compact layer. It was almost entirely composed of cells, the nuclei of which were brilliantly stained with hæmatoxylin, so that it contrasted in colour with the superficial stratum. A large proportion of the cells were fusiform, and contained elliptical or oval, or at times elongated, nuclei. They were, for the most part, arranged with their long axes perpendicular to the free surface of the serotina. Intermingled with the fusiform cells were rounded and nucleated lymphoid corpuscles, which in some localities were more numerous than in others; also a proportion of irregularly-shaped cells, three or four times larger than white blood corpuscles.

The deeper spongy part of the human serotina was thicker than the compact stratum. It owed its spongy appearance to the numerous areolæ it contained, the larger of which were visible to the naked eye. The areolæ varied in form, being in the vertical sections circular or ovoid, or considerably elongated or triangular, or with a greater number than three sides. Sometimes they were closely crowded together, and separated by slender thread-like bands of decidual tissue, but at other times they were further asunder, and broad bands of the tissue intervened. This tissue was continuous with the deeper stratum of the compact layer of the serotina, and, like it, consisted of variously formed cells; but the fusiform corpuscles, more especially in the more slender septa between the areolæ, were more attenuated than in the compact layer. Throughout the serotina the cells were embedded in an indistinctly fibrillated matrix.

The edge of the cut surface of many of the areolæ directed towards the enclosed space was often quite sharp and distinct, as if bounded by a definite coat, but at other times the outline was ragged and wanting in precision, and thin flakes of cells projected from the wall into the areola. Some of the smaller areolæ having a circular outline were unquestionably divided arteries and veins, for they possessed distinct coats and contained red blood corpuscles. But blood corpuscles were also contained in a number of the spaces possessing an elongated triangular, or even more than three-sided form, so that from the nature of their contents they were clearly also divided and dilated blood vessels. The corpuscles were at times so abundant as quite to fill the areolæ, but at other times they only occupied a portion of the space. These larger areolæ did not possess laminated walls, but the decidual tissue formed their apparent boundary, though it is possible that they may have had an endothelial lining. They were obviously colossal capillaries dilated into small sinuses. But in many of the areolæ, cells were intermingled with the red blood corpuscles, the nuclei of which were stained with hæmatoxylin like the proper decidual cells. Some of these cells were white blood corpuscles, others of a more irregular shape were larger than white corpuscles, whilst others were spindle-shaped cells. They were so intermingled with the red corpuscles that they did not seem as if they had got accidentally amongst them in the act of making the section, but rather as if they had formed a portion of the contents of the sinus-like capillary, and it may have been developed from the white blood corpuscles. Many areolæ which contained no blood corpuscles resembled in size, form, and general appearance those in which these corpuscles were situated, and without doubt must be regarded as blood sinuses, out of which the corpuscles had dropped in the process of preparing the section for microscopic examination.

But these blood spaces, though the most numerous, were not the only spaces to be seen in the sections; for in the deeper part of the spongy tissue I occasionally saw sections through tubular structures which possessed a more or less perfect lining of columnar epithelium. Sometimes the cells were in position on the wall, but at other times they were loose in the lumen of the tube. There could be no doubt that these tubes were the utricular glands of the placental area; and on the free surface of the serotina some rounded openings were seen, which were in all probability the mouths of the glands.

In the human placenta at the fifth month the decidua serotina consisted of a compact cellular layer next the placenta, and of a deeper and thicker spongy layer. The spongy character of the latter was due to numerous areolæ, which were irregular in form as in the earlier stages just described, but considerably larger. They were separated from each other by bands of decidual tissue, and though it is possible that some of these areolæ were dilated utricular glands, a large number were unquestionably the dilated blood sinuses of the decidua serotina, for they contained a red injection which had been passed into the uterine vessels. In the human placenta in the ninth month the serotina also consisted of a compact cellular and a spongy layer. It is quite certain that almost the whole, if not the whole, of the spaces in the spongy layer at this stage were dilated blood sinuses, for not only did they contain a red injection passed into them from the uterine vessels, but several of those which lay next the placenta had perforations in their walls through which the placental villi projected into the cavity of the sinus, and the sinus communicated with the maternal blood space in the interior of the placenta. I have obtained no satisfactory evidence of the persistence of the utricular glands in the placental area of the human uterus at and near the full time; and in this respect my observations are more in accordance with those of KÖLLIKER than with those of FRIEDLÄNDER, KUNDRAT and ENGELMANN, LANGHANS and LEOPOLD, who have described enlarged but compressed glands containing epithelium as persisting in the placental area up to the separation of the placenta.

Should this be so, they must be very sparing in numbers. My observations on the structure of the spongy layer of the serotina even in the early weeks of pregnancy show that the blood sinuses of the decidua constitute a greatly preponderating proportion of the areolæ of the spongy layer: much more so than LEOPOLD's observations would lead one to believe. The employment of injections of the uterine vessels in the examination of the decidua in the later months of pregnancy enables one to state with certainty that here also the blood sinuses are the dominating, if not the sole, factors in the constitution of the areolæ.

In *Macacus cynomolgus* the areolæ so abundantly present in the spongy layer of the serotina were obviously not blood sinuses. They did not contain any injection, although the injection had filled not only the vessels in the muscular coat of the uterus, the capillaries, and other vessels of the decidua, but had occupied the maternal blood spaces in the interior of the placenta. In the *Macacus* the veins of the decidua do not expand into elongated sinuses as in the human placenta, but retain the form of cylindrical tubes, and possess definite walls which can be isolated by dissection from the surrounding decidual tissue. The arteries also which penetrate the decidua in their course to the placenta do not in the *Macacus* possess the form of the curling arteries of the human placenta. The areolæ in *Macacus* may probably therefore represent the dilated glands in the placental area of the decidua, although their epithelial lining had lost its glandular character and had assumed for the most part a squamous form.

But the placental part of the decidua was not limited to the layer which lies between the placenta and the muscular coat of the uterus. Both in the human placenta and in that of the Ape, processes of the decidua were prolonged into the interior of the organ. In the human placenta these processes have been referred to, and in some instances imperfectly figured by several observers, as VAN DER KOLK,[*] ECKER,[†] PRIESTLEY,[‡] and KÖLLIKER.[§] In a paper published in 1872 [||] I described and figured in the human placenta at the full term, bands of the decidua extending from the placental layer of the serotina up to the chorion, and separating the lobes or cotyledons of the placenta from each other. I named these bands the primary or inter-cotyledonary decidual dissepiments. Their junction with the chorion was readily seen near the margin of the placenta, and is represented in the plate appended to my paper; but nearer the middle of the organ the junction could not always be traced. I also observed that these dissepiments could be split into two layers, one remaining attached to each cotyledon, and between these layers a utero-placental vein was not unfrequently situated. Smaller prolongations passing into the

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[*] 'Waarnemingen over het Maaksel van de Menschelijke Placenta,' 1851.

[†] 'Icones Physiologicæ,' 1852–1859.

[‡] 'Lectures on the Gravid Uterus,' 1860.

[§] 'Entwicklungsgeschichte,' 1st edition, p. 145, 1861.

[||] 'Journal of Anatomy and Physiology,' vol. vii., p. 133, plate 5.

interior of the cotyledons for a greater or less distance, but not reaching the chorion, were also described as secondary or intra-cotyledonary dissepiments. Since that time I have examined the intra-placental prolongations of the decidua in younger placentæ. In the placenta at the fifth month the arrangement was very instructive. Not only did very definite bands of decidua separate the lobes of the placenta from each other, but numerous intra-cotyledonary or secondary dissepiments passed into each lobe, both from the general surface of the serotina and from the surface of the inter-cotyledonary dissepiments. From these secondary dissepiments numerous still more delicate offshoots branched off in the intervals between the villi of the chorion, where they formed a network in the interspaces of which the villi were contained. Hence the whole interior of the placenta was traversed by prolongations of the decidua serotina, which formed an intervillous network of trabeculæ. The dissepiments had the characteristic structure of the compact layer of the decidua; but the intervillous offshoots, though they contained the large decidua cells close to where they arose from the dissepiments, yet were made up in great part of a delicately fibrillated connective tissue in which scattered corpuscles were imbedded.

Kölliker, in the second edition of his 'Entwicklungsgeschichte' (p. 336, 1876), has recently described these inter-cotyledonary dissepiments by the name of septa placentæ, and his observations on these bands of decidua closely correspond with those I had previously made on the mature placenta. The surface of the decidua, which is directed to the interior of the human placenta, is not therefore a plane surface, but presents an irregularly uneven appearance from the numerous intra-cotyledonary dissepiments which project from it.

In the *Macacus nemestrinus* described by Rolleston, processes of maternal tissue were followed into the placenta. In *M. cynomolgus* the placental surface of the decidua serotina was uneven from the numerous hillock-like elevations which in the description of the organ (p. 534) I have compared with stalagmites: these correspond to the intra-cotyledonary dissepiments of the human placenta. In the *Cynomolgus*, however, though the division into cotyledons is marked by furrows on the chorionic surface of the organ, and though there is a special thickening of the decidua opposite the borders of contiguous lobelets, yet the decidua does not send prolongations between the lobelets up to the surface of the chorion, like the inter-cotyledonary dissepiments of the human placenta. Hence the intra-placental maternal blood spaces in *Macacus* form throughout the interior of the organ a more continuous anastomosing cavernous arrangement than in the human placenta.

The presence of the definite arrangement of stratified cells on the placental surface of the chorion of *M. cynomolgus*, which I have named the sub-chorionic cells, is of particular interest in connexion with some observations made during the last six years on the structure of the human placenta. In 1872 Winkler described and figured diagrammatically,* by the name of *Schlussplatte*, a layer extending over the entire

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* "Zur Kenntniss der Menschlichen Placenta," 'Archiv für Gynäkologie,' iv., 238.

extent of the placental surface of the human chorion, consisting of homogeneous intercellular substance with cells scattered in it. This layer also, he stated, invested the stems of the villi, but was not prolonged on to their finer branches. From its arrangement he regarded it as bounding the maternal blood spaces on the fœtal side of the placenta, and as formed of maternal tissue. This structure has recently been investigated by KÖLLIKER, who named it "decidua subchorialis."[*] Though recognising that the placental surface of the chorion presented throughout a peculiar lamellated structure, KÖLLIKER was unable to trace the decidua subchorialis for more than a short distance from the border of the placenta; neither was he able to see that it gave a covering to the stems of the villi, except to those which arose from the chorion in close proximity to the placental border. Throughout the placenta generally he states that the maternal blood spaces had on their fœtal aspect no other boundary than the chorion itself. LEOPOLD, in his memoir on the human placenta already referred to, says[†] that his observations correspond with those of KÖLLIKER. But a different view of the nature of this structure has been advanced by LANGHANS.[‡] He recognises that about the middle of pregnancy a layer of cells is developed. At first the cells are small in size, but subsequently present a great resemblance to decidual tissue, though in his opinion they do not belong to the decidua. For he states that they are formed between the vascular fibrillar stroma of the chorion and its epithelial covering, and are probably derived from the vascular layer of the chorion itself. He names the structure the "cell layer of the chorion frondosum," employing a term which implies no opinion of the origin of the cells. In most cases the layer is not of equal thickness throughout, but the cells lie in groups, between which a homogeneous intercellular substance is situated, which is directly continuous with the fibrillar tissue of the chorion. At a later stage of placental growth he states that this cell layer becomes converted into canalised fibrine, for after the dissolution of the chorionic epithelium in the last months of pregnancy a deposition from the maternal blood takes place, and the lamellated layer of the chorion frondosum is formed. This cellular and lamellated layer bounds the space within the placenta in which the maternal blood is contained, and it is penetrated by the chorionic villi which project into that blood space.

My attention has also been directed to the placental surface of the chorion in the human placenta. In the placenta in the fifth month this surface was covered by a single layer of cells, which had the aspect of the chorionic epithelium. They were, however, to all appearance in process of degeneration, for they were infiltrated with fine granular particles. In a placenta in the seventh month, although this surface of the chorion was obscured by masses of blood corpuscles, and by a granulated material of indefinite structure, which might have been fibrine, here and there patches of cells

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[*] 'Entwicklungsgeschichte,' ed. 1876, p. 337.

[†] *Op. cit.*, p. 52.

[‡] *Op. cit.*, p. 256.

could be seen, which in size and shape resembled those seen in a placenta in the ninth month. In the last month of pregnancy a definite layer of cells was seen on the placental surface of the chorion, not limited, as KÖLLIKER has stated, to the marginal lobes, but extending generally over its whole surface. These cells were polygonal in shape and in contact with each other by their edges, so that they formed a definite layer. Their nuclei were relatively large, the protoplasm granulated; and in size, shape, and appearance they closely resembled the large cells of the compact layer of the decidua placentalis. In their position and general characters they were like the sub-chorionic cells of the *Macacus*, but instead of possessing as in that animal a stratified arrangement so as to form a membrane thick enough to be seen with the naked eye, they formed, so far as my observations have gone, apparently only a single layer of cells.

There are difficulties in the way of giving a satisfactory account of the mode of origin of the sub-chorionic cells in the human placenta. Their position and characters point to one or other of three sources—modified chorionic epithelium, intra-chorionic cells, or from the decidua serotina. I do not think it likely that they can be modified chorionic epithelium, for they not only differ so much from the cells of that epithelium in appearance and size, but, from LANGHANS's observations and my own, there is every reason to believe that the chorionic epithelium degenerates and disappears in the human placenta about the middle of pregnancy.

Many facts and arguments are advanced by LANGHANS in favour of their origin in the human placenta, where alone he has studied them, from the vascular layer of the chorion, and I must refer to his memoir* for a full discussion of the subject. Here, however, I may state that before his memoir was published I had observed in *Macacus cynomolgus*, in addition to and quite distinct from the sub-chorionic cells, rows of intra-chorionic cells having a granulated protoplasm, such as one finds in the cells of the decidua, which had obviously originated within the chorion (p. 532). These intra-chorionic cells, however, did not form a continuous layer, but were broken up into groups, separated from each other by fibrillated connective tissue, and situated near the amniotic surface.

The sub-chorionic cells in *Macacus* had a different arrangement, not only from the intra-chorionic cells, but from the sub-chorionic cells as described in the human placenta by LANGHANS, or as seen by myself. They were not broken up into groups or patches, but possessed a continuous stratified arrangement over the whole placental surface of the chorion, and invested the stems of the villi so that the fibrillar structure of the chorion was excluded from forming the boundary of the intra-placental maternal blood spaces. The fact that between the deeper layers of these cells the proper tissue of the chorion seemed to penetrate, points to the origin of these deeper layers at least from the chorion, and to their homology with the intra-chorionic cells. The great resemblance between the more superficial layers of these cells and the cells

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[*] 'Archiv für Anat. und Phys.,' 1877, p. 256.

of the decidua was so very striking, that when I first examined them I was of opinion that they were decidua cells covering the placental surface of the chorion, investing the stems of the villi and forming a decidua sub-chorialis, which walled in with maternal tissue the maternal blood space at its chorionic boundary. If this view be accepted then the more superficial of the sub-chorionic cells would have a different origin from those of the deeper layers. Though several arguments might be advanced in favour of such a conclusion, yet as through want of material I have only had the opportunity of studying these sub-chorionic cells in one particular stage of development of the placenta in *Macacus*, I do not definitely commit myself to the theory of the decidual origin of the superficial layers of cells. The more so as the corresponding though not precisely similar arrangement of cells in the human placenta does not apparently exist in the earlier half of placental development, which one would be disposed to say it ought to have done if it had been derived from the decidua.

But whilst there is a difficulty in definitely assigning a decidual origin to the sub-chorionic cells, and to the cells enveloping the stems of the villi as they arise from the chorion, there can be no doubt that the stratified arrangement of cells surrounding the villi at their attachment to the hillock-like elevations of the decidua are derived from the decidua, for they can be traced in direct continuity with it.*

The layer of cells enveloping the lateral and terminal buds and the smaller branches of the villi in the *Macacus* closely resembles the corresponding layer of cells on the villi of the human placenta, and consists of somewhat flattened though not squamous cells, rectangular in outline, and closely applied to each other by their margins so as to form a continuous layer. These cells were apparently first described and figured by Mr. DALRYMPLE† in the human placenta, and since that time have attracted the attention of all who have studied the minute structure of the villi. As regards their homology there is, however, a difference of opinion. By most observers they are regarded as an epithelial layer continuous with the general epithelial covering of the chorion, and belonging therefore to the fœtal part of the placenta. In opposition,

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* When this Memoir was presented to the Royal Society, I had not had the opportunity of reading Signor ERCOLANI's latest and very important memoir, 'Sull' unità del Tipo Anatomico della Placenta nei Mammiferi e nell' Umana specie' (Bologna, 1877), a copy of which he has with great courtesy presented to me. In this memoir Signor ERCOLANI gives a diagrammatic representation (Taf. v., figs. 15, 16) of the embedding in the human placenta of the end of a fœtal villus in a heap of decidual cells, similar to what I have described and figured in *Macacus* (Plate 49, figs. 10, 11). He also speaks most decidedly in support of the decidual origin of the sub-chorionic cells of the human placenta. On theoretical grounds I would also support this view of their origin, but I have hesitated in the text definitely to assign this mode of origin to these cells, as, through the want of specimens in successive stages of development, I have not been able to trace their mode of origin.

† Medico-Chirurgical Transactions, 1842, vol. xxv., p. 21. Mr. DALRYMPLE speaks of them as nucleated cells, resembling an irregular epithelium. E. H. WEBER had previously figured the villi as possessing a pellucid margin, but had not recognised that this pellucid border was cellular. See his drawing in WAGNER's 'Icones Physiologicæ,' and the copies of the figure in R. WAGNER's 'Physiology,' translated by Dr. WILLIS, 1842, fig. cxix.; and Dr. BALY's translation of MÜLLER's 'Physiology,' fig. 231.

however, to this view, it may be stated that the chorionic epithelium apparently degenerates and disappears in the later months of pregnancy, whilst this layer of cells persists throughout the whole period of gestation. The late Professor GOODSIR, in his memoir "On the Structure of the Human Placenta,"* described them as derived from the decidua, and as belonging therefore to the maternal part of the placenta. The observations made from time to time, and quite independently of each other, by Signor ERCOLANI and by myself on the minute structure of the placenta in numerous mammals, have proved that there is interposed between the maternal blood and the fœtal villi a layer of cells derived from the uterine mucous membrane, and belonging therefore to the maternal placenta. The cellular covering of the villi in the human placenta and in that of the *Macacus* has a similar relation on the one hand to the maternal blood, and on the other to the tissue of the villus and to the capillary terminations of the umbilical vessels, which the epithelial cells of the maternal part of the placenta in the diffused, cotyledonary, and zonary forms of placenta have to the maternal and fœtal blood vessels. On this ground, therefore, they might be regarded as homologous with each other.

In order, however, to prove the derivation of the cellular covering of the human villi from the decidua, it would be necessary to trace a prolongation of the decidua around them. In the more advanced of the two early human gravid uteri I have already described (p. 546), I found that the villi were adherent both to the decidua serotina and to the inner surface of the decidua reflexa, from both of which delicate prolongations passed between the villi, so that the fixing and interblending of the two structures seemed to be due to a coincident growth of the villi and of the decidua. It is probable, therefore, that at this early stage of placental development the villi may become ensheathed by the cell structures of the decidua. In the placenta of the fifth month, as I have described at p. 550, the interlocking of the fœtal villi with trabecular prolongations of the decidua was distinctly recognised. Even in the fully-formed placenta the attachment of the terminal parts of the villi to the placental surface of the decidua shows how intimate is the relation of the one structure to the other. I have no observations to offer on the early stages of development of the placenta in *Macacus*, but from the similarity in appearance and arrangement of the cellular covering of the villi to that in the human placenta there can be no doubt that it has a similar origin. Should the more superficial layers of the sub-chorionic cells be derived from the decidua, then an additional argument may be urged in support of the decidual origin of the cellular covering of the villi.

Both in the *Macacus* and in the human female the stems of the villi arose from the placental surface of the chorion, separated from each other by intervals which in the *Macacus* were from one-tenth to two-tenths of an inch, and in the human placenta from two-tenths to three-tenths. In the *Macacus*, the stems, though giving off nume-

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* 'Anatomical and Pathological Observations,' Edinburgh, 1845. 'Anatomical Memoirs,' Edinburgh, 1868.

rous branches, yet diminished but slightly in thickness in their passage to the decidua; but in the human placenta they divided much more completely into branches, and these branches, with their finer offshoots, reached the decidua. In both, the villi were adherent to the decidua, and at the points of adhesion were imbedded in that structure. In the *Macacus*, however, the adhesion seemed to be firmer than in the human placenta, and the decidual cells were prolonged for a greater distance over the villi. The villi are, therefore, in both placentæ attached both at their chorionic and decidual ends, so that numerous checks are interposed to the too great separation of the chorion and the placental decidua from each other, such as might arise if the blood were poured too rapidly into the intra-placental maternal blood spaces, and the villi were floating free. In the human placenta, where on account of its greater magnitude an additional check may be needed, it is supplied by the passage of the inter-cotyledonary dissepiments through the placenta to the chorion.

The relatively large interval between the stems of the villi gives room for the ramifications of the numerous branches of the villi with their lateral and terminal bud-like offshoots. In both the human placenta and that of the *Macacus* the buds closely resembled each other both in form and size, and, in both, the capillary terminations of the umbilical arteries had a similar arrangement in loops and coils within the buds. In both, an extra-villous network of capillaries was situated in the placental part of the chorion in the intervals between the bases of origin of the stems of the villi. This extra-villous network corresponded in its position to the extra-villous capillary network, which I have elsewhere described, in the diffused and cotyledonary forms of placenta.* In neither, however, did I observe an extension of the capillary network into the non-placental parts of the chorion such as occurs in the zonary placenta; but the non-placental part of the chorion of *Macacus* was traversed by the branches of the umbilical arteries and vein, which passed between the placental lobes.

Both in the human placenta and in that of the *Macacus* an intra-placental system of spaces for the circulation of the maternal blood was present. These spaces formed an irregular but freely communicating anastomosis interposed between the villi. They were bounded on the chorionic aspect by the sub-chorionic cells; on the decidual aspect by the decidua placentalis; whilst the spaces between the villi were bounded by the villi themselves with their cellular covering. In the human placenta the curling arteries pass in so tortuous a manner through the placental decidua that they are followed with difficulty. If they previously be injected from the uterine arteries, in specimens where the placenta is attached to the wall of the uterus, they can, with the aid of a lens, be followed through the decidua. The delicate coat of the artery can be slit up with a needle, and the plug of injection can be followed into the interior of the placenta. Another way of observing their mode of opening into the placenta is to cut off the portion of decidua placentalis in which a curling artery is situated, to remove the villi attached to its placental aspect, and then to examine

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* Lectures on the Comparative Anatomy of the Placenta, already cited.

that aspect with a good lens, when the mouth of the artery can be seen to open obliquely on the inner surface of the decidua placentalis. In the *Macacus*, owing to the placental arteries being so much smaller in size, and not exhibiting a characteristic curling appearance, it was difficult to distinguish them from the arteries of supply of the decidua placentalis, and their exact mode of communication with the interior of the placenta was not satisfactorily ascertained.*

In the human placenta, the utero-placental veins form sinus-like dilatations, not only in the serotina occupying the placental area, but in the inter-cotyledonary dissepiments and at the margin of the placenta. These sinuses communicate with the interior of the placenta by cribriform apertures in their walls, through which it is not uncommon to see the placental villi project into the sinus. In the *Macacus*, the corresponding veins are not dilated into sinuses, but preserve, as already pointed out, their cylindrical tubular form.

Both the utero-placental veins and the curling arteries of the human placenta possess a smooth inner surface, due to the presence of an endothelial lining. From the freedom of their communication with the intra-placental maternal blood spaces one would expect that a prolongation of this endothelial lining for a greater or less distance into the placenta would take place. Although such a prolongation is denied by many observers, yet I have occasionally seen an appearance of a lamella on the villi distinct from and external to their epithelial covering, which layer I have interpreted as derived from and representing the endothelial wall of the maternal vessels,† though without committing myself to the statement that it was universally present. LANGHANS has since given a description of an endothelial-like membrane on the placental surface of the decidua serotina in placentæ in the fourth and seventh months, though he cannot speak with the same confidence of its existence in the placenta at the full time. LEOPOLD, though unable to detect an endothelial covering on the villi, has seen it on the placental surface of the decidua between the place of attachment of the villi to that structure. From the free communication of the curling arteries and utero-placental veins with the interior of the placenta, and from the prolongation of their endothelial lining for some distance at least into the organ as a wall for the intra-placental blood spaces, there can be no doubt that these spaces represent maternal blood vessels. They are, I believe, greatly dilated blood capillaries, the endothelial wall of which is in part preserved, though to a large extent it apparently disappears, so that the villi with their cellular covering lie in the maternal blood spaces, which have assumed a cavernous character. That the maternal blood capillaries, even at a very early period in the development of the decidua placentalis, become dilated into sinuses I have already pointed out (p. 548) in my description of the decidua serotina in human uteri from the third to the fifth week of gestation.

I am unable to state if the endothelial lining of the uterine vessels in *M. cynomolgus*

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* See the Appendix for a more explicit statement on this point.

† "On the Placentation of the Sloths," Trans. Roy. Soc. Edinburgh, vol. xxxii., p. 99, 1873.

is prolonged into the interior of the placenta, as the injection of the organ interfered with the use of the nitrate of silver reaction. I could not in this specimen obtain any evidence of a layer on the villi external to their cellular covering, but I may refer to a previous observation* on the structure of the villi in the placenta of the *M. nemestrinus* in the Oxford Museum, in which a layer of apparently flattened cells seemed to be situated external to the proper cellular covering of the villus. But whether an endothelial prolongation of the lining of the maternal vessels be present or not within the placenta, there can I think be no doubt that the inter-villous spaces in *Macacus* are of the same nature as the corresponding spaces in the human placenta. Should, as is most probable, the cellular covering of the villi be derived from the decidua, then in the human placenta and in the *Macacus*, as in the other placental mammals, a layer of cells, derived from the epithelium of the uterine mucous membrane, would be interposed between the maternal blood and the capillary terminations of the fœtal vessels.

The stage of development of the placenta in the *Macacus* was too advanced to enable me to determine if the villi of the chorion had had any relation to the utricular glands.

The comparison that I have now made between the gravid uterus and placenta in *Macacus cynomolgus* and the human female proves that they correspond in the form of the uterus and in the arrangement of the fœtal membranes, and that they both possess a discoid placenta, which in the *Macacus* is divided into two lobes, but is undivided in the human placenta. In the arrangement and relative position of the constituent parts of the placenta they also correspond, and although some differences of detail in the characters of some of the structures occur, yet in the main features of construction, makroscopic as well as microscopic, they have a close resemblance to each other. I have little doubt, if a detailed examination of the placenta in the other genera of Apes were made, that a similar resemblance in structure would be found.

EXPLANATION OF PLATES.

PLATE 48.

Fig. 1. A view of the interior of the gravid uterus of *Macacus cynomolgus*, obtained by making a longitudinal incision through the anterior wall. The fœtus is shown undisturbed in its position.

v. Vagina.

c. Cervix uteri. About one-third less than nature. (Page 527.)

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* Cited in the Memoir "On the Placentation of the Sloths," p. 97.

Fig. 2. Vertical section through the non-placental part of the chorion, the decidua vera, and the muscular wall of the same uterus, the section being made near the os internum. × 300.

am. The cells of the amnion, partly shown in profile and partly with their flat surfaces.

ch. The chorion. Two ridge-like elevations of which may be seen fitting into furrows of the decidua vera.

ce. Columnar epithelium covering the decidua vera.

se. The sub-epithelial cell-tissue of the decidua, the darkly-shaded spots in which represent sections through divided blood-vessels.

s. A space in the sub-epithelial tissue.

mm. Layer of muscularis mucosæ.

sm. Submucous areolar coat.

m. Muscular coat of the uterus, divided into two layers and exhibiting sections through divided blood vessels. (Page 530.)

Fig. 3. Vertical section through the decidua vera of the human uterus at the fifth month. × 20.

e. The compact cellular layer which represents in its position the epithelium ; in it are the depressions of tubular glands.

se. The more superficial part of the spongy layer immediately subjacent to the compact cellular layer.

sp. The deeper and more characteristically spongy part of the spongy layer. (Page 543.)

Fig. 4. Vertical section through a portion of the same human decidua vera. × 300.

e. The stratified cells of the compact layer, representing in their superficial position the epithelium.

se. The subjacent spongy layer in which are situated rows of fusiform cells, which have a general resemblance to the cells of non-striped muscle.

s. A space in this tissue. (Page 544.)

PLATE 49.

Fig. 5. A vertical section through the uterine wall and placenta of *Macacus cynomolgus.* Natural size.

ch. The placental chorion and umbilical cord.

u. The muscular wall of the uterus.

s. The spongy layer of the decidua serotina.

c. The compact layer of the decidua serotina. From the placental surface of the chorion the stems of the villi may be seen to pass through the placenta to reach the compact layer of the decidua. The blue colour is intended to indicate the blue injection in the blood vessels of the chorionic villi. (Page 532.)

Fig. 6. A portion of the chorion, with the stems of two of the villi arising from it dissected out of the placenta of *Macacus cynomolgus*. The uniformity in size of the stems throughout their length, and the numerous branches to which they give origin are represented. The blue colour is intended to show the blood vessels magnified. (Page 532.)

Fig. 7. Vertical section through the placental chorion and a small part of the stem of a villus from the same placenta. × 20.

am. The layer of amnion.
ch. The chorion containing divided blood vessels.
sch. The layer of sub-chorionic cells prolonged on the stem of the villus. (Page 533.)

Fig. 8. Vertical section through a portion of the chorion of the same placenta. × 350.

am. Amnion.
ch. Chorion.
ich. Intra-chorionic cells.
sch. Sub-chorionic cells. (Page 532.)

Fig. 9. A portion of one of the septa between the areolæ in the spongy layer of the decidua serotina of *Macacus cynomolgus*. × 350.

e. The epithelial cells with their large nuclei. At *sc* the epithelium has been shed and the subjacent fusiform corpuscles and lymphoid cells of the sub-epithelial tissue are exposed. The darkly-shaded tube is a blood capillary situated in the sub-epithelial tissue.

Fig. 10. Vertical section through the uterine wall and decidua serotina of the same animal. × 65.

m. Muscular coat containing divided blood vessels.
s. Decidua serotina containing areolæ.
aa. Hillock-like elevations of the serotina projecting into the interior of the placenta.
v. A villus with its contained blood vessel, injected blue, penetrating into the interior of a large hillock of decidua. (Page 534.)

Fig. 11. A section through the compact layer of the decidua serotina and a view of one of the hillock-like prolongations of the same placenta. × 350.

s. Compact layer of decidua serotina.
a. A hillock-like prolongation. The cellular structure of both is shown, and some isolated cells are represented detached from the serotina. The pale blue colour, *v*, shows the injection in the vessels of the stem of a villus embedded in and surrounded by the decidual cells of the hillock, and from it some idea may be formed of the extent

to which the villus penetrates into the hillock. The darker blue colour is in the capillaries of terminal villi adherent to the surface of the decidual hillock, and apparently continuous with the villus embedded in the hillock. The layer of cells, *cc.*, which envelopes the smaller terminal villi, is shown.

Of the drawings from nature from which these figures are taken, fig. 9 is by myself; for fig. 5 I am indebted to Mr. J. D. DUNLOP; fig. 6 to ALFRED H. YOUNG, M.B.; fig. 1 to Dr. J. HALLIDAY SCOTT; and the remainder to JOHN HAYCRAFT, M.B.

APPENDIX.

(November 30, 1878.)

During the month of October I received, through the courtesy of Professor A. H. GARROD, the placentæ of two other monkeys, the examination of which has enabled me to clear up one or two points which I was obliged to leave in doubt in my memoir, and to add some additional particulars.

The one placenta was obtained on October 4th from a *Cercocebus fuliginosus*, which had been impregnated by a *Macacus cynomolgus* living with it in the same cage. The *Cercocebus* had been well advanced in pregnancy, and had met with her death by falling from the topmost bar in one of the lofty cages in the gardens of the Zoological Society of London. The other placenta was shed during parturition on October 14th, along with a living fœtus, by a *Cynocephalus mormon*, which had been impregnated by the same *Macacus cynomolgus*.

The placenta of the Sooty Mangeby was contained in the cavity of the uterus, but the fœtus had been removed before it came into my possession, and a note had not been taken of its position in the uterus. The placenta consisted of two lobes, unequal in size. The larger, 3 inches in diameter and almost circular, was adherent to the posterior wall of the uterus; the smaller, 2½ inches in one diameter by 2 inches in another, had been adherent to the anterior wall, but along with the surrounding chorion had become detached from it. The two lobes were separated from each other by a broad band of intermediate chorion, in which the vessels ramified that passed from one lobe to another. The cord was attached to the centre of the larger lobe. From the lower edge of this lobe to the region of the os uteri the chorion was closely adherent to the inner surface of the uterus.

The placenta of the female *Cynocephalus* was single, almost circular in form, and measured 3½ inches in diameter. All the membranes had been preserved, and no trace of a smaller subordinate lobe was to be seen. The cord joined the placenta near its middle. The presence of only a single placental lobe in *Cynocephalus mormon* is of interest in connexion with an observation made by M. BRESCHET on the placenta in *Cynocephalus sphynx.*[*] M. BRESCHET states that in his specimen the placenta appeared to be single, but, influenced apparently by the idea that the placenta was double in the Monkeys of the old world, and single only in those of the new world, he thought that from the torn condition of the membranes a smaller subordinate lobe had been destroyed by the mother. It is probable, however, that in M. BRESCHET'S specimen, as was certainly so in my own, only a single placenta was present. Hence it would appear that in the Baboons, as in the Chimpanzee, the placenta consists of only a single lobe, so that the opinion which was at one time entertained that the Apes of the old world could be distinguished from those of the new by the invariable presence of a bi-lobed and not a single-lobed placenta is not borne out by more extended inquiry.

Both in the placenta of *Cercocebus* and *Cynocephalus,* as in the previously described *Macacus cynomolgus,* the placenta was divided by furrows into lobelets. The uterine surface of the shed placenta of *Cynocephalus* was covered by a well-defined layer of decidua, which was ragged on its uterine surface, owing, doubtless, to the tearing through of the thin walls between the loculi, similar to those I have described in *Macacus.* Several utero-placental veins, which had been torn across, lay in grooves in the placental decidua, usually in the furrows between the lobelets and opened by obliquely directed mouths into the interior of the placenta, the villi within which could be seen through the venous orifices. Those villi which were placed next the edge of the mouth of the vessel were adherent to its wall, so that a provision existed which would admit of a prolongation of the endothelial wall of the vessel on to the villus. When the placenta had been steeped in spirit, a quantity of the fluid had soaked into its interior, and could by gentle pressure be squeezed out through the mouths of the utero-placental veins, which proved that a free communication existed between these vessels and the interior of the placenta. The torn veins closely resembled those figured by JOHN HUNTER on the uterine surface of the shed placenta of his *Macacus rhesus.*

But in addition to the vessels that I have described as veins, I saw other vessels attached to the uterine surface of the placenta of *Cynocephalus,* which I believe to be the utero-placental arteries. They were smaller in size than the veins, but yet sufficiently large to enable me to pass a pig's bristle into the lumen. One in particular was traced for about an inch, extending with a slightly wavy course in close contact with the uterine surface of the organ. Another could be traced for half an inch covered over by a thin pellicle of decidua to open into the placenta about the middle of a lobelet.

The detached lobe of the placenta of the Sooty Mangeby had also its uterine surface covered by decidua, and a thick layer of decidua, showing numerous loculi, was left on the placental area of the uterus, and in both the torn-across mouths of utero-

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[*] 'Mémoires de l'Institut,' 1845, p. 452.

placental veins could be seen. It is clear, therefore, from this specimen, as well as from the appearance presented by the placenta of the Baboon, that, when the placenta is separated, a portion of the decidua remains attached to the wall of the uterus, and that another portion peels off with the placenta ; hence, to use the terms employed by Dr. ROLLESTON, there is both a deciduous and a non-deciduous serotina.

As it was desirable to make quite certain that the vessels I have described in the whole series of placentæ as utero-placental veins, were veins and not arteries, I directed my assistant, Mr. A. B. STIRLING, to pass a red injection into the uterine arteries of *Cercocebus fuliginosus* and a blue injection into the uterine veins, so as to inject the vessels belonging to the attached lobe of the placenta, and enable me readily to discriminate the one set of vessels from the other. I then found on carefully detaching the lobe from the placental area of the uterus that the vessels corresponding to those which I have throughout this Memoir described as utero-placental veins contained the blue injection, so that the accuracy of my interpretation of their venous nature was thus established. But entering this same placenta were several vessels, which contained a red injection, and may therefore be fairly regarded as utero-placental arteries. Though smaller than the utero-placental veins, they were yet large enough to admit a pig's bristle, and when slit open were seen to have a distinct orifice into the placenta, through which the fœtal villi could be seen. Hence it is probable that the fine arteries, which I saw in the decidua serotina of *Macacus cynomolgus* (p. 536), were not for the placenta, but for the supply of the decidual tissue.

By means, therefore, of these two placentæ I have been able to establish much more definitely than in the placenta of *Macacus cynomolgus* the distinction between the utero-placental arteries and veins, and to show that each kind of vessel has a free communication with the interior of the placenta, so as to provide for a free circulation of blood throughout the interior of the organ.

In tearing away the non-placental chorion of *Cercocebus* from the wall of the uterus, I observed a number of very tortuous arteries, filled with red injection, passing from the uterine wall to that layer of the decidua which remained attached to the chorion. These vessels were so slender as to appear to the naked eye not larger than fine threads.

The outer surface of the chorion of the shed placenta of *Cynocephalus* was irregularly covered by a thin flocculent membrane, which was, as a rule, closely adherent to the chorion, though in places it was flocculent and partially separated from it. In its structure it consisted for the most part of white fibrous connective tissue, in which many fusiform corpuscles were situated. Occasional patches of polygonal cells, set together after the manner of an epithelium, were also seen. This membrane obviously represented the decidue reflexa of previous writers on this subject ; but for the reasons I have already advanced (p. 545), it is difficult to say how far the membrane was a decidua vera or a decidua reflexa.

XVII. *Observations on the Nervous System of* Aurelia aurita.

By EDWARD ALBERT SCHÄFER, *Assistant Professor of Physiology in University College, London.*

Communicated by W. SHARPEY, *M.D., LL.D., F.R.S.*

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[PLATES 50–51.]

LAST August I undertook, at the request of my friend Mr. G. J. ROMANES, an investigation with the view of proving the presence or absence of histologically differentiated nervous structures in the Medusæ. Mr. ROMANES' experiments* have shown the existence of a central nervous apparatus in the marginal bodies of these animals, and probably also of nervous tracts (lines of discharge) over the lower surface of the nectocalyx or umbrella. But up to the present time the anatomical proof of the existence of a nervous system in this class has rested chiefly upon the authority of HAECKEL, who has described† in two genera of the craspedote Medusæ a ring of nerve-fibres lying on the inner side of the marginal canal, and provided with a ganglionic enlargement at the base of each lithocyst. From each of these ganglia four nerves are described as passing—one to the polypite, and the others to the adjacent tentacles and lithocyst.

My observations have been chiefly confined to *Aurelia aurita*, partly because, at the time of year chosen, this species furnished the most abundant material, partly on account of its having been the subject of the greater number of Mr. ROMANES' experiments, and also because the nervous structures can be more readily made out in this species than in any other that could be readily got. But preparations of a species of *Chrysaora* make it abundantly evident that ganglion cells somewhat similar to those immediately to be described as underlying the muscular sheet in *Aurelia* are found also in this genus, and it is probable that what is true of these two genera will apply with but little modification to other genera of acraspedote Medusæ. Whether a similar distribution of nerves exists also in the Craspedota, is a matter which must for the present remain undecided; my own observations, which were confined to species of *Thaumantias* and *Tiaropsis*, have so far yielded a negative result.‡

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* Phil. Trans., 1876, pp. 269, et seq.; and Phil. Trans., 1877.

† Beiträge zur Naturgeschichte der Hydromedusen.

‡ See postscript.

The nervous system of *Aurelia* consists (1) of the *marginal bodies* or *lithocysts*, the discharging function of which has been clearly demonstrated by Mr. ROMANES; (2) of certain tracts of peculiarly modified epithelium in their vicinity, which we may term the *nerve-epithelium;* (3) of an interlacement of nerve-fibres, which covers the whole of the under surface of the muscular sheet, lying between the muscular fibres and the ectoderm-cells, and partly amongst the latter, and which may be termed the *subumbrellar plexus.*

Subumbrellar Nervous Plexus.

The plexus can be distinctly seen even in the fresh tissue if care be taken to bring the subumbrella flat and uninjured under microscopic observation, and the fibres then have very much the appearance of the sympathetic fibres, or fibres of Remak, of the Vertebrata, and, like these, show an indistinct longitudinal striation. Here and there oval or fusiform swellings occur in the course of the fibres, and in the larger of the swellings a vesicular nucleus, and a distinct bright nucleolus may be detected. These appearances are so obvious as to allow of no question that we have before us undoubted nerve-fibres, and bipolar ganglion-cells. The tissue which they underlie being just as clearly muscular, with well-characterised cross-striæ, it is interesting to observe, even so low down in the metazoic scale as the Medusæ, that the textures, which in the higher animals are generally looked upon as the most highly differentiated, should have already attained a degree of structural complexity and of functional activity in many respects scarcely inferior to the nervous and muscular tissues of Vertebrates.

The subumbrellar nerves. Structure of the nerve-fibres.—The reagents which are ordinarily employed for the demonstration of nervous tissues, and especially the chloride of gold, bring the structures in question to view in the most striking manner possible. As in the higher animals, the nerve-fibres and the substance of the ganglion-cells become stained of a deep violet colour by this reagent, so that the fibres may be followed with ease over large tracts of the surface (Plate 50, figs. 1–7; Plate 51, figs. 11–16). Osmic acid preparations lack this distinctiveness of colouration, so that the nerves are scarcely better exhibited *in situ* than in the fresh preparation. But whereas after treatment by the chloride of gold method the fibres appear markedly smaller than in the fresh tissue, and seem to have become somewhat shrunken, in the preparations made with osmic acid they preserve their original size and for the most part their pristine appearance (Plate 50, fig. 8). If the nerve-fibres which have been stained with chloride of gold be attentively observed, it may be seen that they are surrounded by a clear space which separates them from the tissue in which they lie. This space may have been produced by the expression of fluid from the fibre during its supposed shrinkage, or on the other hand it may represent a previously existing homogeneous sheath, surrounding an axis-cylinder; and, if this were the case, certain small nuclei, which are occasionally seen adhering to the fibres (figs. 13 and 14, *n.*), might be regarded as belonging to such a sheath. The fact of the distinction between fibre and sheath not being visible in the unstained preparation would be accounted for, if we suppose them to possess much the same index of refraction.

Course and distribution of the nerve-fibres.—The nerve-fibres are not uniformly distributed over the whole subumbrella. They are met with in least abundance near the margin, but are not wholly absent even here, and I have sometimes thought I could trace a delicate fibre into a tentacle. But I have not been able to detect anything of the nature of a marginal ring of nerve-fibres like that described by HAECKEL in the craspedote forms.

At certain parts the fibres come together to form wonderfully intricate interlacements (fig. 7), whilst at other places only a few fibres may be seen crossing the field of the microscope (fig. 3). Neither individually nor collectively do the fibres appear to have any special relation to the nutritive tubes. Nor are they especially numerous in the neighbourhood of the lithocysts, although a certain number of nerve-fibres may generally be traced converging towards each of these structures (fig. 4). I have never been able to follow them directly, either into the marginal bodies or into the nerve-epithelium near; at the same time it is probable that some of these converging nerves, if they do not enter or issue from the lithocyst itself, at least come into relation with the thick nerve-epithelium which is found near the base of that organ. The difficulties of observation are here very considerable, owing partly to the fineness of the converging fibres, and partly to the fact that the tissues near the margin, and especially near the lithocysts, are apt, more than elsewhere, to become uniformly darkly stained by the reagent employed.

At the central part of the umbrella some of the fibres may be observed to sweep round the margins of the four genital sacs, and to pass between these towards the polypite, but it has not been possible to trace them actually into the latter.

The general direction of the fibres over the whole subumbrella is radial, but this direction is greatly obscured by bendings and intercrossings. Moreover, a few nerves pass in the direction of the muscular fibres, and therefore parallel with the circumference of the swimming-bell.

If we trace out the course of the individual nerve-fibres more closely (as has been done with the fibre marked *xx.* in figs. 11–16), we are struck with certain remarkable facts. In the first place, each fibre is entirely distinct from, and nowhere structurally continuous with, any other fibre. Secondly, each fibre is provided with a bipolar nerve-cell (fig. 13), which is interpolated in or near the centre of the fibre, each end of the fibre representing the prolongation of one of the poles of the nerve-cell. Thirdly, each nerve-fibre is of limited length (seldom exceeding 4 millims. from end to end), and in most cases tapers at either extremity to a gradual termination. Lastly, it may be mentioned that the fibres are rarely branched; and where they are so (as in fig. 12) the branches do not join with other nerve-fibres, but after a longer or shorter course end in a tapering extremity like the unbranched fibres.

It seemed at first sight almost incredible that with such a prodigious number of nerve-fibres, exhibiting so close an interlacement, there should be no actual junctions of the intercrossing nerves. And it was especially difficult of credence because some of the experiments of Mr. ROMANES, performed with the view of testing the amount of section which the tissue could endure without loss of nervous (or excitational) continuity, seemed to point to the existence of a structurally continuous network of nerve-fibres. Nevertheless, there can be no doubt that the fibres do not come into anatomical continuity. On the other hand, it can readily be seen that each nerve-fibre comes at one or more points of its course into very close relations with other nerve-fibres. Two fibres, for example, may sometimes be observed to bend towards each other out of their previous course, in order to run closely side by side for a greater or less distance, and in such cases one fibre may hook round the other (fig. 6), or they may even be two or three times intertwined. At other places a number of fibres come together from different parts and join in a very close entanglement, the fibres in which run for the most part parallel (figs. 7 and 15), and it is only with difficulty that the individual fibres can be followed. So that although there is no actual anatomical continuity, abundant opportunity is afforded for inductive action, whether electrical or of some other kind. That physiological continuity is thus maintained it seems as yet premature to conjecture.

Mode of termination of the nerve-fibres.—Most of the fibres end, as already mentioned, in a gradually tapering extremity, which is sometimes bifid (fig. 16, x^{vi}.). This, so far as could be seen, does not actually enter a muscular fibre, although since there is no sarcolemma it comes in close contact with the muscular substance. In some instances the nerve-ending takes the form of a fusiform dilatation (fig. 11, *x.*), which may contain a small nucleus; in other cases the dilatation is more marked, and may even form a triangular flattened expansion within which a number of nuclei can be detected (fig. 8, *p.*). These expansions of the end of the nerve appear to represent a primitive form of the motorial end plate.

The nerve-cells.—The cells vary in size, but for the most part are roundly fusiform bodies, averaging ·05 millim. in length, and ·015 millim. in diameter. As stained with gold they are of characteristic appearance, and present a marked resemblance to the bipolar cells which are met with in the spinal ganglia of fish, or to those which are found interpolated in the course of the nerves distributed to the ciliary muscle in the human eye. The cell-substance is granular in the gold preparation, but extraordinarily clear and pellucid when examined fresh; in this respect resembling the ganglion cells of the human retina. An appearance of striation radiating from the poles into the substance of the cell is sometimes faintly visible. The nucleus, which is not coloured by the gold salt, is either spherical or ovoid: it is clear and vesicular, and contains a very characteristic nucleolus. The nucleus is generally situate nearer to one of the extremities of the cell, rarely in the centre. The cell itself is not always evenly fusiform, but is often bulged out at one end; it passes at either pole sometimes abruptly, but more often gradually into the nerve-fibre.

Cells are frequently met with, especially in young specimens, which closely resemble the nerve-cells just described, with the exception that the nerve-fibre processes are of much more limited length, or even, in some cases, altogether absent. They are, no doubt, ectoderm cells, in process of development into nerve-cells and nerve-fibres: they are usually more abundant in the neighbourhood of the polypite than elsewhere (fig. 3). The arms of the polypite itself generally show a number of cells of this description, but they are entirely destitute of nerve-fibre prolongations.

Function of the subumbrellar nerves.—Each nerve-fibre may be regarded as serving to connect a part of the muscular sheet nearer the circumference with other parts nearer the centre of the swimming-bell, and thus to bring the contraction of different zones of muscle more nearly in correspondence. Further, as a result of the interlacements which occur, and the closely parallel course which the fibres take in them, it is reasonable to conclude that nervous impulses are transmitted by some means or another from fibre to fibre. If so, the result would be the same as if an actual net-work of nerves existed, viz., the production of a general co-ordination in the contractions—not absolute, it is true, but often nearly so. Lastly, since the nerve-cells are brought into close relation with the exterior of the body through the medium of the ectodermic epithelial cells, amongst which they lie, any stimulus affecting these is communicated by means of the nerve-fibre processes to the parts of the muscular sheet to which these fibres themselves are distributed, and, if there is any means of transmission from fibre to fibre, it will rapidly spread over the general subumbrellar plexus to the whole of the contractile tissue.

The Lithocysts and Tracts of Nerve-epithelium.

The lithocysts of *Aurelia* are eight thumb-shaped projections (fig. 17) at the circumference of the umbrella, each being situate in one of the marginal bodies and having a horizontal direction with a slight upward curvature. They are covered above by a cushion-like prominence (fig. 17, *c.*) of the upper edge of the umbrella, and are further hidden by a tongue-shaped fold on either side, into which a cœcal protrusion of the marginal canal extends.*

At the base of the cushion above referred to, there is seen a deep pit which extends down towards the attached end of the lithocyst. This pit it will be convenient to distinguish by the name of *fovea nervosa superior* (figs. 17, 18, *f.n.s.*). It is lined by a

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* These diverticula of the nutritive canal, after passing a certain distance laterally into the substance of the fold, contract abruptly and the narrowed portion then passes a variable distance towards the end of the fold. The larger portions of the diverticula often present, on one or both sides, a solid cellular axis, projecting into the lumen of the canal, and formed apparently by an infolding of the entodermal wall, but its purpose and the precise mode of its formation I have been unable to elucidate.

layer of thickened ectoderm, the ciliated columnar cells of which (fig. 10, A, B) are very narrow and elongated, each having a clear oval nucleus, which causes the portion of the cell where it is situated to be bulged out. For convenience of adaptation, therefore, the nuclei of adjoining cells are on different levels, and this gives an appearance as if there were several layers of nuclei (fig. 18). The upper or free end of the cell is peculiar in presenting a distinct, highly refractive thickened border (fig. 10, A, *b.*), convex outwardly, and from the top of the convexity a single long cilium springs (fig. 10, A, *c.*). The attached ends of the cells rest upon a fibrous stratum, often granular looking in section, which in fact seems to be formed by the interlacement and union of fine fibres prolonged from the branching fixed ends of the columnar cells (fig. 10, A, *f.*). These fibres have a certain resemblance to delicate nerve-fibres; they pass from the cells, at first perpendicularly through the stratum in question (fig. 18), but they then turn abruptly down towards the bottom of the fovea, passing beneath and between the branching processes of the other cells, under which they aid in forming in like manner a fibrous stratum. Towards the upper limit of the fovea the fibrous stratum becomes thinner and at last altogether ceases, and the elongated cells pass by a gradual transition into the small epithelium cells which cover the upper surface of the umbrella.

Immediately beneath and on the oral side of the attached end of the lithocyst is another deep depression of the surface. This, which may be termed *fovea nervosa inferior* (fig. 17, *f.n.i.*), is applied for the greater part of its extent to the process of the nutritive canal which passes to the lithocyst. It is lined by a thick epithelium with a nervous substratum, quite similar in appearance to that found in the superior fovea; towards the lithocyst this epithelium passes into the ectodermal covering of that structure. It is possible that the subumbrellar nerve-fibres, which were before described as converging towards the attachment of the lithocyst, end in the fibrous substratum of this inferior fovea.

The lithocyst itself consists of three parts, which may be distinguished as the *basal* (fig. 17, *b.*), the *intermediate* (fig. 17, *i.*), and the *terminal* portions (fig. 17, *t.*), the last-named projecting freely under the cushion before mentioned. They are marked off from one another by two shallow grooves. The terminal part contains the clump of calcareous crystals which give the name of lithocyst to the whole organ. The intermediate part has a deep brown colour on the up-turned surface, especially in and near the groove which marks it off from the terminal portion, the colour being situated in the ectodermic cells.*

The interior of the lithocyst is occupied throughout its whole extent by a radial prolongation from the marginal nutritive canal (fig. 17, *n.c.*), which is, of course, lined

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* The presence of calcareous crystals (otoliths ?) in the marginal bodies of the Medusæ has led to their being commonly regarded as auditory organs, and the presence of pigmented spots (ocelli ?) is supposed to indicate the existence of a dim visual perception, but there has been hitherto no experimental proof of the actual possession of these functions in the case of *Aurelia*.

by entoderm cells. This prolongation remains hollow in the basal and in the intermediate portions, but into the terminal part it is continued as a solid projection, which expands into the clump of otolithic cells. These, therefore, are continuous with, and are doubtless derived from the entoderm, for they have no connexion whatever with the ectoderm, being separated from it by a thin layer of the jelly-like tissue (mesoderm ?). This is a noteworthy fact, for in nearly all animals in which an auditory organ containing otoliths is developed, these particles are, so far as is at present known, formed in connexion with cells derived solely from the ectoderm. The lithocyst consists then (1) of a central part or core derived from the entoderm, and (2) of a covering formed of ectodermic cells, the two being separated from one another by a thin layer of the jelly-like mesoderm.*

Entodermic part or core of the lithocyst.—As already mentioned, the central canal of the lithocyst is prolonged directly from the circular nutritive canal of the margin, and it extends as far as the junction of the intermediate with the terminal portion. It is lined by a columnar ciliated epithelium, the cells of which are much longer than those found in the ordinary nutritive canals. Its lumen is partially filled with rounded cells, which towards its extremity entirely occlude it, so that the canal is prolonged into the terminal portion of the lithocyst by a somewhat narrowed, solid stalk, which widens out as the otolothic clump. Of the cells which form the stalk the external are much elongated, and are disposed in a radiating manner, whilst those in the centre are rounded or polygonal, and the more distal already have small crystals developed in their interior. The peripheral otolithic cells of the clump itself have also a radiate arrangement and an oblong appearance in longitudinal section (fig. 17), whilst the internal ones are more irregularly packed, and many of them almost regularly dodecahedral in shape. In teased (osmic) preparations their flattened sides sometimes are seen to have the bases of columnar cells applied to them, the ends of the latter tapering to fine branching processes (fig. 10, E). These (nerve fibre ?) processes seem to become lost in a network having a granular appearance in section, underlying the tapering ends of the columnar entodermal cells, like the stratum already described as existing under the ectodermal cells of the foveæ nervosæ and that immediately to be described in connexion with the ectodermic part of the lithocyst. Opposite the foveæ the fibrous stratum under the ectodermic cells comes for a certain

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* In one specimen of *Aurelia*, sections of which were prepared, there was observed a small supplementary organ (fig. 20, *l'*) a short distance from one of the ordinary lithocysts, remarkable for several reasons. In the first place, it projected from the *upper* edge of the umbrella-margin, and in place of being covered with a cushion it was free superiorly, resting below on the prolonged lower edge of the umbrella. In the second place it was of simple structure, consisting merely of a bell-shaped ectodermal prominence, enclosing a diverticulum of the marginal canal; both its ectodermal and entodermal layers (which were separated by a thin layer of jelly) being distinctly thickened. But the most interesting fact was that the entoderm at the apex of the bell was prolonged *through the ectodermic covering*, in the form of a short stalk, bearing a small bunch of otolithic cells, which could thus project, naked and without an ectodermic covering, into the surrounding medium.

distance into contact with the corresponding stratum just described as underlying the entodermic cells of the lithocystic canal (fig. 18). Moreover, indications can sometimes be obtained of fibres bridging across the mesoderm which separates the ectoderm of the fovea nervosa superior from that covering the base of the lithocyst, so that all the tracts of modified epithelium with fibrous substratum, whether ectodermic or entodermic in their origin, are probably to be regarded as forming collectively a rudimentary nervous centre.

The ectodermic covering of the lithocyst.—The cells which compose the ectodermic covering of the lithocyst form a single layer, which is continuous with the general ectodermic covering of the under surface of the umbrella. But the layer varies very greatly in thickness, in conformity with the varying length of the epithelium cells and with the greater or less development of the granular (fibrous) stratum which lies immediately beneath them. This is by far most strongly marked over the basal and intermediate portions of the lithocyst, gradually disappearing in the terminal portion, where the ectoderm is represented by a layer of small flattened cells covering the clump of entodermal otolithic cells, and separated from them by a very thin mesodermic stratum. Some of these cells contain the brown pigment before mentioned, and near the distal of the two furrows which encircle the lithocyst there are transitional forms between them and the greatly elongated pigmented cells of the intermediate portion immediately to be described.

The epithelium of the basal portion quite resembles that already described as lining the foveæ. The fixed branched extremities of the cells pass as before through the granular-looking sub-epithelial stratum, and having reached its deeper portion turn sharply round to course either peripherally or centrally, and some, it may be, also laterally, within the layer in question. How these and the other fibres of the similar layers before described terminate must for the present remain undecided. They seem gradually to shade off and disappear beneath the neighbouring epithelium cells. They are very much less defined than the fibres of the general nervous plexus of the subumbrella; moreover, they unite as before said into an irregular network, and altogether have far less decidedly the character of nerve fibres. The stratum which they form resembles in its half-granular half-fibrous appearance the neuroglia of the grey matter of the Vertebrate brain; and the tracts in question with their elongated ciliated cells and the granular substratum bring forcibly to mind the appearance presented by the central nervous system of the Vertebrate embryo at the time when differentiation into cells and fibres is just beginning. Altogether there can be very little doubt that we here meet with the first beginnings (in a phylogenetic sense) of a central nervous apparatus. It is interesting to remark that although characteristic nerve-cells are met with on the nerves of the subumbrella, yet in these specialized parts, which may be regarded as the representatives not only of a central nervous system but also of sense-organs, definite nerve-cells are altogether wanting.

The ectodermic cells of the intermediate portion of the lithocyst resemble, on the whole, the cells of the basal portion. But in those which are pigmented, and which we may suppose to be concerned with visual perception, we meet with some modifications (fig. 10, C D). The outer extremity of the cell is much enlarged, and contains the whole of the pigment, and from the middle of this wide and generally convex free end there extends a very long filament of exquisite fineness. The fixed end of the cell is prolonged from the nuclear dilatation as a straight, very delicate thread, which usually shows one or more minute varicosities. Its branches become lost in the network of cell processes which forms the subjacent fibrous stratum.*

October, 1877.

Description of the Figures.

Fig. 1. Part of the plexus of nerves covering the muscular sheet, taken about midway between the margin and the polypite. Fusiform bipolar nerve-cells are seen on some of the fibres. The direction of the muscular fibres is indicated by the slightly curved lines. Drawn under a magnifying power of about 140 diameters, but not to scale.

Fig. 2. Part of the same plexus, from over a nutrient tube. The vertical striation is intended for shading only; not, like the transverse lines, to indicate the direction of the muscular fibres.

Fig. 3. Part of the same plexus, from near the margin of one of the genital sacs. Three cells are observed amongst the nerve-fibres which somewhat resemble the nerve-cells, but are of irregular shape, and possess no nerve-fibre processes.

Fig. 4. Margin of an *Aurelia* near a lithocyst, showing the manner in which the nerve-fibres of the subumbrellar plexus converge towards the attachment of the lithocyst. The lithocyst has been removed from the place marked *l* in the figure. Less magnified than the three preceding figures.

Fig. 5. Part of the submuscular plexus of *Aurelia*, more highly magnified.

c′. A tripolar nerve-cell. These occur but rarely.

c, *c*. Bipolar nerve-cells.

Drawn under a power of about 300 diameters.

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* Since this article was written, a paper ('Ueber das Nervensystem und die Sinnesorgane der Medusen,' Jena. Zeitschr., 1877), by O. and R. Hertwig, has come to hand, which contains a preliminary notice of microscopical observations upon the nervous system and sense-organs of the Medusæ. The authors appear to have chiefly studied these structures in the craspedote forms, but they also record observations upon two or three genera of the acraspedote Medusæ. The extensively distributed nerve-fibres and bipolar cells here described as covering the muscular sheet in the latter seem to have escaped their observation, but their account of the structure of the lithocysts and the modified epithelium in their neighbourhood in other species of Medusæ, so far as can be made out from the brief preliminary statement, corresponds in the main with the observations here recorded upon the similar structures in *Aurelia*.

Fig. 6. Two nerves of the submuscular plexus bending towards and looping round one another. Drawn under a power of about 600 diameters.

Fig. 7. A very dense interlacement of nerve-fibres situate between two of the nutritive tubes. Drawn under a power of about 100 diameters.

Fig. 8. Central part and ends of a nerve-fibre from the submuscular plexus of *Aurelia* (osmic acid preparation). The fibre took a nearly straight course across the direction of the muscular fibres. The whole of the fibre is not given: it was about 3 millims. in length.

a. End of fibre nearest polypite with motorial end-plate enlargement, *p*.
b. Middle of fibre with bipolar ganglionic enlargement.
c. End of fibre nearest to margin of umbrella.

Drawn under a power of about 600 diameters.

Fig. 9. Vertical section through the ectoderm covering the under surface of the umbrella of *Chrysaora hyoscella*.

c. Ganglion-cell.
e. Layer of epithelium-cells.
m. Layer of muscular fibres.

Drawn under a power of about 400 diameters.

Fig. 10. Isolated cells of the nerve-epithelium of *Aurelia aurita*.

A. Three of the long ciliated cells in conjunction.
  b. Thickened border.
  c, Cilia.
  f. Fixed extremity branching into fine fibres.
  n. Nuclear dilatation.
B. A single cell, similar to the foregoing.
C D. Isolated cells from the pigmented patch on the intermediate portion of the lithocyst.
  h. Fine hairlet.
  n. Nuclear dilatation.
  p. Enlarged free end of the cell loaded with brown pigment.
  v. Varicosities on central end of the cell.
E. An otolithic cell with two adherent columnar cells. The ends of these, which are turned away from the otolith, are very delicate and branched.

Figs. 11–16 represent a series of views (drawn under a magnifying power of about 600 diameters, but not to scale) of various parts of the same nerve-fibre, but only about half of the whole length of the fibre is represented. Any other fibres that happened to be visible in the same field of the microscope are also delineated. The clear margin around each fibre giving an appearance resembling a sheath is distinctly seen in all. In all the series the fibre is marked *xx*. The direction of the subjacent muscular fibres is in all parallel to the upper and lower margins of the paper, and is sketched in at one or two places.

Fig. 11. End of fibre nearest to margin: near its commencement is seen a fusiform nucleated swelling.

Fig. 12. The next part of the fibre. It exhibits two lateral branches, which take the direction of the muscular fibres, and soon become lost amongst them.

Fig. 13. The central part of the fibre with its interpolated nerve-cell. A small nucleus adheres to one end of the cell, which perhaps belongs to a sheath represented by the clear surrounding of the cell.

Fig. 14. The next succeeding portion of the fibre. Here there are two such nuclei adhering to the sides of the fibre. A nerve-fibre termination, *t*, occurs in this field. Also a nerve-cell, which is only prolonged at *one* end into a nerve-fibre, *c′*.

Fig. 15. The nerve-fibre now comes into relation with a large number of other fibres; it then takes a sharp bend, nearly at right angles, and very shortly after can be traced to its termination in a bifid extremity. This is represented in fig. 16.

Fig. 17. Meridional section through the margin of the umbrella of *Aurelia aurita*, the section passing through the middle of a lithocyst (semi-diagrammatic).

b, Basal; *i*, intermediate; and *t*, terminal portion of the lithocyst.
c. Cushion covering the lithocyst.
ect. Ectodermic covering of lithocyst.
ect′. Ectodermic covering of under surface of umbrella with the cut ends of the muscular fibres, *m.f.*
ect″. Ectodermic covering of upper surface of umbrella.
f.n.i. Fovea nervosa inferior.
f.n.s. Fovea nervosa superior.
m. Jelly-like mesoderm; *m′*, layer of mesoderm between ectoderm and entoderm of lithocyst.
n.c. Nutritive canal passing to the interior of the lithocyst and continuous at its extremity through the stalk *n*, with the clump of the otolithic cells, *o.c.* None of my sections show a communication of this canal with the exterior, as commonly described (see HUXLEY's 'Invertebrates,' p. 135).
p. Pigmented patch on upper surface of intermediate portion.

Fig. 18. Vertical section (tangential to the margin of the umbrella) through the base of the lithocyst and the fovea nervosa superior. Letters as in previous figure. The fibrous stratum under the ectoderm of the fovea nervosa superior is seen to be in continuity with that under the entoderm of the nutritive canal. In the engraving of the figures this fibrous stratum has been imperfectly rendered, and in some places has been altogether omitted.

Fig. 19. A small portion of entoderm lining the nutritive canal near the fovea nervosa inferior, with two delicate processes of (mesodermic?) cells, simulating nerve-fibres, passing towards the epithelium of the canal.

Fig. 20. Vertical meridional section through the margin of an *Aurelia* near one of the lithocysts, showing the rudimentary organ described in the text (note to p. 569). The extremely dark appearance imparted to it is a mistake of the engraver.

ect. Bell-shaped projection formed of thickened ectoderm and occupied by a prolongation of the nutritive canal, *n.c.*; the thickened entoderm of which is seen to pierce its ectodermic covering and project as an otolithic clump, *o.c.*

ect′. Ectoderm of under surface.

ect″ Ectoderm of upper surface of umbrella.

m. Mesoderm.

r.n.c. Radial nutritive canal cut obliquely. This is seen to be connected to other canals by a layer of entodermic cells, *end.*, indicating an adhesion of originally separated tracts, such as are seen in *Chrysaora.*

POSTSCRIPT.

(Received October 16, 1878.)

It was not originally my intention, in submitting my observations upon the nervous system of *Aurelia aurita* to the Royal Society, to add to the length of the communication by introducing an account of the general literature of the Medusæ, and for this reason: that in looking through the various papers upon the subject to which I had access I could find no observations (beyond those of HAECKEL, referred to at the beginning) that had any direct bearing upon the immediate subject of this article. I was fully aware of the fact that fibres had been described *in the jelly-like tissue*, and, more than this, I was familiar with the aspect of these fibres; but since both from their situation and appearance there could be no idea of their nervous nature, or of their having any connexion with those described in this paper, it seemed to me unnecessary to refer specially to them. And the same with regard to previous observations upon the "marginal bodies," which had not before, so far as I knew, been submitted to the more modern methods of histology, and especially had not been examined by means of sections, which are necessary for throwing light upon the intimate structure of such objects. But my friends thought differently, and, yielding to their advice, I was preparing an account of the literature of the Medusæ to take the form of an Appendix to this article, when the appearance, in the spring of this year, of the monograph by the Brothers HERTWIG,[*] in which the literature of the whole subject is treated of in an exhaustive manner, rendered superfluous any further action on my part. As to the work of the Brothers HERTWIG it is difficult to do justice to the carefulness of the descriptions and illustrations, and the philosophical way in which the subject is treated. Unfortunately, their supply of *Aurelia* seems to have been limited, and altogether their observations upon the Acraspeda have been comparatively few, although the types studied were well selected. This will probably account for the fact that the subumbrellar plexus which I have described in *Aurelia* has escaped their notice. Moreover, the nervous character of the epithelium of the foveæ near the lithocysts is not alluded to by them, probably because in the acraspedote species which they chiefly examined the epithelium of these parts has not so distinctly the character in question as it has in *Aurelia*. In *Chrysaora hyoscella*, for example, the ectoderm of the superior foveæ is throughout like that of the rest of the upper surface of the umbrella.[†] I had hoped to have continued and extended my researches this year, and indeed spent some little time on the coast of Brittany with this object, enjoying the hospitality of M. LACAZE-DUTHIER's experimental laboratory at Roscoff. But my hopes were disappointed by the adverse weather which prevailed during my stay there, and I succeeded in obtaining but few specimens. Nevertheless I was enabled to place one fact beyond doubt—the existence, namely, of nerve-fibres in the velum of some at least of the Craspedota. On page 128 of their work, the Brothers HERTWIG speak of having only by the most careful search been able to make out the existence in two genera (*Trachynema* and *Cunina*) of fine fibrils, *perhaps of nervous nature*, taking a radial direction in the velum beneath the epithelium of the under side. In a specimen of *Thaumantias* (species ?) I observed in the living animal distinct nerve-fibres like those I had seen in the subumbrella of *Aurelia*. They passed outwards from the nervous ring, becoming gradually finer until lost near the free edge of the velum, and since they exhibited no nucleated enlargement I conclude with the Brothers HERTWIG that they spring from ganglion-cells in the ring.

October, 1878.

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[*] Das Nervensystem und die Sinnesorgane der Medusen. Monographisch dargestellt von OSCAR HERTWIG und RICHARD HERTWIG. Mit 10 lithographirten Tafeln. Leipzig, 1878.

[†] CLAUS (Studien über Polypen und Quellen der Adria. Denkschriften der Wiener Akademie, 1877) seems to have noticed the peculiar character of the ectodermic cells in this fovea in *Aurelia*. He has advanced the hypothesis that it represents a rudimentary olfactory organ, and has given it a name in accordance with this hypothesis ; but it would be difficult to put the opinion to experimental proof. The ganglion-cells in the subumbrella of *Chrysaora* are also noticed by CLAUS, but his statement as to the existence of ganglion-cells under the nerve-epithelium I am unable to confirm.

XVII. *On the Modifications of the Simple and Compound Eyes of Insects.*

By B. Thompson Lowne, *F.R.C.S., Lecturer on Physiology at the Middlesex Hospital Medical School, Arris and Gale Lecturer on Anatomy and Physiology in the Royal College of Surgeons, &c.*

Communicated by Prof. W. H. Flower, *F.R.S.*

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[Plates 52–54.]

Although the compound eyes of the Arthropoda have been examined and described with great care in former times by J. Müller,[*] Leydig,[†] Gottsche,[‡] and Claparede,[§] and more recently by Max Schultze[||] and Dr. R. Grenacher,[¶] the improved methods and instruments of the present time have enabled me to add considerably to the published descriptions of the eyes of insects.

My attention was first directed to this subject by a paper from the pen of Dr. Grenacher. My observations do not accord well with the observations of this author, but I think this is chiefly from the fact that he has used the eyes of immature insects, which differ greatly from those of the mature insect, and from the difficulty there has hitherto been in preparing sections of sufficient thinness to allow the minute structure of the pigmented portion of the eye to be observed. I have been enabled to overcome this difficulty by imbedding the head of the insect in cocoa butter, in the manner first devised by Mr. Schafer, and used by him in the investigation of the early conditions of the mammalian ovum; in this way I have been enabled to obtain sections of the requisite thinness.

In the present communication the principal types of eye are described which I have found in the class Insecta. Reserving the distribution of these types in the class for a future communication, I shall merely indicate the Orders in which each type is found; and in so doing would especially draw attention to the fact that the number of species and genera which I have at present examined is far too small to enable me

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[*] Verg. Phy. der Gesichtssinnes, 1826.

[†] Müller's Archiv., 1855. Lehrbuch der Histologie, 1857.

[‡] Müller's Archiv., 1852.

[§] Kol. Zeitsch., band viii.

[||] Archiv., band iii., 1867.

[¶] Zehender Monatsblatt, 1877.

to state that the eyes described are typical examples of the structure in the Order in question.

I shall conclude with some remarks on the function of the compound eye.

All the preparations, except where the contrary is stated, were prepared from insects hardened in a 2 per cent. solution of chromic acid. I have not found the peroxide of osmium so good in their preparation, and have only used it in a few instances.

I. *On the Structure of the Stemmata of* Eristalis tenax. (Plate 52, fig. 1.)

I have but little to add to what is already known of the structure of the simple eye. I have at present only examined it in a few of the Diptera, but have found such complete accordance between the descriptions of the authors already named and my observations, that I shall only briefly describe the structure of the ocellus of this insect, as it affords the best starting point for the correct interpretation of the structure of the compound eye.

Fig. 1 represents the ocellus. It consists of a very convex lens, rather more convex on its inner than its outer surface. Immediately behind the lens are the recipient structures, rods (fig. 1, *a*), consisting of an outer and an inner segment. The outer segment (*a*), which is next the lens, is a cylindrical, highly-refractive rod; the inner (*b*) is a fusiform nucleated cell. The inner segments are surrounded and separated from each other by an orange-coloured granular pigment.

The outer segment of each rod is from $\frac{1}{1000}$th to $\frac{1}{500}$th of an inch in length, and $\frac{1}{6000}$th of an inch in diameter; it is finely striated in the longitudinal direction. These rods are not closely packed together, but seem to lie in a fluid; this may, however, be a *post-mortem* change. Those at the periphery of the eye appear to be twice as long as those at the centre. I have not found them to be doubly refractive, nor have I ever observed any transverse division into disks.

The inner extremity of each rod-cell is connected with a fusiform cell (fig. 1, *c*), or with several fusiform cells arranged one beyond the other, and these are connected with the central nervous ganglion by fine nerve fibres. The fibres are surrounded by a few minute granules of a highly refractive substance. The nerves of the three ocelli unite into a single trunk.

The principal fact to which I would draw special attention is the apposition of the recipient elements of the retina with the lens, and the entire absence of anything like a vitreous body. In the young eye, the percipient structures are separated from the lens by a layer of cells. I have not observed this condition in the present species, but it is seen in the ocellus of the larva of *Dyticus*, *Acilius*, &c.

The great convexity of the lens in the ocellus of *Eristalis* must give it a very short focus, and it is manifestly but ill adapted for the formation of a picture. The comparatively small number of rods must further render the production of anything like a perfect picture, even of very near objects, useless for purposes of vision. I strongly suspect that the function of the ocelli is the perception of the intensity and the direction of light rather than vision in the ordinary acceptation of the term.

II. *On the Structure of the Compound Eye in* Tipula. (Figs. 2 to 5.)

The eye in *Tipula oleracea* is intermediate in structure between a true compound eye and a collection of ocelli.

The curvature of the common cornea is nearly hemispherical. It is divided into a number of strongly convex hexagonal facets, each of which is $\frac{1}{1000}$th of an inch in diameter, and $\frac{1}{2000}$th of an inch in thickness in its thinnest part. The outer surface is more strongly curved than the inner. The axes of the adjacent lenses make an angle of from four to five degrees with each other. Each lens is surrounded by a deeply pigmented portion of the cornea, which forms a black hexagonal framework between the lenses.

Beneath each lens there are sixteen rod-like cells (a''), which are easily distinguished in the immature imago.

In the mature imago these cells are so strongly pigmented with deep black pigment, that even in the thinnest sections I have been unable to detect the divisions between them; neither do they exhibit any transparent openings in transverse sections. I have found this to be the case both in specimens hardened in chromic and osmic acids.

Between each of these opaque cells and the facet of the cornea is a minute highly-refractive globule (a'), of a bright purple colour. These cells bring to mind the highly pigmented retinal cells of the Pigeon.*

Beneath the rod-cell layer is an elongated chamber containing a very remarkable structure, the "retinula" of Dr. GRENACHER, which I shall name the facellus (f). The facellus consists of seven fusiform cells, the outer extremities of which terminate in fine hair-like points, which appear to pass into the rod-like cells of the more superficial layer. The points of the cells of the facellus are the extremities of fine highly-refractive threads, which pass through the fusiform cells of which it is composed, and are prolonged through the long cylindrical organ which connects the facellus with the ganglionic retina. These axial threads are easily distinguished in transverse sections through the facellus (figs. 4 and 4*a*).

The cells of the facellus appear to become chitinous in the fully-formed imago, and are yellow in specimens hardened in chromic acid; like all the highly-refractive structures of the eye, including the cornea, they resist the action of solutions of caustic alkali for a considerable time. In the immature imago they are slightly granular, especially near their surface, but they contain no pigment.

A strong chitinous membrane (m) separates the parts already described from the

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* MAX SCHULTZE, Archiv., bd. iii.

deeper structures of the eye, but it is perforated beneath each facellus, so that the latter is in continuity with the deeper structures.

A membranous flask-like sac extends from the inner extremity of each facellus to the edge of the corresponding facet of the cornea; this is lined with deep black pigment cells.

Between each facellus and the ganglionic retina is a long compound rod, larger at its outer extremity than at its inner extremity. It is usually spoken of as the rod of the compound eye, but I shall call it the stemon (*st*), as I think I shall be able to show that it cannot be considered as the homologue of the rod-like structure of the true compound eye.

The stemonata, corresponding to the outer facets of the eye (fig. 5), are very short and conical, being very much larger at their outer than at their inner extremities. In the immature imago the stemon can be seen to consist of seven cells, but in the mature insect, and especially in the centre of the eye, these are so perfectly fused together that the component cells of the stemon can be no longer recognised.

The stemon is surrounded at its outer extremity by a very dense sheath of pigment, but this is deficient at its inner end. The stemon contains minute black granules of pigment, and these are arranged in four thread-like lines, which, with comparatively low powers, have an appearance which induced LEYDIG* to describe them as muscular elements; beside these, minute scattered black pigment granules are seen in the protoplasm of the stemon.

Some of the stemonata remain distinct throughout their whole course, but others unite with each other, so that three or four are fused at their inner extremities into a single thread. At their inner extremities all the stemonata branch, and are connected with pigmented stellate cells.

The highly refractive threads of the facellus are seen in transverse sections at the outer end of the stemon, but I have been unable to distinguish them at the inner attenuated extremity.

The stellate cells already alluded to are situated between two fine chitinous membranes. The outer of these (m^1) sends delicate sheaths over the stemonata; the inner (m^2) is perforated by the fine processes of the stellate cells, which communicate with the round cells (g) within the second membrane. The round cells are supported in a fine network of neuroglia, also apparently given off from the inner surface of the second membrane. Beneath the round cells are several layers of fusiform cells (c), which appear to be situated at the outer extremity of the optic nerve, and to be in continuity with its fibres.

III. *On the Compound Eye of* Vespa vulgaris *and* Vespa rufa. (Figs. 7 and 8.)

I have used these two species indiscriminately in the investigation of the compound eye, as I have found no difference in its structure.

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* *Loc. cit.*

The type of the compound eye in the Wasp is the same as that of the eye of *Tipula*, but the two differ in the following points :—

The curvature of the general cornea is so slight that the visual axes of adjacent facets in the centre of the cornea only make angles of 8′ with each other. The facets are only $\frac{1}{2000}$th of an inch in diameter, but they are $\frac{1}{1000}$th of an inch in thickness, and consist of numerous layers. The approximate radius of curvature of the outer surface of a facet is $\frac{1}{1500}$th, and that of the inner surface is $\frac{1}{2500}$th of an inch.

The refractive index of the material of which the compound cornea is formed does not differ materially from that of Canada balsam: this is easily seen in specimens mounted in fluid balsam. In order to determine the index of refraction with the greatest accuracy, I found the focal lengths of the lenticular facets of the cornea of a Hornet first in air and then in water, thus eliminating the radii of curvature. By this means I calculated the refractive index to be 1·53. The great difficulty is the determination of the real focus with sufficient accuracy, but the results are sufficiently accurate to give an approximate idea of the position of the focus in the eye. These results give $\frac{1}{200}$th of an inch as the distance of the focus behind the inner surface of the cornea, so that the rays may be considered as approximately parallel to the axes of the rod-cells.

As in *Tipula*, there are sixteen rod-cells (a'') behind each facet. There is also a small highly refractive globule of a dark purple colour, and a facellus (f) very similar to that in the eye of *Tipula* behind the rod-cells. All these structures are surrounded by so much deep violet pigment in my preparations that the details can only be observed with considerable difficulty.

IIIA. *On the Compound Eye of* Formica rufa. (Figs. 6, 7*a*, and 8*a*.)

My description and figures of the eye of this insect are taken from the eye of the mature female imago.

The eye of this Ant differs but little from that of the Wasp. The corneal facets are larger, measuring $\frac{1}{1500}$th of an inch in diameter, but are not more than $\frac{1}{2000}$th of an inch in thickness. The spherules beneath the cornea are colourless. The rod-cells (a'') are imbedded in a large quantity of deep violet pigment (fig. 7*a*); they are $\frac{1}{10000}$th of an inch in diameter. The facellus (f) is shorter and wider, and consists of more rod-like cells than I have observed in the facellus of any other insect: there are at least twelve cells; it is surrounded by a layer of deep purple pigment. The chamber in which the rod-cells lie is lined by deeply pigmented rod-like cells which differ from those in the centre of the chamber in the extent of their pigmentation and in not being connected with the facellus, so that in some of my sections in which the facellus and the deeper parts have been torn away the pigmented rods of the periphery of the chamber alone remain. Under these circumstances the eye appears to be provided with a chamber like the eye of a true dipterous insect, surrounded with palisade-like rods of pigment.

The stemon (*st*) is much shorter than that of the Wasp. Each has four elongated cells attached to its surface; these, as well as the stemon itself, are coloured with violet pigment. This pigment is in fine granules, and, like that of the rods in the eye of the Lobster, according to KUHNE,* and that of the eye in all the insects which I have examined, is unaffected by light. The transverse section of the stemonata (fig. 8*a*) shows that they are cylindrical and not prismatic; they exhibit four or more bright spots on their periphery, and are surrounded with granules of purple pigment. I am at a loss to understand the bright spots, but am inclined to view them as the result of molecular change; they may, however, be the indications of highly refractive threads. I have not, however, been able to detect any such threads in the vertical sections.

The stemonata rest on a limiting membrane of chitin (*m*).

I have been more fortunate in the examination of the ganglionic retina of the Ant than in that of the Wasp. The stemon is connected with the nuclear layer by a single thick nerve fibre (*n*); but from what I have seen in the Lepidoptera I have no doubt that by appropriate preparation this would be found to consist of a large number of component fibrillæ. My preparations of the eye of this insect were made from specimens which were killed some two or three years ago by immersion in spirit, and which had been put away and forgotten. The ganglionic retina (*g*) consists exclusively of small nuclei, or perhaps of very small round cells: these are connected with the deeper ganglia by bundles of nerve fibres.

I have not detected any stellate cells, nor have I found the fusiform cells so universally present; but I have not obtained sections of the deeper ganglia; neither have I as yet, in any of the insects which have a semi-compound eye like the Ant, detected the presence of any decussation of the fibres of the optic nerve. I have not, however, obtained a thoroughly satisfactory section of all the parts of the nervous structures connected with the optic tract, owing to the difficulty of getting a section in the plane which includes them all, if such a plane exists, as it certainly does in many of the Diptera and Lepidoptera. I am inclined to believe, however, from what I have seen, that no such decussation occurs in these insects.

IV. *On the Structure of the Compound Eyes of* Eristalis tenax, Syrphus, Musca vomitoria, Stomoxys, *and* Tabanus. (Figs. 9 to 20.)

The eyes of all the Brachycerous Diptera which I have examined are formed on one type, which differs entirely from that on which the eyes hitherto described are formed. They all have a cavity beneath each facet of the cornea containing a slightly coagulable fluid. At the inner extremity of this cavity, which is conical, there is a body consisting of four nuclei or small cells, and beyond this a rod-like structure which apparently differs but little from the stemon. I shall, however, distinguish it

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* *Loc. cit.*

by the term rhabdion, as I think that there is evidence that it does not in any way correspond to the structure which I have named the stemon. I am rather inclined to regard it as the representative of the rod-like cells in the eyes hitherto described. If a facellus exist at all, it is placed beneath this structure—a fact that is clearly indicated by the position of the facellus in the eyes of the Lepidoptera, in which there can be no doubt of its presence. As will be seen, there is a structure in the nervous retina of the flies which resembles the facellus very closely, but a true facellus is entirely wanting.

Eristalis. (Figs. 9 to 13.)—The eye in *Eristalis* does not differ in any way that I have been able to discover from that of *Syrphus,* but the parts of the latter are often more easily made out from their greater transparency. I shall describe the eye of the former insect, and refer to that of the latter when I have found the parts more distinct in it.

The cornea is about $\frac{1}{1500}$th of an inch in thickness, and the facets average $\frac{1}{800}$th of an inch in diameter. In the centre their adjacent axes make an angle of about 1° with each other. Those in the centre of the cornea are hexagonal and small: usually $\frac{1}{1000}$th of an inch in diameter; those at the edges are square, and as much as $\frac{1}{750}$th of an inch across. The facets in the immature imago and at the periphery of the cornea are surrounded by nuclei of a bright brown tint (the so-called nuclei of SEMPER) (fig. 9 *a*). These appear to be adherent to the substance of the cornea. In the mature imago and in the centre of the cornea the facets are surrounded by a framework of deep black pigment which conceals the nuclei, and is probably developed in or around them. Immediately beneath each corneal facet is a deep cup-like cavity (fig. 9) surrounded by flat cells filled with bright orange-coloured pigment; at the bottom of this cup there are four nucleated cells (a') which rest upon the extremity of a quadrangular rod (figs. 10 *a* and 10 *b*). These parts attain a very high development in *Acridium* and in the Diurnal Lepidoptera. I shall call the four cells the tetrasome, and the quadrangular rod on which they rest the tetraphore.

Between the tetrasome and the nervous retina is the rhabdion (a''). This consists of a protoplasmic sheath, containing a bundle of four fine highly-refractive threads, which are united together at the outer end of the rhabdion into an apparently single axial thread, which enlarges to form the tetraphore. I have been unable to make out the fourfold nature of this structure, but suspect that it consists of four elements. The outer extremity of the rhabdion is cylindrical, and is surrounded by a number of pigment cells (p), forming a structure which has been called the iris. I shall speak of these cells as the outer pigment cells of the rhabdion. In this region the rhabdion is seen to be grooved longitudinally, the grooves being filled by prolongations of the pigment cells (fig. 10 *e*). These details are best seen in sections of the eye of *Syrphus.* Beyond the region of the outer pigment cells the rhabdion is triquetrous, or more rarely quadrangular (figs. 10 *f*, and 11); a double bundle of fine moniliform pigmented fibres lies at each angle. These pigmented fibres are partly derived from the outer pigment cells of the rhabdion, and partly from four pigmented nuclei which are situated at the inner extremity of this structure (fig. 12). These pigmented fibres become very tortuous when they are acted on by water, and apparently produce the contorted conditions of the rhabdion which have been attributed to the elasticity of its axial structure. The interspaces between the prismatic portions of the rhabdia are occupied by large sac-like tracheal tubes. These are, so far as I can tell, confined to the Diptera, and are quite characteristic in this group.

The inner extremities of the rhabdia rest on a strong chitinous membrane (fig. 12, *m′*), which is perforated for the rhabdia to communicate with the nervous retina beneath, and for the tracheal tubes. The rhabdia appear to be continuous with the thick outer processes of the large stellate cells of the nervous retina (fig. 12). In a few cases I believe I have seen two rhabdia connected with one cell. I have been unable to trace any continuation of the axial structure of the rhabdion into the nerve cells of this region, but in some specimens I have seen four fine processes continued from the rhabdion into the region of the nervous retina, but in these the cells had disappeared in the preparation, so that I cannot state whether these were mere connective elements, or whether they belong to the proper structure of the nervous apparatus.

Fig. 12 shows the inner extremity of the rhabdia and their relation to the nerve structures beneath them. The oval cells are probably embryonic, as I have not found them in the adult imago. The drawing is from the eye of a small species of *Syrphus*, from which I succeeded in getting a very beautiful series of preparations.

The rhabdia of the two peripheral rows of facets are united into bundles at their inner extremities, four or more forming a compound structure, which is surrounded by elongated pigment cells (fig. 10 *l*). These compound rhabdia have six or more pigmented nuclei at their inner extremities. The form of the transverse section of the rhabdia is very variable (fig. 10, *d* to *i*).

The Nervous Retina of Eristalis *and* Syrphus.—As my most successful investigations of the nervous retina have been made in these insects, and as the modifications of the other parts are best understood when the nervous retina is included in the description, I shall describe the nervous retina in these insects, and afterwards state the points in which the same part appears to differ from it in other insects.

Fig. 13 represents this structure. From without inwards there are (*g′*) two layers of ganglion cells, (*n*) a layer of small round cells, (*f′*) a very remarkable layer of bundles of fusiform cells, so like the cells of the facellus in the eyes of the insects already described that it can hardly be regarded as anything but its physiological representative; and (*g″*) a third layer of stellate ganglion cells. These structures form the outer ganglionic retina, and are connected by a decussating optic nerve (*n′*) with a still deeper layer of staff-shaped cells, or, rather, with several layers of fusiform cells (*c*) superimposed one on the other. This inner ganglion is connected with the supraœsophageal ganglion by a distinct peduncle.

The intercommunication of the elements of the external ganglion or ganglionic retina is very difficult to determine, but I have no doubt, from the examination of many hundred preparations, that the ganglion cells of the outer layer are continuous with the protoplasm of the rhabdia by their outer processes, and that the stellate cells of the two outer layers form a complex network with each other by their lateral processes. I have been unable to determine whether the inner processes of these cells pass into the small round cells of the third layer, but I suspect they do; they certainly communicate with the fusiform cells of the fourth layer.

Figs. 10, *k*, and 13, f', represent the bundles of cells in this layer. The first is a transverse section through a bundle from a stained specimen. I shall call this the facelloid layer of the retina. The bundles of cells consist of five or six cells. (I am at a loss to explain this deviation from the number of structures in the rhabdion, but it will be remembered that the number of cells in the facellus of *Tipula* is not the same as the number of rod-like elements.)

The innermost or fifth layer of the ganglionic retina (g'') is formed of stellate nerve cells like those of the outer layer. These rest on a membrane of extreme tenuity: the inner limiting membrane. This is connected with the outer or basal membrane on which the rhabdia rest by a fine connective network, or neuroglia, in the spaces of which the elements already described are situated. The number of layers of elements is very much reduced in those portions of the retina which correspond to the peripheral portions of the eye. The outer ganglion cells are reduced to a single layer, and the facelloid layer exhibits fewer sets of cells.

The optic nerve (n') consists of clear, often varicose fibres. These unite the inner and outer ganglia, and form a complete decussation from above downwards, as well as from behind forwards. The inner half of each of these fibres is surrounded by a vast number of minute nuclei, which refract light highly. I have been unable to satisfy myself of their connection with the fibres, but I am inclined to the belief that they are united with them, as they move with them when the glass cover is shifted, and are only separated from them with great difficulty. The inner ganglion (c) consists of five or six layers of fusiform cells of granular protoplasm.

Musca vomitoria (figs. 14 to 17).—In this insect the chamber of the eye (fig. 14) is shorter than in *Eristalis*, and the tetrasome (a') is placed in a small ovoid cavity at its inner extremity, surrounded by a dense layer of pigment, so that only its apex is exposed to the light. The segments of the tetrasome are finely striated in a longitudinal direction.

The cornea has the curvature of an epicycloid in section (fig. 51, page 596). The facets are $\frac{1}{1000}$th of an inch in diameter; the radius of curvature of the outer surface is $\frac{1}{1000}$th of an inch, and that of the inner surface is $\frac{1}{750}$th of an inch. The focal length of the lens is $\frac{1}{400}$th of an inch, measured in air. The distance of the outer extremity of the tetrasome to the inner surface of the cornea is as nearly as possible $\frac{1}{1000}$th of an inch. The focal length of the lens is given from the same surface of the cornea, so that the tetrasome lies considerably within the focus of the lens.

The rhabdia are less regular in size and structure than those of the Syrphidæ, as they intercommunicate with each other in the manner represented in fig. 15. The axial threads vary from four to twelve after the intercommunication of the rhabdia. The communicating branches contain only two axial threads, and the rhabdia near the tetrasomes contain four axial threads (fig. 15*a*). I do not think the axial threads intercommunicate. Fig. 16 represents the rhabdia and axial threads in transverse section.

Beside the pigmented moniliform fibres of the rhabdia, which are like those in the eye of *Syrphus*, there is a network of stellate pigment cells between the rhabdia, which contain a brilliant rose-coloured pigment; this becomes darker as the age of the insect advances.

A quantity of granular orange-coloured pigment is collected at the inner extremity of each rhabdion in a small spherical mass (fig. 17). The inner extremities of the rhabdia seen *in situ* have the appearance of a layer of polygonal epithelium.

Stomoxys calcitrans.—The only difference that I have been able to observe in the eyes of this insect as compared with those of *Musca vomitoria* is that the corneal facets are smaller: a condition which appertains in all small insects. They are only $\frac{1}{1600}$th of an inch in diameter.

Tabanus bovinus.—I have only examined the eye of *Tabanus* in dried specimens, so that I can only speak of the chitinous framework by which the various parts of the eye are supported. This attains a very remarkable development in *Tabanus* (figs. 18, 19, and 20). Not only are the chambers surrounded by chitinous hexagons, but the rhabdia are invested by chitin, and are connected by membranous septa, which divide the spaces from each other in which the trachea lie. These septa are strengthened by transverse thickenings. The most remarkable deviation from the ordinary structure of the dipterous eye is seen in the structure of the rhabdion, which appears to consist of two separate halves divided from each other by a fissure, each having its own sheath. I have found nothing like this in any other insect; but the structure needs investigation in the recent insect. It is apparently identical with a condition described by Dr. GRENACHER as existing in the rhabdia of some Coleoptera.*

V. *The Structure of the Eye in* Agrion puella. (Figs. 21 to 24).

In *Agrion puella* the type of the eye does not differ greatly from that in the Diptera. The chamber is much deeper and is filled with a gelatinous fluid. It is prismatic in form, and has a ring of four very transparent cells immediately under the cornea (fig. 21*a*); but in *Æschna* there are from eight to twelve cells.† The walls of the chamber (fig. 21) are not surrounded by pigment cells, but sixteen are found around the tetrasome. Long, exceedingly fine processes are given off from

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* *Loc. cit.*

† CLAPAREDE.

these cells which line the chamber. They are moniliform, with small granules of dark brown pigment. The walls of the chamber are chitinous.

The rhabdia are hexagonal in transverse section in their outer extremity (fig. 23, *a*), and in the young imago at least are easily seen to be made up of six cells surrounding the central highly refractive threads. The inner portions of the rhabdia are round in transverse section (fig. 23, *b* and *c*); these organs are everywhere pigmented with fine black pigment. In many of my preparations they contain bright globules like oil; I suspect this is due to degenerative changes during the preparation of the specimens. I have observed the same in the stemonata of *Formica*.

The rhabdia are surrounded by a network of stellate cells containing black pigment (fig. 22).

The trachea of the rhabdia form a network in the spaces between them; but there is nothing like the large blind tracheal sacs found in the same region of the eye in the Diptera.

The external ganglionic retina (fig. 24) differs from that of the Diptera in the large quantity of black pigment developed in it: this is contained in the stellate cells of the neuroglia. The granules or round cells (*n*) are more numerous than in the Diptera, and form several layers, and the place of the facelloid layer of the Dipterous eye is occupied by a triple layer of large prismatic cells (*f* and *n'*). These also contain a large amount of pigment. I have not been able to make out the structures of this portion of the eye with the same clearness as in the Diptera, owing to the pigment in the cells of the neuroglia.

VI. *On the Structure of the Eye in* Acridium (Stenobothrus). (Figs. 25, 26, and 27.)

The cornea is not divided into facets in this insect, but both its surfaces are continuously curved. Beneath the cornea is a framework of chitinous chambers like the cells of a honeycomb; these are $\frac{1}{1000}$th of an inch in diameter. In each chamber there is an exceedingly complex tetrasome; this consists of two parts, which I shall call the tetrasome (*t*) and the tetraphore (*t'*) (figs. 25, 26, and 27).

The tetrasome is placed immediately beneath the cornea. In the young *Acridium*, just before the development of the wings, it consists of four transparent nucleated cells (fig. 25); but in the adult insect these are developed into four spherical highly refractive bodies containing numerous minute vacuoles* (figs. 26 and 27). They are supported on the sides of a square rod-like body formed of four segments, which are enlarged below into the body of the tetraphore.

The tetraphore in the adult insect consists, like the tetrasome, of a highly refractive substance, probably chitin; but in the immature insect it consists of four cells, which first become chitinous where they are in contact with each other, or they develope a

* Similar vacuoles exist in the tetraphore of *Vanessa*; these have been described by CLAPAREDE, *loc. cit.*

rod-like chitinous structure between them, which gradually takes their place. I am entirely inclined to the former view, and regard the segments of this organ as modified cells.

The inner extremity of the tetraphore rests on the outer extremity of the rhabdion, which is swollen into an ovoid enlargement (a). The highly refractive continuation of the tetraphore is plainly seen to be continued as a thread-like process in the axis of the rhabdion (a'). In transverse sections this is easily seen to be composed of four separate fibres. The thread-like axis of the rhabdion is enlarged into two fusiform swellings at the outer extremity of the organ.

The outer extremity of the rhabdion is surrounded by a number of pigment cells; these send fine moniliform pigment threads over it. The pigment is of an olive-brown colour. The rhabdia are cylindrical and straight.

I have at present been unsuccessful in the investigation of the ganglionic retina in this insect.

VII. *On the Structure of the Compound Eye in* Vanessa atalanta. (Figs. 28 to 34.)

The eyes of this insect are similar to the last-described form, but present important differences in the presence of lenticular facets to the cornea, in the structure of the tetraphore, and in the presence of a distinct facellus upon which the rhabdion rests.

The corneal facets are strongly convex on their outer, and slightly concave on their inner, surface; they are $\frac{1}{1000}$th of an inch in diameter.

The tetrasome consists of four nucleated cells in the immature, and of four highly refractive spheres (t) containing vacuoles in the mature insect. It is placed immediately beneath the cornea. The tetraphore consists of an outer very transparent globe (t''), enclosing an ovoid highly refractive body (t') containing vacuoles. An exceedingly fine prolongation of this body connects it with the rhabdion, and the whole floats in the fluid of the chamber (figs. 28 and 30). The chamber is prismatic, as in the last form; its pigment cells are arranged in two sets: eight surround the edge of the corneal facet, and a second set is situated at the inner extremity of the chamber. Numerous fine moniliform pigmented processes are given off from these cells, those from the outer set interdigitating with those of the inner, and so forming the pigmented lining of the chamber, as in the eye of *Agrion.*

The rhabdia are quadrangular in section, and are of smaller diameter at their outer than at their inner extremities. In transverse section some of these rhabdia appear to consist of five cylinders, but in the majority four of these are fused into a single investing sheath, enclosing an axial structure (fig. 31). Each of the external portions has a pigmented thread, which is easily separated from the rhabdion; it is connected with a pigmented nucleus at its inner extremity.

At the inner extremity of each rhabdion (figs. 32 and 33, a'') there is a cylindrical cavity (a''') formed by a membranous sheath from the basal membrane; the walls of these cavities are deeply pigmented. Between these cylindrical cavities are others of smaller diameter. Each of the larger cylinders contains seven rod-like cells ; the smaller ones transmit tracheal tubes. I suspect the rod-like cells represent the facelli in the eye of *Vespa* and *Tipula*—a view strengthened by, and indeed entirely resting upon, the condition of the eye in the Crepuscularia. (See fig. 36.)

Immediately beneath the basal membrane there is a grouping together of the nerve-fibres into bundles, which are deeply pigmented with dark brown pigment. Amongst these bundles are a number of large stellate cells (*p*),* all more or less strongly pigmented, but bearing a very close resemblance to the stellate nerve-cells of the outer ganglion or nervous retina; together with small round and stellate nerve cells (*g*). Beneath these are numerous elongated fusiform cells (*c*), arranged in bundles like those of the facelloid layer in the retina of the Diptera, but more nearly resembling the fusiform cells of the deep ganglion of those insects. The fibres of the decussation of the optic nerve, which unite the outer and inner ganglia, are arranged in bundles which have the appearance of large nerve-fibres.

VIII. *On a Modification of the Eye in the Diurnal Lepidoptera.* (Fig. 35.)

The only modification I have observed in the Diurnal Lepidoptera is one in which the tetraphore is placed near the bottom of an elongated chamber; this appears to occur in *Pieris, Colias,* and *Gonepteryx:* the only three genera in which I have examined the eye, except *Vanessa.* Eight very delicate transparent cells (c' and c'') appear to fill this chamber (fig. 35). The lens of the corneal facet has a much less curve on its outer surface in this form of eye. Owing to an accident in the process of preparation, I regret that I am unable to determine from which of these three genera the figure is taken, but they are all very much alike in structure. I believe, however, that it is a drawing from the eye of *Colias.*

Claparede represents a semi-diagrammatic section of the partially developed eye of the pupa of *Vanessa;* it shows the original condition of the chamber of the eye filled with eight cells, in the interior of which the hard structures of the tetrasome are developed. The researches of Claparede on the development of the eye in this genus are very complete, and throw great light on the morphology of the compound eye.†

IX. *On the Structure of the Eye in the Sphingidæ.* (Fig. 36.)

I have not been able to examine the eye in the recent insect, but Prof. Flower placed at my disposal a very fine pupa of a *Sphinx* which had been many years preserved in spirit, from which I obtained some very excellent preparations.

[*] These cells are figured by Claparede (*loc. cit.*) in a drawing of the parts in the mature pupa. Judging from his figure, they are probably nervous elements.

[†] *Loc. cit.*

The eye in *Sphinx* is quite intermediate in structure between that of the Nocturnal and of the Diurnal Lepidoptera.

Immediately beneath the cornea, which was still in an undeveloped state in the pupa examined, are four small cells containing nuclei (a'); these rest upon a hard cone, consisting of four segments (a). This structure is characteristic of the eyes of the Nocturnal Lepidoptera, and it is perfectly clear in recent specimens. It persists in the dried insect, like the other chitinous structures. It had assumed an amber colour in the *Sphinx* pupa from which this description is taken (fig. 36, a). Immediately beneath the cone, as I shall call this body, is the rhabdion (a''). This differs in no way from that of *Vanessa,* except in its greater length, and in the fact that its outer end was much contorted. Beneath the rhabdia is a layer of undoubted and true facelli (f): one to each ocellulus. Each facellus consists of seven cells, the slender prolongations of which pass into the corresponding rhabdia. The facelli are surrounded by nucleated pigment cells and are continuous with the nerve-fibres. These are gathered together into bundles and unite into nerve-trunks (st); the fibres from thirty or forty facelli being united into a single trunk. They are deeply pigmented with violet-coloured pigment. At their inner extremity the bundles of nerve-fibres branch, and are connected with stellate nerve-cells. The other structures of the retinal ganglion were not distinguishable. The nerve-fibre bundles appear to represent the stemon of the semi-compound eye of *Vespa* and *Tipula.*

X. *On the Structure of the Eye in the Noctuid Moths.* (Figs. 37 to 42.)

At present I have only examined the eye in the true Noctuids. As in the Crepuscularia, there are four cells (a') immediately beneath the cornea, but in some species these cells each contain one or two bright highly-refractive nuclei, which appear to be formed of the same material as the deeper cone (figs. 38 and 42). The nuclei are rod-like, and have their long axes at right angles to the corneal facet. These cells rest upon a cone (a) formed of four segments, like that in the eye of the Crepuscularia. In some Moths this cone is surrounded by pigment cells which form four lines, one adjacent to each segment of the cone (fig. 38), and give off numerous moniliform fibres, which entirely surround the cone; in other species the cells are reduced to the magnitude of minute granules, from which the pigment fibres of the chamber are given off (fig. 37).

The inner extremity of the cone is continued inwards, in the form of four exceedingly fine, highly refractive threads (a''). These are, I believe, always surrounded by a protoplasmic sheath in the recent condition; but in a great many of my preparations the sheath has disappeared, and nothing is left but the highly refractive axial threads. They form the rhabdion. It is not uncommon to find these rhabdia united into bundles which form a network, and in some species the rhabdia are united into complex bundles, which are enclosed in chitinous sheaths surrounded by a large amount of pigment (figs. 39 and 40).

In the eye of the Herald moth (fig. 38) I have found some very remarkable drop-like appendages at the inner extremities of some of the cones (a'''), but I have not been able to make out their nature. I almost suspect they are the result of the rupture of the axial threads of the rhabdion, and are produced by the contraction of these threads, which, if such is the case, are viscous in the recent condition.* The thread-like prolongations of the cone are seen to end at their inner extremities in very long fusiform cells (c), which like the rhabdia are sometimes contained in tubular sheaths of chitin (fig. 41). The inner extremities of the fusiform cells are connected with stellate ganglion cells; but the whole of the deeper structures in the few species I have examined are so deeply pigmented that I am not able to give any satisfactory details concerning the ganglionic retina.

XI. *General Remarks on the Morphology of the Eyes of Insects.*

Three forms of eye have been recognised in the Arthropoda since the time of J. Müller's investigation of the subject: the simple, the aggregate, and the compound eye.

In the simple eye there is no difficulty in recognising the signification of the rod-like elements which are situated beneath the cornea, or their epithelial origin.

There can be little doubt but that we have the highest development of the aggregate eye in the so-called compound eye of the Nematocerous Diptera and of the Hymenoptera.

As far as the cornea is concerned, these eyes do not differ from the true compound eyes of other insects and of many crustaceans; but as I have shown, the deeper parts are similar to those of the simple eye in a high condition of differentiation. This form of eye is therefore to be regarded as a highly developed form of a connecting link between the simple and compound eye.

I am at present unable to point out in a satisfactory manner the nature and morphological relations of the facellus. Although this structure is present in the eyes of many Lepidoptera, it is apparently absent in the true Diptera, unless the facelloid layer of the retina can be regarded as its representative. The morphological representative of the facellus is more probably found in the pigmented bundles at the inner extremity of the compound rhabdia of the periphery of the eye in these insects.

A comparison of the parts in the aggregate eyes of the Nematocerous Diptera and Hymenoptera with the structures in the true compound eye is not difficult, but it can only be made in a tentative manner until the development of the aggregate eye has been more thoroughly worked out and a comparison has been made with the

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* A similar contraction has been observed by Max Schultze in the inner extremities of the rods of Vertebrates, as a *post-mortem* condition. (Archiv., band iii., p. 220.)

development of the compound eye: a task which I shall hope to commence at least next summer.

CLAPAREDE's paper, which is most accurate, gives very valuable details on the development of the true compound eye.* He has also described and figured the various stages of the development of the eye of *Formica*, but he has neither detected the rod-cells of the chamber nor the facellus: I cannot help thinking that this has been from the manner in which the stemon separates from the facullus. I strongly suspect that CLAPAREDE's preparations represent only a part of the eye, and that in the more advanced stage the chamber is only partially represented. Unless this is the case, the remarkable deviation in the Hymenoptera from the more usual form of the eye in insects is developed from an eye which differs in no important particular from the compound eye in its simplest condition.

The true compound eye of insects is seen in three very different forms in the fully developed insect; but the observations of CLAPAREDE show that in the undeveloped condition of the eye these are all most probably identical, or nearly so. In this condition each segment of the eye consists of thirteen principal cells: eight form the cone (Krystalkegel) and five represent the rhabdion. Beside these there is a variable number of pigment cells.

The highly refractive structures of the axis of these parts are probably formed by the deposit of chitin in the substance of the primitive cells; but I think that further investigation of the whole subject is needed. Although my observations on the development of the compound eye agree in the main with those of CLAPAREDE, I have yet to work out the subject with the increased knowledge which I now possess.

The view that the hard and highly refractive parts of the cone or chamber are formed in the interior of the primitive cells is borne out, however, by the condition of the cone in the Nocturnal and Crepuscularian Lepidoptera, and according to LEYDIG† in the eye of *Cantharis melanura*, *Elater noctiluca*, and *Hyperia*, where the whole of the large cells of the primitive cone are replaced by the hard scleral cone.

I shall call the form of eye typical of the Nocturnal Lepidoptera the conic eye: and shall speak of the conic eye as proto-conic in its embryonic condition, and sclero-conic in the form it assumes in the Nocturnal Lepidoptera, and many other insects.

At present I have not found the proto-conic form of eye in any fully developed insect, but I have not yet examined the eyes of the Coleoptera, in which the writings of previous observers render it highly probable that such eyes exist, at least amongst the Pentamera. The work of LEYDIG shows that it exists in Melolontha.

Starting from the conic eye as the nearest approach to the primitive eye, there are two very remarkable and opposite deviations. In one, the cone is replaced by fluid, and the recipient structures are reduced to their simplest condition, as in the eyes

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* *Loc. cit.*

† *Loc. cit.*

of the Brachycerous Diptera, and the Dragon-flies. I shall speak of this as the hydro-conic eye.

In the other form the cone is highly modified, and appears as a very complex tetrasome and tetraphore. I shall speak of this as the tetraphoric eye.

I shall conclude this portion of the subject by indicating very briefly the probable distribution of the three forms of compound eye, as well as that of the highly complex semi-compound eye, which for brevity may be called micro-rhabdic.

I have at present found the micro-rhabdic eye in the Nematocerous Diptera, the Hymenoptera, and Hemiptera, although I have only examined the eyes of a few members of the Order, and these far from exhaustively. I believe that it will also be found in the eyes of many Coleoptera, such for instance as Bryaxis.

The conic eye is the usual form of the compound eye in the Crustacea: at least it is found in the Lobster, Palæmon, and Hyperia. As already stated, it is found in the Nocturnal Lepidoptera, in the Sphingidæ, and probably in all the Pentamerous Coleoptera at least.

Judging from LEYDIG's descriptions, it is also found in the Cursorial Orthoptera.

I have at present found the tetraphoric eye only in *Vanessa, Colias, Pieris, Gonepteryx,* and *Acridium;* but I have not examined any other species of the Orthoptera and Diurnal Lepidoptera. The hydroconic eye occurs in all the Brachycerous Diptera and in the Dragon-flies.

XII. *On the Theory of Mosaic Vision.*

The structure of the compound eye appears to favour the view long ago expounded by JOH. MÜLLER. This view is supported by the absence of lenticular facets in many species of Arthropods; by the relative sharpness of vision, not only in different species, but in different parts of the field of the same eye, as well as by the behaviour of a beam of light in passing through a highly refractive rod immersed in a less highly refractive medium, or surrounded by black pigment.

It is, further, the only theory which has been hitherto advanced that is competent to explain the phenomena when we bear in mind the relation of the recipient structures of the compound eye to the nerve elements beneath them.

On the passage of a ray of light through a highly refractive rod of small dimensions.—In order to arrive at some knowledge of the manner in which light passes through the highly refractive rods of the eye in the Arthropoda, I made the following experiments.

I took a capillary glass tube about $\frac{1}{100}$th of an inch in diameter, and placed it upright in a small transparent trough under the microscope, and filled both the vessel and the tube with water. The tube was an inch in length and was examined with an inch objective. I found that no light passed through the lumen of the tube, but that the section of the wall of the tube was brilliantly illuminated. I next placed a few fine glass threads, drawn from glass rod, in the interior of the capillary tube; these were as nearly as possible the same length as the tube, and measured $\frac{1}{1000}$ of an inch in diameter. The end of each of these fine rods appeared as a small bright disk in the deep black lumen of the tube, and when the light was shut off from the rest of the field reminded me of the appearance of the disk of a planet seen through a telescope, although the illumination was not by any means powerful, but was obtained from an ordinary gas burner ten feet from the microscope. When the focus of the microscope was altered so that the ends of the rods lay beyond it, the circles of light enlarged, showing that the rays left the rod in a divergent direction.

The same phenomena were observed when the diameter of both the rods and the containing tube was somewhat increased, but they were not so brilliant. In some cases when the ends of the rods were beyond the focus of the object glass, the central spot of light was surrounded by grey rings from interference. When the extremities of the rods were lenticular, as well as when the lower end of the rod was enlarged by fusing it into a globule of glass, or when it was drawn into a cone, the same phenomena occurred; so the brilliancy of the upper extremity of the rods was increased when the light, falling on the lower extremity, was convergent, so long as the axis of the ray was in the same direction as the axis of the rod, although oblique pencils produced only very feeble illumination of the upper extremity of the rod, even when the obliquity was only slight—at most, three or four degrees from the axis of the rod.*

It appears pretty evident that the appearances described are due in some way to total internal reflection.

In order to estimate the effect of the pigment I used glass rods, covered, except at their ends, with a layer of black varnish; and I found that even with rods $\frac{1}{500}$th of an inch in diameter, and only half an inch long, it was very difficult to transmit any light at all, unless the rays were absolutely parallel with the axis of the rod. With longer lengths of rod, or rods of smaller diameter, no light was transmitted, and the ends of the rods appeared quite black.

From these facts I think the inner extremities of the fine rod-like structures of highly refractive material which extend from the cornea, or from the inner end of the chamber into the deeper structures of the eye, may be regarded in the light of luminous points, illuminated by the light of the central pencil transmitted through the lens, and having as their function the excitation of the nerve ending in which they are embedded.

The focus of the lenticular facet in all the insects which I have examined lies considerably deeper than the outer extremity of the rhabdion in the true compound eye, and much below the surface of the rod-like recipient structures in the micro-rhabdic eye, so that for objects removed only the tenth of an inch or less from the

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* OSCAR SCHMIDT has since recorded some similar experiments with glass rods in KÖLLIKER, 'Zeitsch. für Wissensch. Zoo.', Bd. 30. Beiblatt.

facet, we have to do with convergent rays and not with the focal point. This points to some mode by which the stimulation of the nerve ending is brought about, other than the union of homocentric pencils in a point beneath the compound cornea. In *Hydrophilus piceus*, however, according to Exner,* the focal point for parallel rays lies within the crystal cone.

Whether each facet in the compound eye corresponds to one or to four distinct luminous impressions must at present, at least, remain a matter of doubt. I think, however, there can be no doubt that several distinct luminous impressions are transmitted from each facet in the micro-rhabdic eye of *Tipula* and *Vespa;* and there can be no doubt that a number of distinct luminous impressions are received by the ocellus or simple eye. I cannot, however, believe that the ocelli of insects can produce anything worthy of the name of an image, in the Diptera and Hymenoptera at least. The few retinal elements, their near approach to the lens, and the strong curves of the surfaces of the latter, are but ill adapted for more than the perception of light and the direction in which it is most intense.

In the compound eye the curvature of the cornea and the number of facets agree well with Müller's theory. It is true that Claparede has expressed the opposite opinion, but I think I shall be able to show that this is based on an incorrect assumption.

Claparede has stated that if Müller's theory were true, a hive Bee should be unable to perceive objects of less than eight or nine inches in diameter at a distance of 20 feet as distinct; but he comes to this conclusion by assuming that the acuity of vision is the same over the entire field. This is far from being the case in any insect which I have examined, with the single exception of *Tipula*, where it is approximately so, perhaps. In all the other insects which I have examined, the axes of vision for adjacent facets make a very small angle with each other in the central portion of the visual field, and a much larger one at its circumference. And although I have not had the opportunity of examining the cornea of the hive Bee critically, in the humble Bee, the Wasp, Tabanus, and the great Dragon-flies, the angles made by the axes of adjacent facets are not more than from eight to fifteen minutes: a condition which would enable objects of from half an inch to an inch in diameter to be seen as distinct at a distance of twenty feet—an acuity of vision quite sufficient to account for all the observed phenomena of vision in insects.

The method which I have adopted in calculating the acuity of vision is as follows:—

A magnified image of the entire cornea is thrown upon a sheet of paper by means of a camera lucida attached to the microscope. By using low powers and appropriate illumination, the error from distortion of the image can be reduced to a minimum. The profile of the various meridians was then sketched. By drawing tangents to the curve, the radii of curvature of different parts of the curve are readily found. The ratio of the diameter of the facets to these radii gives the sine of the angle subtended

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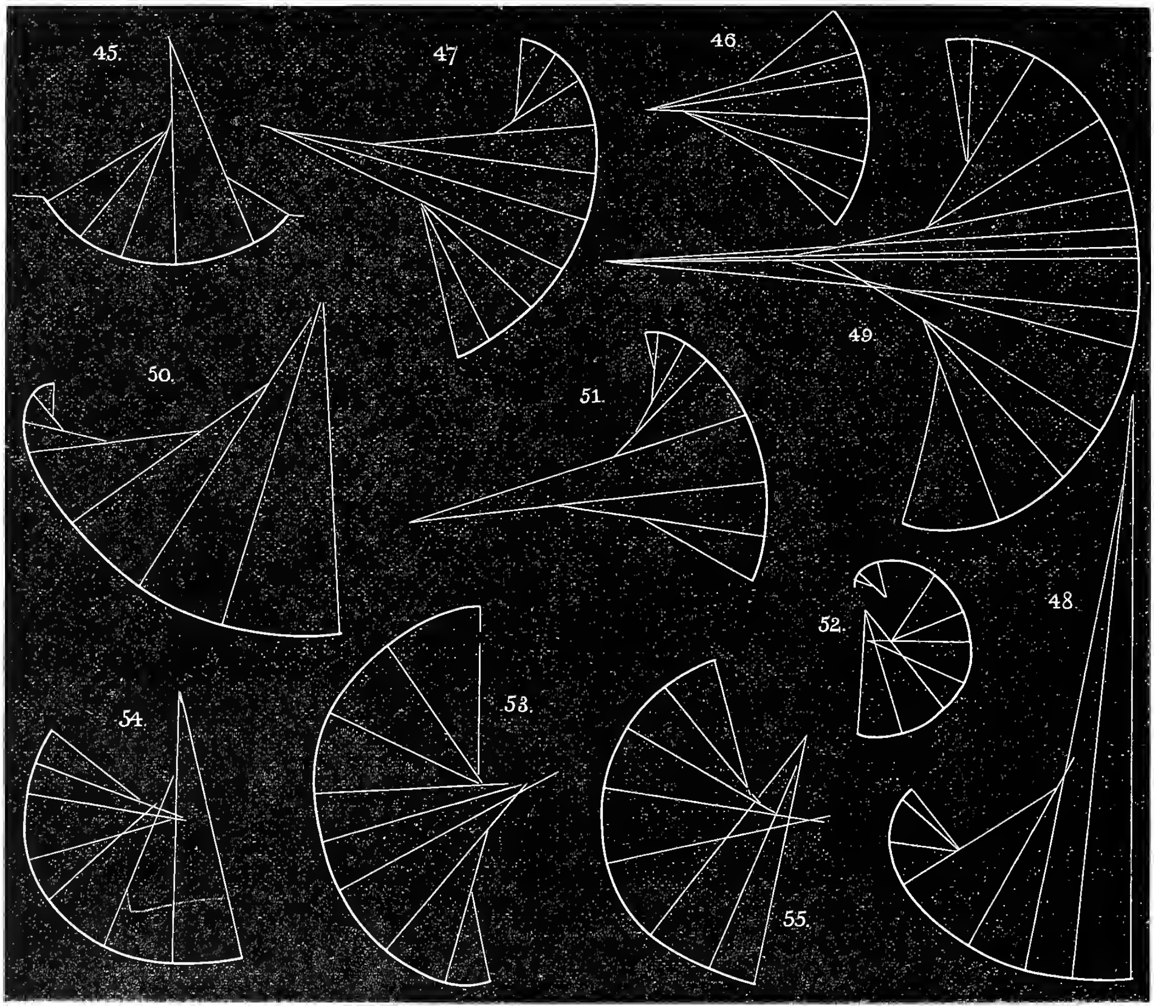
* Wien Sitzungsberichte, 1876.

by each facet. It will be seen by the accompanying figures (figs. 45 to 55)* that the curves of the meridians of the compound cornea approach more or less closely to an epicycloid.

The average angles subtended by the facets in the region of most distinct vision in different insects are given in the following table, in which I have added the greatest angles subtended by the facets at the periphery, the diameter of the corneal facets and the acuity of vision, both in the centre and at the periphery; according to SNELLEN'S system, the unit of vision being the power of perceiving an object at twenty feet, which has a diameter corresponding to an angle of five minutes: an angle of one minute being taken as the mean size of a visual perception in man. The signification of the fraction in the fourth and fifth columns is, that an object appears in the same detail to the insect as it does to man, when the distance of the object from the eye is



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* See description of plates, p. 602.

measured by the denominator of fraction for man, and by the numerator for the insect. Thus a Dragon-fly would see an object 20 feet from its eye in the same detail that a man would perceive if it were seen at a distance of 160 feet.

	Least angle.	Greatest angle.	Diameter of facets.	Greatest sharpness of vision.	Least sharpness of vision.
Æschna grandis	$8'$	$30'$	$\frac{1}{750}$	$\frac{20}{160}$	$\frac{20}{600}$
Vespa rufa, worker	$8'$	$85'$	$\frac{1}{2000}$	$\frac{20}{160}$	$\frac{20}{1700}$
,, *vulgaris*, worker	$8'$	$85'$	$\frac{1}{2000}$	$\frac{20}{160}$	$\frac{20}{1700}$
Bombus muscorum, female..	$8'$	$30'$	$\frac{1}{1000}$	$\frac{20}{160}$	$\frac{20}{600}$
Tabanus bovinus, male	$18'$	12°	$\frac{1}{1000}$	$\frac{20}{360}$	$\frac{20}{14400}$
Syrphus, sp.	1°	4°	$\frac{1}{1000}$	$\frac{20}{1200}$	$\frac{20}{4800}$
Musca vomitoria	1°	6°	$\frac{1}{1000}$	$\frac{20}{1200}$	$\frac{20}{7200}$
Colias edusa..	1°	2°	$\frac{1}{1000}$	$\frac{20}{1200}$	$\frac{20}{2400}$
Noctua, sp.	2°	12°	$\frac{1}{1000}$	$\frac{20}{2400}$	$\frac{20}{14400}$
Tipula oleracea	4°	5°	$\frac{1}{1000}$	$\frac{20}{4800}$	$\frac{20}{6000}$

The region of the most distinct vision extends from the visual line or the perpendicular to the centre of the least curved portion of the cornea, to a distance of from twelve to fifteen degrees in the horizontal, and from twenty to thirty degrees in the vertical meridian; so that the region of the most distinct vision for each eye is approximately half an ellipse, with its long axis vertical in front. But when the two eyes are taken it is approximately a circle in front of the insect; the two fields do not overlap in this direction.

It will be seen that the acuity of vision, according to MÜLLER's theory, must vary directly with the radius of curvature of the surface of the cornea, and inversely as the diameter of the corneal facets. In many of the Diptera, as in *Tabanus*, the facets of the peripheral region of the cornea are three times the diameter of those in its centre.*

The size of the corneal facets varies in different insects from $\frac{1}{750}$th to $\frac{1}{2000}$th of an inch. They seem to bear a relation to the size of the insect, as the largest are found in the largest and the smallest in the smallest insects; but I have found none less than $\frac{1}{2000}$th of an inch, although I have examined the eyes of many Diptera, of a line or less in length. As the radii of curvature in very small insects are also very short, the vision of such insects is less distinct than that of larger insects; at least, the distance at which objects can be seen distinctly must be very small.

J. MÜLLER has pointed out that the flight of insects depends on their power of vision, and this will account for the distances which large insects sweep through when

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* In the genus *Æschna* the facets of the upper third of the compound cornea are twice the diameter of those of the lower two-thirds; there is apparently no difference in the other part of the eye, except a proportionate increase in size.

disturbed, whilst the smaller species are confined, as a rule at least, to short flights, and remain hovering around a single branch or twig, unless carried away by currents of air.

The direction of the visual line is also a point of considerable importance. The fields of most acute vision are so combined in *Tabanus* that the visual line is directed forwards in the horizontal plane of the insect. In the pollen-feeding Diptera, and in most Lepidoptera, the visual lines diverge from each other to the extent of from fifteen to thirty degrees, and are directed downwards at an angle of thirty degrees, instead of lying in the horizontal plane.

In the Wasp the line of vision is directed forwards; in a species of *Noctua* it is directed almost directly downwards; and in the great Dragon-flies, where a very large field exists in which the visual power must be very great, the visual lines are directed forwards in the plane of the insect, diverging from each other to the extent of about thirty degrees.

In all the Coleoptera which I have examined, although the corneal facets are small, the radii of curvature of the cornea are very short; so that they cannot see objects distinctly in detail at any great distance. The same is true of the ants.

J. MÜLLER has stated that the vision will be the same for distant as for near objects, and this is true if measured by the angle under which the smallest object is seen as a distinct visual impression; but it will make a great difference in the details which can be perceived whether the object, as for instance another insect, subtends an angle of only one degree, or of from fifty to sixty degrees. By means of the fourth column in the table given above we may also estimate the distinctness with which near objects are seen by the species of insects in question. I have often been struck with the fact that the mimicry of the Diptera to the Hymenoptera is only sufficiently close when the insects are seen at a distance to be likely to afford any protection to the Diptera; but if the view of MÜLLER is the true one, and the acuity of vision is expressed in the above table, it would be sufficiently close to deceive the majority of other insects even at close quarters. I have frequently observed that Flies give place to both Wasps and the Syrphidæ which resemble them.

Under the supposition that MÜLLER's theory is the true one, I was for a long time much puzzled to account for the lenticular facets of the cornea. That they are not essential to the vision of insects is apparent from the frequent absence of such facets both in insects and crustaceans, which give the strongest evidence of very considerable acuity of vision. My experiments, however, with glass rods seem to point to the explanation that these facets enable a larger pencil of rays to reach the inner extremity of the highly refractive parts of the eye; for instance, when a cone exists, the axial cone of light entering the rhabdion will be much larger with a lenticular facet than when no lens exists.

There are some very interesting facts with regard to the distribution of lenticular facets in the Insecta. Thus some Noctuids have practically no lenticular facet, or, at least, a very feeble one, whilst others have a very convex lens. The same is true in the Diurnal Lepidoptera. In Hydrometra the corneal facets are composed of two parts of different refractive powers. The outer portion of the cornea is more strongly refractive than the inner portion ; it is also more convex on its inner surface than the inner surface of the corneal facet, so that it presents the condition of a very powerful bi-convex lens in apposition with a second lens, which is concavo-convex, the two fitted together like an achromatic object glass. In *Dyticus* there is also a remarkable arrangement: the corneal facets have a scleral cone adherent to their inner surfaces. I have at present examined only dried specimens, but hope to continue the investigation.

The region of binocular vision.—In most insects the field of vision in the two eyes has a common portion in the peripheral region in the vicinity of the mouth ; in this region the radius of curvature of the cornea is very short. It is, therefore, only adapted for the acute vision of very near objects. It is chiefly developed in predaceous insects. It probably serves the insect in judging of the distance of objects from the mouth.

J. Müller, in his classical work 'On the Comparative Physiology of Vision,' states that no portion of the compound eye in any of the insects he examined corresponded in the direction of the axis of the facets with the eye of the opposite side ; but he does not appear to have examined the eyes with sufficient minuteness to have been able to detect the slight overlapping of the two fields which I have described.

Description of the Plates.

PLATE 52.

Fig. 1. The ocellus of *Eristalis*.
Fig. 1*a*. One of the rod cells of the same.
Fig. 2. A vertical section of a portion of the compound eye of the common Crane-fly.
Fig. 3. A vertical section of a portion of the chamber of the compound eye of an immature Crane-fly, showing the rod cells and the facellus.
Fig. 3*a*. One of the facets of the same eye seen from without, showing the extremities of the rod cells beneath.
Fig. 4. A transverse section through the middle of the facellus.
Fig. 4*a*. A section through the lower extremity of the facellus.
Fig. 5. The stemon and nervous retina of the same.
Fig. 6. A vertical section of a portion of the compound eye of the female of *Formica rufa* (from a specimen preserved in spirit).
Fig. 7. The rod cells, facellus, and stemon of the eye of the Wasp.

Fig. 7*a*. A transverse section immediately below the cornea of the eye of *Formica rufa*.
Fig. 8. Transverse section through the rod cells of the eye of the Wasp.
Fig 8*a*. A transverse section through the stemonata of the eye of *F. rufa*.
(Figs. 9 to 13, inclusive, are in Plate 53.)
Fig. 14. The chamber from the eye of the Blowfly.
Fig. 15. The rhabdia of the same.
Fig. 15*a*. A portion of one of the rhabdia of the same.
Fig. 16. Transverse sections through the rhabdia of the same.
  a and *b*. Compound rhabdia.
  c and *d*. Simple rhabdia.
Fig. 17. The inner extremities of the rhabdia and the nuclear masses of pigment seen from the retinal surface of the rhabdia.
Fig. 18. The rhabdia and chambers of *Tabanus*, from a dried insect.
Fig. 19. A portion of two rhabdia from the same.
Fig. 20. The chambers of the eye of *Tabanus*.

The details in all the figures are drawn with a Nachet $\frac{1}{16}$th immersion.

PLATE 53.

Fig. 9. Section through one of the elements of the compound eye of *Eristalis*.
Fig. 9*a*. Four facets of the cornea, showing SEMPER'S nuclei.
Fig. 10. Details of the compound eye of *Eristalis*.
  a. Transverse section through the tetrasome.
  b. Transverse section immediately below the tetrasome.
  c. Section through the lower end of one of the rhabdia from near the periphery of the eye.
  d. Section just above the pigment cells of the rhabdion.
  e. Section just below the pigment cells.
  f, *g*, and *i*. Sections through the rhabdia.
  h. Lower end of one of the rhabdia, with six pigmented nuclei from near the periphery of the eye.
  k. Section through a facellus from the facelloid layer of the retina.
  l. Lower end of a compound rhabdion from near the outer edge of the eye.
Fig. 11. A portion of one of the triquetrous rhabdia.
Fig. 12. Lower end of the rhabdia to show their connexion with the nervous retina.
Fig. 13. The nervous retina of *Eristalis*.
Fig. 21. Two chambers from the eye of *Agrion virgo*.
Fig. 21*a*. Four facets from the same eye seen from the outer surface.
Fig. 22. A portion of one of the rhabdia.

Fig. 23. Transverse sections of the rhabdia.
  a. Section throughout the outer extremity of a rhabdion.
  b and *c*. Through the middle of the rhabdion.
Fig. 24. The ganglionic retina of the same insect.
(For figs. 25, 26, and 28, see Plate 54.)
Fig. 27. The tetrasome of *Acridium* seen from the surface of the cornea.
Fig. 29. The tetrasome of *Vanessa atalanta*.

PLATE 54.

Fig. 25. The chamber of the eye of the nymph of *Acridium*.
Fig. 26. The same, from an imago of the same.
Fig. 28. Two chambers from the eye of *Vanessa atalanta*.
Fig. 30. Four chambers seen from the surface of the cornea. For the sake of clearness, all the parts are only represented in one of the segments.
Fig. 31. The rhabdion of the same.
Fig. 32. Transverse section through some of the facelli of the same eye.
Fig. 33. The ganglionic retina.
Fig. 34. Vertical section through one of the facelli of the same eye.
Fig. 35. The chamber of the eye of *Colias*.
Fig. 36. A vertical section of a portion of the eye of a Hawk Moth from the pupa.
Fig. 36*a*. One of the cones of the same.
Fig. 37. One of the cones of the compound eye of a Noctuid.
Fig. 38. A vertical section of a portion of the eye of another species of Noctuid.
Fig. 39. A similar portion of the eye of a third species of Noctuid.
Fig. 40. A transverse section through the compound part of the rhabdion, from the same insect as the last preparation.
Fig. 41. A transverse section through sheaths of the rhabdia of a Noctuid Moth.
Fig. 42. The cones seen from the cornea, from the same specimen as fig. 38.
Fig. 43. An outline of the eye of *Vespa* to show the direction in which the principal meridians were drawn.

DIAGRAM. (Page 596.)

Fig. 44. Scales of fractions of an inch to which the succeeding diagrams are drawn.
(For figs. 44 to 54, see page 596.)
Fig. 45. The principal horizontal section of the cornea of *Vespa* (scale A).
Fig. 46. The vertical meridian of the same eye (scale A).
Fig. 47. The curvature of the principal vertical meridian of a species of *Syrphus* (scale B).
Fig. 48. The curvature of the principal horizontal meridian of *Æschna grandis* (scale A).
Fig. 49. The principal vertical meridian of the same (scale A).
Fig. 50. The horizontal meridian of *Tabanus bovinus* (scale A).
Fig. 51. The principal vertical meridian of *Musca vomitoria* (scale B).
Fig. 52. The vertical meridian of the eye of a Noctuid Moth (scale B).
Fig. 53. The principal vertical meridian of the eye of *Colias edusa* (scale B).
Fig. 54. The same, a meridian 30° from the last (scale B).
Fig. 55. The same, a meridian at right angles to the last (scale B).

XIX. *A Tenth Memoir on Quantics.*

By A. Cayley, *F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge.*

Received June 12,—Read June 20, 1878.

The present Memoir, which relates to the binary quintic $(* \between x, y)^5$, has been in hand for a considerable time: the chief subject-matter was intended to be the theory of a canonical form discovered by myself and which is briefly noticed in Salmon's 'Higher Algebra,' 3rd Ed. (1876), pp. 217, 218; writing $a, b, c, d, e, f, g \ldots u, v, w$ to denote the 23 *covariants* of the quintic, then a, b, c, d, f are connected by the relation $f^2 = -a^3d + a^2bc - 4c^3$; and the form contains these covariants thus connected together, and also e; it, in fact, is $(1, 0, c, f, a^2b - 3c^2, a^2e - 2cf \between x, y)^5$.

But the whole plan of the Memoir was changed by Sylvester's discovery of what I term the Numerical Generating Function (N.G.F.) of the covariants of the quintic, and my own subsequent establishment of the Real Generating Function (R.G.F.) of the same covariants. The effect of this was to enable me to establish for any given degree in the coefficients and order in the variables, or as it is convenient to express it, for any given deg-order whatever, a selected system of powers and products of the covariants, say a system of "segregates:" these are asyzygetic, that is, not connected together by any linear equation with numerical coefficients; and they are also such that every other combination of covariants of the same deg-order, say every "congregate" of the same deg-order, can be expressed (and that, obviously, in one way only) as a linear function, with numerical coefficients, of the segregates of that deg-order. The number of congregates of a given deg-order is precisely equal to the number of the independent syzygies of the same deg-order, so that these syzygies give in effect the congregates in terms of the segregates: and the proper form in which to exhibit the syzygies is thus to make each of them give a single congregate in terms of the segregates: viz., the left hand side can always be taken to be a monomial congregate $a^\alpha b^\beta \ldots$ or, to avoid fractions, a numerical multiple of such form; and the right hand side will then be a linear function, with numerical coefficients, of the segregates of the same deg-order. Supposing such a system of syzygies obtained for a given deg-order, any covariant function (rational and integral function of covariants) is at once expressible as a linear function of the segregates of that deg-order: it is in fact only necessary to substitute therein for every monomial congregate its value as a linear function of the segregates. Using the word covariant in its most general sense, the conclusion thus is that every covariant can be expressed, and that in one way only, as a linear function of segregates, or say in the segregate form.

Reverting to the theory of the canonical form, and attending to the relation $f^2=-a^3d+a^2bc-4c^3$, it thereby appears that every covariant multiplied by a power of the quintic itself a, can be expressed, and that in one way only, as a rational and integral function of the covariants a, b, c, d, e, f, linear as regards f: say every covariant multiplied by a power of a can be expressed, and that in one way only, in the "standard" form: as an illustration take $a^2h=6acd+4bc^2+ef$. Conversely an expression of the standard form, that is, a rational and integral function of a, b, c, d, e, f, linear as regards f, not explicitly divisible by a, may very well be really divisible by a power of a (the expression of the quotient of course containing one or more of the higher covariants g, h, &c.), and we say that in this case the expression is divisible, and has for its divided form the quotient expressed as a rational and integral function of covariants. Observe that in general the divided form is not perfectly definite, only becoming so when expressed in the before-mentioned segregate form, and that this further reduction ought to be made: there is occasion, however, to consider these divided forms, whether or not thus further reduced, and moreover it sometimes happens that the non-segregate form presents itself, or can be expressed, with integer numerical coefficients, while the coefficients of the corresponding segregate form are fractional.

The canonical form is peculiarly convenient for obtaining the expressions of the several derivatives (GORDAN's *Uebereinanderschiebungen*) $(a, b)^1$, $(a, b)^2$, &c. (or as I propose to write them $ab1$, $ab2$, &c.), which can be formed with two covariants, the same or different, as rational and integral functions of the several covariants. It will be recollected that in GORDAN's theory these derivatives are used in order to establish the system of the 23 covariants: but it seems preferable to have the system of covariants, and by means of them to obtain the theory of the derivatives.

I mention at the end of the Memoir two expressions (one or both of them due to SYLVESTER) for the N.G.F. of a binary sextic.

The several points above adverted to are considered in the Memoir; the paragraphs are numbered consecutively with those of the former Memoirs upon Quantics.

The Numerical and Real Generating Functions.—Article Nos. 366 to 374, and Table No. 96.

366. I have in my Ninth Memoir (1871) given what may be called the Numerical Generating Function (N.G.F.) of the covariants of a quartic ; this was

$$A(x)=\frac{1-a^6x^{12}}{1-ax^4 .\ 1-a^2x^4 .\ 1-a^2 .\ 1-a^3 .\ 1-a^3x^6},$$

the meaning being that the number of asyzygetic covariants $a^\theta x^\mu$, of the degree θ in the coefficients and order μ in the variables, or say of the deg-order $\theta.\mu$ is equal to the coefficient of $a^\theta x^\mu$ in the development of this function. And I remarked that the formula indicated that the covariants were made up of (ax^4, a^2x^4, a^2, a^3, a^3x^6) the quartic itself, the Hessian, the quadrinvariant, the cubinvariant, and the cubicovariant, these being connected by a syzygy a^6x^{12} of the degree 6 and order 12. Calling these covariants a, b, c, d, e, so that these italic small letters stand for covariants,

Deg-order.	
1.4	a
2.0	b
2.4	c
3.0	d
3.6	e

then it is natural to consider what may be called the Real Generating Function (R.G.F.) : this is

$$\frac{1-e^2}{1-a\ .\ 1-b\ .\ 1-e\ .\ 1-d\ .\ 1-e};$$

the development of this contains, as it is easy to see, only terms of the form $a^\alpha b^\beta c^\gamma d^\delta$ and $a^\alpha b^\beta c^\gamma d^\delta e$, each with the coefficient $+1$, so that the number of terms of a given deg-order $\theta.\mu$ is equal to the coefficient of $a^\theta x^\mu$ in the first-mentioned function : and these terms of a given deg-order represent the asyzygetic covariants of that deg-order : any other covariant of the same deg-order is expressible as a linear function of them. For instance, deg-order 6.12, the terms of the R.G.F. are a^3d, a^2bc, c^3 : there is one more term e^2 of the same deg-order ; hence e^2 must be a linear function of these : and in fact $e^2=-a^3d+a^2bc-4c^3$, viz., this is the equation $\Phi^2=-U^3J+U^2IH-4H^3$.

367. SYLVESTER obtained an expression for the N.G.F. of the quintic: this is

$$\frac{\begin{array}{rl} a^0\,. & 1 \\ +a^3\,. & x^3+x^5+x^9 \\ +a^4\,. & x^4+x^6 \\ +a^5\,. & x+x^3+x^7-x^{11} \\ +a^6\,. & x^2+x^4 \\ +a^7\,. & x+x^5-x^9 \\ +a^8\,. & x^2+x^4 \\ +a^9\,. & x^3+x^5-x^7 \\ +a^{10}. & x^2+x^4-x^{10} \\ +a^{11}. & x+x^3-x^9 \\ +a^{12}. & x^2-x^8-x^{10} \\ +a^{13}. & x-x^7-x^9 \\ +a^{14}. & x^4-x^6-x^8 \\ +a^{15}. & -x^7-x^9 \\ +a^{16}. & x^2-x^6-x^{10} \\ +a^{17}. & -x^7-x^9 \\ +a^{18}. & 1-x^4-x^8-x^{10} \\ +a^{19}. & -x^5-x^7 \\ +a^{20}. & -x^2-x^6-x^8 \\ +a^{23}. & -x^{11} \end{array}}{1-ax^5\,.\,1-a^2x^2\,.\,1-a^2x^6\,.\,1-a^4\,.\,1-a^8\,.\,1-a^{12}}$$

viz., expanding this function in ascending powers of a, x, then if a term is $\mathrm{N}a^\theta x^\mu$, this means that there are precisely N asyzygetic covariants of the deg-order $\theta.\mu$.

368. It is known that the number of the irreducible covariants of the binary quintic is $=23$; representing these by the letters a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w (a the quintic itself), the deg-orders of these, and the references to the tables which give them are

Deg-order.		Tab.	Mem.
1.5	a		
2.2	b	13	2
,,.6	c	14	,,
3.3	d	15	,,
,,.5	e	16	,,
,,.9	f	17	,,
4.0	g	18	,,
,,.4	h	19	,,
,,.6	i	20	,,
5.1	j	21	,,
,,.3	k	22	,,
,,.7	l	23	,,
6.2	m	83	8
,,.4	n	84	,,
7.1	o	90	9
,,.5	p	91	,,
8.0	q	25	,,
,,.2	r	92	,,
9.3	s	*	
11.1	t	94	9
12.0	u	29	3
13.1	v	95	9
18.0	w	29A	5

Starting from the foregoing expression of the N.G.F. of the quintic, we can, instead of each term $a^\theta x^\mu$, introduce a covariant or product of covariants of the proper deg-order $\theta.\mu$: the mode of doing this depends of course on the different admissible partitions of θ, μ, and it is for some of the terms very indeterminate: for instance, a^5x^{11} is ai, bf, or ce. I found it possible to perform the whole process so as to satisfy a condition which will be presently referred to; and I found

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* See end of Memoir. The S of Table 93 has the value $-96(\text{D}, \text{M})+16\,\text{BO}-7\text{GK}$, but it is better to use the simple value $-(\text{D}, \text{M})$; and the S of the present Memoir has this value, say $\text{S}=-(d, m)$.

R.G.F. of quintic=		Deg-orders.
1 .	$1-b^5$	0.0—10.10
$+d$.	$1-ag^2$	3.3—12. 8
$+e$.	$1-b^2$	3.5— 7. 9
$+f$.	$1-b$	3.9— 5.11
$+h$.	$1-ag^2$	4.4—13. 9
$+i$.	$1-b^2g$	4.6—12.10
$+j$.	$1-ag^2$	5.1—14. 6
$+k$.	$1-b^2$	5.3— 9. 7
$+l$.	$1-bg$	5.7—11. 9
$+m$.	$1-ag^2$	6.2—15. 7
$+n$.	$1-b^2g$	6.4—14. 8
$+o$.	$1-b^3$	7.1—13. 7
$+p$.	$1-b^2g$	7.5—15. 9
$+r$.	$1-b^2g$	8.2—16. 6
$+dj$.	$1-ag^2$	8.4—17. 9
$+s$.	$1-abg$	9.3—16.10
$+hj$.	$1-ag^2$	9.5—18.10
$+j^2$.	$1-ag^2$	10.2—19. 7
$+jk$.	$1-b^2g$	10.4—18. 8
$+t$.	$1-b^3$	11.1—17. 7
$+jm$.	$1-ag^2$	11.3—20. 8
$+jo$.	$1-bg$	12.2—18. 4
$+v$.	$1-b^5$	13.1—23.11
$+js$.	$1-bg$	14.4—20. 6
$+jt$.	$1-g$	16.2—20. 2
$+w$.	$1-a$	18.0—19. 5

$$1-a\ .\ 1-b\ .\ 1-c\ .\ 1-g\ .\ 1-q\ .\ 1-u$$

where observe that each negative term of the numerator is equal to a positive term into a power or product of terms a, b, g, contained in the denominator: this is the condition above referred to. The expansion thus consists only of terms each with the coefficient $+1$; for instance, a part of the function is

$$\frac{s\,(1-abg)}{1-a\ .\ 1-b\ .\ 1-c\ .\ 1-g\ .\ 1-q\ .\ 1-u},\ =\frac{s}{1-c\ .\ 1-q\ .\ 1-u}\cdot\frac{1-abg}{1-a\ .\ 1-b\ .\ 1-g},$$

where the first factor is the entire series of terms $sc^{\delta}q^{\epsilon}u^{\zeta}$, and the second factor is the series of terms $a^{\alpha}b^{\beta}g^{\gamma}$ omitting only those terms which are divisible by abg: and in the product of the two factors the terms are all distinct, so that the coefficients are still each $=1$. The same thing is true for every other pair of numerator terms: and (since the terms arising from each such pair are distinct from each other) in the expansion of the entire function the coefficients are each $=+1$. Hence (as in the case of the quartic) for any given deg-order, the terms in the expansion of the R.G.F. may be taken for the asyzygetic covariants of that deg-order; and if there are any other terms of the same deg-order, each of these must be a linear function, with numerical coefficients, of these asyzygetic covariants: thus deg-order 6.11, the expansion contains only the terms a^2h, acd, bc^2; there is besides a term of the same deg-order, ef, which is not a term of the expansion, and hence ef must be a linear function of a^2h, acd, bc^2; we in fact have $ef=a^2h-6acd-4bc^2$.

The terms in the expansion of the R.G.F. may be called "segregates," and the terms not in the expansion "congregates;" the theorem thus is: every congregate is a linear function, with determinate numerical coefficients, of the segregates of the same deg-order.

369. I stop to remark that the numerator of the R.G.F. may be written in the more compendious form

$$\begin{aligned}
&(1-b^5)(1-v)+(1-b^3)(o+t)+(1-b^2)(e+k)+(1-b)f\\
&+(1-ag^2)(d+h+j+m+dj+hj+j^2+jm)\\
&+(1-bg)(l+jo+js)\\
&+(1-b^2g)(i+n+p+jk)\\
&+(1-abg)s\\
&+(1-g)jt\\
&+(1-a)w\,;
\end{aligned}$$

but the first-mentioned form is, I think, the more convenient one.

370. It is to be noticed that the positive terms of the numerator are unity, the seventeen covariants $d, e, f, h, i, j, k, l, m, n, o, p, r, s, t, v, w$, and the products j into (d, h, j, k, m, o, s, t) where j^2 is reckoned as a product; in all, 26 terms. Disregarding the negative terms of the numerator the expansion would consist of these 26 terms, each multiplied by every combination whatever $a^\alpha b^\beta c^\gamma g^\delta q^\epsilon u^\zeta$ of the denominator terms a, b, c, g, q, u (which for this reason might be called "reiterative"): the effect of the negative terms of the numerator is to remove from the expansion certain of the terms in question, thereby diminishing the number of the segregates: thus as regards the terms belonging to unity, any one of these which contains the factor b^5 is not a segregate but a congregate: and so as regards the terms belonging to d, any one of these which contains the factor ag^2 is a congregate: and the like in other cases.

For a given deg-order we have a certain number of segregates and a certain number of congregates: and the number of independent syzygies of that deg-order is precisely equal to the number of congregates: viz., each such syzygy may be regarded as giving a congregate in terms of the segregates: we have on the left hand side a congregate, or, to avoid fractions, a numerical multiple of the congregate, and on the right hand side a linear function, with numerical coefficients, of the segregates.

371. The syzygy is irreducible or reducible; and in the latter case it is, or is not, simply divisible: viz., if the congregate on the left hand side contains any congregate factor (the other factor being literal) then the syzygy is reducible: it is, in fact, obtainable from the syzygy (of a lower deg-order) which gives the value of such congregate factor. But there are here two cases; multiplying the lower syzygy by the proper factor, the right hand side may still contain segregates only, and then no further step is required: the original syzygy is nothing else than this lower syzygy, each side multiplied by the factor in question, and it is accordingly said to be simply divisible (S.D.). But contrariwise, the right hand side, as multiplied, may contain congregates which have to be replaced by their values in terms of the segregates of the same deg-order: the resulting expression is then no longer explicitly divisible by the introduced factor: and the original syzygy, although arising as above from a lower syzygy, is not this lower syzygy each side multiplied by a factor: viz., it is in this case not simply divisible.

For example (see the subsequent Table No. 96, under the indicated deg-orders) (6.6), from the syzygy $9d^2=aj-b^3+2bh-cg$, we deduce (7.11) the syzygy $9ad^2=a^2j-ab^3+2abh-acg$, which (all the terms on the right hand being segregates) requires no further reduction: it is a reducible and simply divisible syzygy. But we have (6.8) a syzygy giving de, and also (6.10) a syzygy giving e^2; multiplying the former of these by e or the latter of them by d, we obtain a value of de^2, but in each case the right hand side contain terms which are not segregates, and have thus to be further reduced; the final formula (9.13) is

$$3de^2=-4a^2bj+3a^2dg+4ab^4-8ab^2h+4abcg-12b^2cd,$$

which is not divisible by any factor: the syzygy is thus reducible, but not simply divisible.

A syzygy which is not in the sense explained reducible, is said to be irreducible.

372. The number of irreducible syzygies is obviously finite: it has, however, the large value 179 as appears from the annexed diagram, showing the congregates determined by these several syzygies, and the deg-orders of the syzygies:—

		1	d	e	f	h	i	j	k	l	m	n	o	p	r	s	t	v	w	j^2	
0.0	1	b^5 10.10	ag^2 12.8	b^2 7.9	b 5.11	ag^2 13.9	b^2g 12.10	ag^2 14.6	b^2 9.7	bg 11.9	ag^2 15.7	b^2g 14.8	b^3 13.7	b^2g 15.9	b^2g 16.6	abg 16.10	b^3 17.7	b^5 23.11	a 19.5		1
3.3	d		6.6	6.8	6.12	7.7	7.9	ag^2 17.9	8.6	8.10	9.5	9.7	10.4	10.8	11.5	12.6	14.4	16.4	21.3	13.5	d
3.5	e			6.10	6.14	7.9	7.11	8.6	8.8	8.12	9.7	9.9	10.6	10.10	11.7	12.8	14.6	16.6	21.5		e
3.9	f				6.18	7.13	7.15	8.10	8.12	8.16	9.11	9.13	10.10	10.14	11.11	12.12	14.10	16.10	21.9		f
4.4	h					8.8	8.10	ag^2 18.10	9.7	9.11	10.6	10.8	11.5	11.9	12.6	13.7	15.5	17.5	22.4	14.6	h
4.6	i						8.12	9.7	9.9	9.13	10.8	10.10	11.7	11.11	12.8	13.9	15.7	17.7	22.6		i
5.1	j							ag^2 19.7	b^2g 18.8	10.8	ag^2 20.8	11.5	bg 18.4	12.6	13.3	bg 20.6	g 20.2	18.2	23.1	15.3	j
5.3	k								10.6	10.10	11.5	11.7	12.4	12.8	13.5	14.6	16.4	18.4	23.3	15.5	k
5.7	l									10.14	11.9	11.11	12.8	12.12	13.9	14.10	16.8	18.8	23.7		l
6.2	m										12.4	12.6	13.3	13.7	14.4	15.5	17.3	19.3	24.2	16.4	m
6.4	n											12.8	13.5	13.9	14.6	15.7	17.5	19.5	24.4		n
7.1	o												14.2	14.6	15.3	16.4	18.2	20.2	25.1	17.3	o
7.5	p													14.10	15.7	16.8	18.6	20.6	25.5		p
8.2	r														16.4	17.5	19.3	21.3	26.2		r
9.3	s															18.6	20.4	22.4	27.3	19.5	s
11.1	t																22.2	24.2	29.1	21.3	t
13.1	v																	26.2	31.1		v
18.0	w																		36.0		w

Each term inside this diagram is a deg-order indicating the congregate determined by an irreducible syzygy: viz., the congregate is the product of the outside covariants in the line and column containing the deg-order, and of the literal factor (if any) placed immediately above the deg-order. Thus, line d and column i, 7.9 indicates the congregate di, but, same line and column j, 17.9 indicates the congregate $dj.\ ag^2, =adg^2j$.

Observe as regards the foregoing diagram, that dj^2 is irreducible (since neither dj nor j^2 is segregate) and similarly j^2h, j^3, &c., are irreducible: we have thus the last or j^2 column of the diagram.

The simply divisible syzygies are infinite in number, as are also the reducible syzygies not simply divisible. There is obviously no use in writing down a simply divisible syzygy; but as regards the reducible syzygies not simply divisible, these require a calculation, and it is proper to give them as far as they have been obtained.

373. The following Table, No. 96, replaces Tables 88 and 89 of my Ninth Memoir. The arrangement is according to deg-orders, and the table is complete up to the deg-order 8.40: it shows for each deg-order the segregate covariants, and also the congregate covariants (if any), and the syzygies which are the expressions of these in terms of the segregates. When there are only segregates these are given in the same horizontal line with the deg-order; for instance, $\mid 5.9 \mid ab^2, ah, cd,$ shows that for the deg-order 5.9 the only covariants are the segregates ab^2, ah, cd; but when there are also congregates, the segregates are arranged in the same horizontal line with the deg-order, and the congregates, each in its own horizontal line, together with its expression as a linear function of the segregates: thus $\left|\begin{array}{c|c|cc} 5.11 & * & ai & ce \\ \hline & bf & -1 & +1 \end{array}\right.$, the segregates are ai, ce, and there is a congregate bf which is a linear function of these, $=-ai+ce$. The table gives the irreducible syzygies and also the reducible syzygies which are not simply divisible, but the simply divisible syzygies are indicated each by a reference to the divided syzygy which occurs previously in the table.

374. Any syzygy might of course be directly verified by substituting for the several covariants contained therein their expressions in terms of the coefficients and facients of the quintic. But it is to be remarked that among the syzygies, or easily deducible from them, we have (6.18) the before-mentioned equation $f^2=-a^3d+a^2bc-4c^3$, and also a set of 17 syzygies, the left hand sides of which are the covariants $g, h \ldots u, v, w$, each multiplied by a or a^2, and which lead ultimately to the standard expressions of these covariants respectively, viz., each covariant multiplied by a proper power of a can be expressed as a rational and integral function of a, b, c, d, e, f, linear as regards f: supposing them thus expressed, a far more simple verification of any syzygy would consist in substituting therein for the several covariants their expressions in the standard form, reducing if necessary by the equation $f^2=-a^3d+a^2bc-4c^3$: but of course, as to the syzygies used for obtaining the standard forms, this is only a verification if the standard forms have been otherwise obtained, or are assumed to be correct.

The 17 syzygies above referred to are

Deg-ord.

6.10 $a^2g = 12abd+4b^2c+e^2,$

6.14 $a^2h = 6acd+4bc^2+ef,$

5.11 $ai = -bf+ce,$

6.6 $aj = b^3-2bh+cg+9d^2,$

6.8 $ak = -2bi+3de,$

6.12 $al = 2ci-3df,$

7.7 $am = -2b^2d-cj+3dh,$

7.9 $an = b^2e-6bl-2ck-fg,$

8.6 $ao = 2bn+ej,$

8.10 $ap = -2cn-fj,$

9.5 $aq = -2b^2j+bdg-12dm+hj,$

9.7 $ar = b^2k+bp-co+hk,$

10.8 $as = 3bdk+3dp+2im,$

12.6 $at = bjk+jp-2mn,$

13.5 $18au = 2agq+b^2gj+6bmj-6dj^2-ghj+no,$

14.6 $3av = 2b^3q-8b^2j^2-2b^2gm+6bdgj-12bm^2+3et,$

19.5 $18aw = 3b^2gt+b^2qo-4bj^2o-bgmo+18bmt+3dgjo-18djt-3ght-6m^2o,$

the last four of these being, however, beyond the limits of the table: the expressions of g, h, i are here in the standard form: the standard forms of the other covariants j, k . . . u, v, w, will be given further on.

Table No. 96 (Segregates, Congregates, and Syzygies).

Deg ord.	Congs.	Segregates.
1. 1		1
3		
5		a
2. 0		
2		b
4		
6		c
8		
10		a^2
3. 1		
3		d
5		e
7		ab
9		f
11		ac
13		
15		a^3

Table No. 96 (continued).

Deg-ord.	Congs.	Segregates.
4. 0		g
2		
4		b^2, h
6		i
8		ad, bc
10		ae
12		a^2b, c^2
14		af
16		a^3c
18		
20		a^5
5. 1		j
3		k
5		ag. bd
7		be, l
9		ab^2, ah, cd
11	*	ai, ce
	bf	-1 $+1$
13		a^2d, abc
15		a^2e, cf
17		a^3b, ac^2
19		a^2f
21		a^3c
23		
25		a^5
6. 0		
2		bg, m
4		n
·6	*	aj, b^3, bh, cg
	$d^2.9$	$+1$ -1 $+2$ -1
8	*	ak, bi
	$de.3$	$+1$ $+2$
10	*	a^2g, abd, b^2c, ch
	e^2	$+1$ -12 -4 .
12	*	abe, al, ci
	$df.3$	. -1 $+2$
14	*	a^2b^2, a^2h, acd, bc^2
	ef	. $+1$ -6 -4
16	*	a^2i, ace
	abf	. . . . S.D. 5.11, bf

Table No. 96 (continued).

Deg-ord.	Congs.	Segregates.					
6. 18	*	a^3d,	a^2bc,	c^3			
	f^2	−1	+1	−4			
20		a^3e,	acf				
22		a^4b,	a^2c^2				
24		a^3f					
26		a^4c					
28							
30		a^6					
7. 1		o					
3		bj,	dg				
5		bk,	eg,	p			
7	*	abg,	am,	b^2d,	cj		
	$dh.3$	0	+1	+2	+1		
9	*	an,	bl,	ck,	fg		
	b^2e	+1	+6	+2	+1		
	$di.3$	0	+1	+1			
	eh	0	+4	+2	+1		
11	*	a^2j,	ab^3,	abh,	acg,	bcd	
	ad^2	.	.	.	.	.	. . . . . S.D. 6.6, d^2
	ei	.	.	−1	+1	−6	
13	*	a^2k,	abi,	bce,	cl		
	ade	.	.	.	.		. . . . . S.D. 6.8, de
	b^2f	.	.	.	.		. . . . . S.D. 5.11, bf
	$fh.3$	−1	−2	+3	−6		
15	*	a^3g,	a^2bd,	ab^2c,	ach,	c^2d	
	ae^2	.	.	.	.	.	. . . . . S.D. 6.10, e^2
	fi	.	+1	−1	+1	−6	
17	*	a^2be,	a^2l,	aci	c^2e		
	adf	.	.	.	.		. . . . . S.D. 6.12, df
	bcf	.	.	.	.		. . . . . S.D. 5.11, bf
19	*	a^3b^2,	a^3h,	a^2cd,	abc^2		
	aef	.	.	.	.		. . . . . S.D. 6.14, ef
21	*	a^3i,	a^2ce,	c^2f			
	a^2bf	.	.	.			. . . . . S.D. 5.11, bf
23	*	a^4d,	a^3bc,	ac^3			
	af^2	.	.	.			. . . . . S.D., 6.18, f^2

Table No. 96 (continued).

Deg-ord.	Congs.	Segregates.							
25		a^4e,	a^2cf						
27		a^5b,	a^3c^2						
29		a^4f							
31		a^5c							
33									
35		a^7							
8. 0		g^2,	q						
2		r							
4		b^2g,	bm,	dj,	gh				
6	*	ao,	bn,	gi					
	$dk.3$	+1	−3	−1					
	ej	+1	−2						
8	*	abj,	adg,	b^4,	b^2h,	bcg,	cm		
	bd^2								S.D. 6.6, d^2
	ck	−4	+3	+4	−6	+2			
	$h^2.3$	+4	−3	−4	+8	−1	+12		
10	*	abk,	aeg,	ap,	b^2i,	cn			
	bde								S.D. 6.8, de
	$dl.9$	+2	.	+3	+1	+3			
	fj	.	.	−1		−2			
	$hi.3$	+1	.	.	+2	−3			
12	*	a^2bg,	a^2m,	ab^2d,	acj,	b^3c,	bch,	c^2g	
	adh								S.D. 7.7, dh
	be^2								S.D. 6.10, e^2
	cd^2								S.D. 6.6, d^2
	el	.	−1	−2	+1	−2	+2		
	fk	.	+1	.	−1	+4	−6	+2	
	i^2	.	.	−1	.	+1	−2	+1	
14	*	a^2n,	abl,	ack,	afg,	bci			
	ab^2e								S.D. 7.9, b^2e
	adi								S.D. 7.9, di
	aeh								S.D. 7.9, eh
	bdf								S.D. 6.12, df
	cde								S.D. 6.8, de
16	*	a^3j,	a^2b^3,	a^2bh,	a^2cg,	$abcd$,	b^2c^2,	c^2h	
	a^2d^2								S.D. 6.6, d^2
	aei								S.D. 7.11, ei
	bef								S D. 6.14, ef
	ce^2								S.D. 6.10, e^2
	$fl.3$	+1	−1	+2	−1	−3	−6	+6	

Table No. 96 (continued).

Deg-ord.	Congs.	Segregates.							
8. 18	*	a^3k,	a^2bi,	$abce$,	acl,	c^2i			
	a^2de								S.D. 6.8, de
	ab^2f								S.D. 5.11, bf
	afh								S.D. 7.13, fh
	cdf								S.D. 6.12, df
20	*	a^4g,	a^3bd,	a^2b^2c,	a^2ch,	ac^2d,	bc^3		
	a^2e^2								S.D. 6.10, e^2
	afi								S.D. 7.15, fi
	bf^2								S.D. 6.18, f^2
	cef								S.D. 6.14, ef
22	*	a^3be,	a^3l,	a^2ci,	$abcf$,	ac^2e			
	a^2df								S.D. 6.12, df
24	*	a^4b^2,	a^4h,	a^3cd,	$a^2b^2c^2$,	c^4			
	a^2ef								S.D. 6.14, ef
	cf^2								S.D. 6.18, f^2
26	*	a^4i,	a^3ce,	ac^2f					
	a^3bf								S.D. 5.11, bf
28	*	a^5d,	a^4bc,	a^2c^3					
	a^2f^2								S.D. 6.18, f^2
30		a^5e,	a^3cf						
32		a^6b,	a^4c^2						
34		a^5f							
36		a^6c							
38									
40		a^8							
9. 1		gj							
3		bo,	gk,	s					
5	*	ag^2,	aq,	b^2j,	bdg,	hj			
	dm.12	.	−1	−2	+1	+1			
7	*	ar,	beg,	bp,	co,	gl			
	b^2k.3	+1	.	−5	−1	+1			
	dn.3	−1	.	−1	−1				
	em.3	+2	.	+2	+1	−1			
	hk.3	+2	.	+2	+4	−1			
	ij	.	.	+1	+1				
9	*	ab^2g,	abm,	adj,	agh,	b^3d,	bcj,	cdg	
	bdh.3	.	+1	.	.	+2	+1		
	d^3.27	.	+2	+3	.	+1	+2	−3	
	en	+1	+6	.	−1	.	+2		
	ik	.	−1	.	.	.	−3	+3	

Table No. 96 (continued).

Deg-ord.	Congs.	Segregates.										
9. 11	*	a^2o,	abn,	agi,	b^2l,	bck,	ceg,	cp				
	adk	.	.	.	.	.	.	.				S.D. 8.6, dk
	aej	.	.	.	.	.	.	.				S.D. 8.6, ej
	b^3e	.	+1	−1	+6	+2	+1					
	bdi	.	.	.	.	.	.	.				S.D. 7.9, bi
	beh	.	.	−1	+4	+2	+1					
	bfg	.	.	.	.	.	.	.				S.D. 5.11, bf
	d^2e.9	+1	−3	−1	+2	+2						
	fm.3	+1	+3	+1	.	−3	.	−3				
	hl.3	+1	−3	−1	+2	+5	.	−6				
13	*	a^2bj,	a^2dg,	ab^4,	ab^2h,	$abcg$,	acm,	b^2cd,	c^2j			
	abd^2	.	.	.	.	.	.	.	.			S.D. 6.6, d^2
	aek	.	.	.	.	.	.	.	.			S.D. 8.8, ek
	ah^2	.	.	.	.	.	.	.	.			S.D. 8.8, h^2
	bei	.	.	.	.	.	.	.	.			S.D. 7.11, ei
	cdh	.	.	.	.	.	.	.	.			S.D. 7.7, dh
	de^2.3	−4	+3	+4	−8	+4	.	−12				
	fn	−2	+1	+2	−3	+1	−2	.	+2			
	il	−1	.	+1	−2	+1	−3	−3	+3			
10. 0												
2		bg^2,	bq,	gm,	j^2							
4	*	br,	gn,	jk								
	do.3	+2		−1								
6	*	agj,	b^3g.	b^2m,	bdj,	bgh,	cg^2,	cq				
	d^2g	.	.	.	.	.	.	.				S.D. 6.6, d^2
	co	+1	+2	+12	−12	−2						
	hm.3	−1	+1	+ 6	+12	−2	+1	−3				
	k^2	+1	−1	− 4	−12	+2	−1					
8	*	abo,	agk,	as,	b^2n,	bgi,	cr					
	bdk	.	.	.	.	.	.					S.D. 8.6, dk
	bej	.	.	.	.	.	.					S.D. 8.6, ej
	deg	.	.	.	.	.	.					S.D. 6.8, de
	dp.9	−5	.	+3	+15	+5	− 6					
	hn.3	−6	+1	.	+18	+5	−12					
	im.3	+1	.	.	− 3	−1	+ 3					
	jl.3	−5	.	+3	+15	+5	−12					
10	*	a^2g^2,	a^2q,	ab^2j,	$abdg$,	ahj,	b^3h,	b^2cg,	bcm,	cdj,	cgh.	
	adm	.	.	.	.	.	.	.	.	.	.	S.D. 9.5, dm
	b^5.8	.	+1	+10	−9	+3	+12	.	+32	−12	− 4	
	b^2d^2.72	.	−1	− 2	+9	−3	+ 4	−8	−32	+12	+ 4	
	bek.2	.	+1	+ 2	−3	+3	.	+4	+32	−12	− 4	
	bh^2.6	.	−1	− 2	+3	−3	+ 4	−2	− 8	+12	− 4	
	d^2h.27	.	−1	− 2	+3	.	+ 1	−2	− 8	+12	+ 1	
	e^2g	.	.	.	.	.	.	.	.	.	.	S.D. 6.10, e^2
	ep	.	.	.	.	−1	.	−2	−12	+ 6	+ 2	
	fo.2	.	+1	+ 2	−3	+7	.	+4	+24	−24	− 4	
	in.4	.	−1	− 2	+3	−5	.	.	.	+12		
	kl.4	.	+1	+ 2	−1	−1	.	.	+16	+12		

Table No. 96 (concluded).

Deg.-ord.	Congs.	Segregates.							
11. 1		go,	t						
3		bgj,	dg^2,	dq,	jm				
5	*	b^2o,	bgk,	bs,	eg^2,	eq,	gp		
	dr 18	-2	$+5$	-6	.	-3	$+3$		
	ho.3	-2	$+11$	-24	.	-3	$+6$		
	jn.1	$+1$	-3	$+6$	.	$+1$	-2		
	km.6	-2	$+5$	-12	.	-3	$+3$		
12. 0		g^3,	g^2q,	u					
2		gr,	jo						
4	*	b^2g^2,	b^2q,	bgm,	bj^2,	dgj,	g^2h,	hg	
	ko	.	-2	-2	-4	$+3$			
	m^2.12	.	$+2$	$+1$	$+4$	-3	.	-3	
13. 1		g^2j,	jq,	v					
3	*	bgo,	bt,	g^2k,	gs,	kg			
	jr.2	.	-2	.	$+1$	-1			
	mo.2	.	-4	.	$+1$	-1			
14. 0									
2		bq^3,	bgq,	bu,	g^2m,	gj^2,	mq,	o^2	
4	*	bgr,	bjo,	g^2n,	gjk,	js,	nq		
	dgo	.	.	.	.	.	.	.	S.D. 10.4, *do*
	dt.18	$+1$	$+2$	.	-1	$+6$	$+3$		
	mr.12	$+1$	$+2$	.	-1	.	$+3$		

Theory of the Canonical Form.—Article Nos. 375 to 381, and Tables Nos. 97 and 98.

375. As the small italic letters have been used to represent the covariants, different letters are required for the coefficients of the quintic, and (using also new letters for the facients) I take the quintic to be $(\mathrm{a, b, c, d, e, f} \between \xi, \eta)^5$. Considering a linear transformation of $\frac{1}{\mathrm{a}}(\mathrm{a, b, c, d, e, f} \between \xi, \eta)^5$, viz.

$$\frac{1}{\mathrm{a}}(\mathrm{a, b, c, d, e, f} \between \xi - \mathrm{b}\eta, \mathrm{a}\eta)^5$$

this is

$$
= \begin{array}{rl|cccccc}
 & & \xi^5 & 5\xi^4\eta & 10\xi^3\eta^2 & 10\xi^2\eta^3 & 5\xi\eta^4 & \eta^5 \\ \hline
 & & 1 & -b & +b^2 & -b^3 & +b^4 & -b^5) \\
+b & (& & 1 & -2b & +3b^2 & -4b^3 & +5b^4) \\
+ac & (& & & 1 & -3b & +6b^2 & -10b^3) \\
+a^2d & (& & & & 1 & -4b & +10b^2) \\
+a^3e & (& & & & & 1 & -5b) \\
+a^4f & (& & & & & & 1)
\end{array}
$$

which is

$$
= \left(\begin{array}{c|c|cc|cc|cc|cc}
1 & 0 & ac & +1 & a^2d & +1 & a^3e & +1 & a^4f & +1 \\
 & & b^2 & -1 & abc & -3 & a^2bd & -4 & a^3be & -5 \\
 & & & & b^3 & +2 & ab^2c & +6 & a^2b^2d & +10 \\
 & & & & & & b^4 & -3 & ab^3c & -10 \\
 & & & & & & & & b^5 & +4
\end{array}\right)\!\!\left(\xi, \eta\right)^5
$$

The values of a, b, c, d, e, f considered for a moment as denoting the leading coefficients of the several covariants ultimately represented by these letters respectively, are

a	b	c	d	e	f
a $+1$	ae $+1$	ac $+1$	ace $+1$	a^2f $+1$	a^2d $+1$
	bd -4	b^2 -1	ad^2 -1	abe $+5$	abc -3
	c^2 $+3$		b^2e -1	acd $+2$	b^3 -2
			bcd $+2$	b^2d $+8$	
			c^3 -1	bc^2 -10	

satisfying, as they should do, the relation $f^2 = -a^3d + a^2bc - 4c^3$.

Hence forming the values of $a^2b - 3c^2$ and $a^2e - 2cf$, it appears that the value of the last-mentioned quintic function is

$$
(1,\ 0,\ c,\ f,\ a^2b - 3c^2,\ a^2e - 2cf\,)\!\!(\xi, \eta)^5.
$$

Writing herein x, y in place of ξ, η, and now using a, b, c, d, e, f to denote, not the leading coefficients but the covariants themselves (a the original quintic, with ξ, η as facients), we have the form

$$
\mathrm{A},\ = (1,\ 0,\ c,\ f,\ a^2b - 3c^2,\ a^2e - 2cf\,)\!\!(x, y)^5,
$$

a new quintic, which is the canonical form in question: the covariants hereof (reckoning the quintic itself as a covariant) will be written A, B, C . . . V, W; and will be spoken of as capital covariants.

376. The fundamental property is: Every capital covariant, say I, has for its leading coefficient the corresponding covariant i multiplied into a power of a: and this follows as an immediate consequence of the foregoing genesis of A. The covariant i of the form $\frac{1}{\mathrm{a}}(\mathrm{a, b, c, d, e, f} \between \xi, \eta)^5$ has a leading coefficient $=\frac{1}{\mathrm{a}^4}(\mathrm{a^2cf - a^2de} + \&\mathrm{c.})$ which, when $a, b, c, d, e, f \ldots i$ denote leading coefficients, is $=i$ into a power of a: and upon substituting for the quintic the linear transformation thereof

$$(1,\ 0,\ c,\ f,\ a^2b-3c^2,\ a^2e-2cf \between \xi,\ \eta)^5,$$

(observing that in the transformation ξ, η into $\xi - \mathrm{b}\eta$, $\mathrm{a}\eta$ the determinant of substitution is $=\mathrm{a}$), the value is still $=i$ into a power of a; or using the relation $a=\mathrm{a}$, say the value is $=i$ into a power of a. Now the covariant i is the same function of the covariants a, b, c, d, e, f that the leading coefficient i is of the leading coefficients a, b, c, d, e, f; hence the italic letters now denoting covariants the leading coefficient still is $=i$ into a power of a: which is the above-mentioned theorem.

377. To show how the transformation is carried out, consider, for example, the covariant B: this is obtained from the corresponding covariant of $(\mathrm{a, b, c, d, e, f} \between \xi, \eta)^5$ that is

$$\left(\begin{array}{cc|cc|cc} \mathrm{ae} & 1 & \mathrm{af} & 1 & \mathrm{bf} & 1 \\ \mathrm{bd} & -4 & \mathrm{be} & -3 & \mathrm{ce} & -4 \\ \mathrm{c}^2 & +1 & \mathrm{cd} & +1 & \mathrm{d}^2 & +3 \end{array} \between \xi, \eta\right)^2.$$

by changing the variables, and for the coefficients

$$\begin{array}{lcccccc} & \mathrm{a}, & \mathrm{b}, & \mathrm{c}, & \mathrm{d}, & \mathrm{e}, & \mathrm{f} \\ \text{writing} & 1, & 0, & c, & f, & a^2b-3c^2, & a^2e-2cf\,; \end{array}$$

thus the coefficients are

First.	Second.		Third.
$1(a^2b-3c^2)$	$1(a^2e-2cf)$		$-4c(a^2b-3c^2)$
$+3c^2$	$+2cf$		$+3f^2$
$=a^2b$	$=a^2e$	$=$	$-4a^2bc+12c^3$
			$+3(-a^3d+a^2bc-4c^3)$
		$=$	$a^2(-3ad-bc)$

and we have thus the expression of B (see the Table No. 97); and similarly for the other capital covariants C, D . . . V, W: in every case the coefficients are obtained in the standard form; that is, as rational and integral functions of a, b, c, d, e, f, linear as regards f.

378. It will be observed that there is in each case a certain power of a which explicitly divides all the coefficients and is consequently written as an exterior factor:

disregarding these exterior factors, the leading coefficients for B, C, D, E, F are b, c, ad, e, f respectively; that for G is $12abd+4b^2c+e^2$, which must be $=g$ into a power of a, and (in Table 97) is given as $=a^2g$, similarly that for H is $6acd+4bc^2+ef$, which must be $=h$ into a power of a, and is given as $=a^2h$: and so in the other cases. The index of a is at once obtained by means of the deg-order, which is in each case inserted at the foot of the coefficient.

For A, B, C, E, F there is no power of a as an interior factor, and for the invariants G, Q, U we may imagine the interior factor thrown together with the exterior factor, ($\mathrm{G}=a^6g$, &c.): whence disregarding the exterior factors, we may say that for A, B, C, E, F, G, Q, U the standard forms are also "divided" forms. But take any other covariant—for instance, D: the leading coefficient is ad, having the interior factor a; and this being so it is found that all the following coefficients will divide by a (the quotients being of course expressible only in terms of the covariants subsequent to f): thus the second coefficient of D is $-bf+ce$, and (5.11) we have $-bf+ce=ai$, or the coefficient divided by a is $=i$; and so for the other coefficients of D; or throwing out the factor a, we obtain for D an expression of the form $(d, i, \ldots \between x, y)^3$, see the Table 98: this is the "divided" form of D: and we have similarly a divided form for every other capital covariant. All that has been required is that each coefficient of the divided form shall be expressed as a rational and integral function of the covariants $a, b, c \ldots v, w$: and the form is not hereby made definite: to render it so the coefficient must be expressed in the segregate form. But there is frequently the disadvantage that we thus introduce fractions; for instance, the last coefficient of D is $=-ci+df$, where to get rid of the congregate term df we have (6.12), $3df=-al+2ci$, and the segregate form of the coefficient is $=-\tfrac{1}{3}al+\tfrac{2}{3}ci$.

379. We have in regard to the Canonical form a differential operator which is analogous to the two differential operators $xd_y-\{xd_y\}$, $yd_x-\{yd_x\}$ considered in the Introductory Memoir (1854). Let δ denote a differentiation in regard to the constants under the conditions

$$\begin{aligned}
\delta a &= 0,\\
\delta b &= e,\\
\delta c &= 3f,\\
\delta d &= \frac{1}{a}(-bf+ce),\ (=i),\\
\delta e &= -6ad-10bc,\\
\delta f &= 2a^2b-18c^2,
\end{aligned}$$

which (as at once verified) are consistent with the fundamental relation

$$f^2=-a^3d+a^2bc-4c^3\,;$$

then it is easy to verify that

$$\left(x\frac{d}{dy}-4cy\frac{d}{dx}-\delta\right)\mathrm{A}=0\,;$$

and this being so, any other covariant whatever, expressed in the like standard form, is reduced to zero by the operator

$$x\frac{d}{dy}-4cy\frac{d}{dx}-\delta\,;$$

and we have thus the means of calculating the covariant when the leading coefficient is known.

Thus, considering the covariant B, the expression of which has just been obtained, $=(\mathrm{B}_0, \mathrm{B}_1, \mathrm{B}_2 \between x, y)^2$, suppose: the equation to be satisfied is

$$\begin{array}{lll} x\ (\mathrm{B}_1x+2\mathrm{B}_2y & &) \\ -4cy\ (& 2\mathrm{B}_0x+\mathrm{B}_1y) & \\ -x^2\delta\mathrm{B}_0 & -xy\delta\mathrm{B}_1-y^2\delta\mathrm{B}_2 & =0, \end{array}$$

viz., we have

$$\begin{array}{r} \mathrm{B}_1 \qquad\quad -\delta\mathrm{B}_0=0, \\ 2\mathrm{B}_2-\delta c\mathrm{B}_0-\delta\mathrm{B}_1=0, \\ -4c\mathrm{B}_1-\delta\mathrm{B}_2=0\,; \end{array}$$

which (omitting, as we may do, the outside factor a^2) are satisfied by the foregoing values B_0, B_1, $\mathrm{B}_2=b,\ e,\ -3ad-bc$. And if we assume only $\mathrm{B}_0=b$, then the first equation gives at once the value $\mathrm{B}_1=e$, the second equation then gives $\mathrm{B}_2=-3ad-3bc$; and the third equation is satisfied identically, viz., the equation is

$$-4ce+\delta(3ad+bc)=0,$$

that is

$$\begin{array}{lll} -4ce & = \quad -4ce & =0. \\ +c\delta b & \quad\ +\ c\ .\ e & \\ +b\delta e & \quad\ +\ b\ .\ 3f & \\ +3a\delta d & \quad\ +3(-bf+ce) & \end{array}$$

which is right.

Of course every invariant must be reduced to zero by the operation δ: thus we have Table No. 97,

$$\begin{array}{rl} a^2g= & 12\ abd \\ + & 4\ b^2c \\ + & 1\ e^2, \end{array}$$

and thence

$$\begin{array}{llllll|ccc} & & & & & & ade & b^2f & bce \\ \hline a^2\delta g= & (12ad+8bc)\delta b, & = & (12ad+8bc) & e, & = & +12 & & +8 \\ & +4b^2 & .\ \delta c & +4b^2 & .\ 3f & & & +12 & \\ & +12ab & .\ \delta d & +12b(-bf+ce) & & & & -12 & +12 \\ & +2e & .\ \delta e & +2e(-6ad-10bc) & & & -12 & & -20 \end{array}$$

which is $=0$, as it should be.

380. As already remarked, the leading coefficients of H, I, J, &c., are each of them equal to a power of a into the corresponding covariant $h, i, j \ldots$; hence, supposing these leading coefficients, or, what is the same thing, the standard expressions of the covariants $h, i, j \ldots v, w$ to be known, we can calculate the values of $\delta h, \delta i, \delta j, \ldots \delta v, \delta w$ ($=0$, since w is an invariant): and the operation δ instead of being applicable only to the forms containing a, b, c, d, e, f, becomes applicable to forms containing any of the covariants. The values of $\delta a, \delta b, \ldots \delta v, \delta w$ can, it is clear, be expressed in terms of segregates; and this is obviously the proper form: but for δr, δt, and δv, for which the segregate forms are fractional, I have given also forms with integer coefficients. The entire series is

Deg-order.		
2.8	$\delta a = 0,$	
3.5	$\delta b = e,$	
3.9	$\delta c = 3f,$	
4.6	$\delta d = i,$	
4.8	$\delta e = -6ad-10bc,$	
4.12	$\delta f = 2a^2b-18c^2,$	
5.3	$\delta g = 0,$	
5.7	$\delta h = 2be-4l,$	
5.9	$\delta i = -2ab^2+2ah-18cd,$	
6.4	$\delta j = -n,$	
6.6	$\delta k = -2aj+6b^3-9bh+3cg,$	
6.10	$\delta l = -3abd-7b^2c+7ch,$	
7.5	$\delta m = -bk-p,$	
7.7	$\delta n = 4cj,$	
8.4	$\delta o = b^2g+6bm-6dj-gh,$	
8.8	$\delta p = 8abj-5adg-10b^4+15b^2h-5bcg+10cm,$	
9.3	$\delta q = 0,$	
9.5	$\delta r = \tfrac{1}{2}(aq+6b^2j-5bdg-jh),$	$=2b^2j-2bdg-6dm,$
10.6	$\delta s = -2agj+2b^3g+3b^2m+21bdj-4bgh+2cg^2-3cq,$	
12.4	$\delta t = \tfrac{1}{2}(bgm+4bj^2-3dgj-hq),$	$=-b^2q+hq+6m^2,$
13.3	$\delta u = 0,$	
14.4	$\delta v = \tfrac{1}{6}(-5bgr-10bjo+5gjk-12js-9nq),$	$=-6dt-6mr+nq.$
19.3	$\delta w = 0.$	

It is obvious that for every covariant whatever written in the denumerate form $(I_0, I_1 \ldots \between x, y)^a$, the second coefficient is equal to the first coefficient operated upon by δ; so that the foregoing formulæ give, in fact, the second coefficients of the several covariants.

381. It is worth noticing how very much the formulæ of Table No. 97 simplify

themselves, if one of the covariants b, c, d, e vanishes, in particular if b vanishes. Suppose $b=0$; writing also (although this makes but little difference) $a=1$, we have

$$\begin{aligned}
a &= 1,\\
b &= 0,\\
c &= c,\\
d &= d,\\
e &= e,\\
f^2 &= -d-4c^3,\\
g &= e^2,\\
h &= 6cd+ef,\\
i &= ce,\\
j &= 9d^2+ce^2,\\
k &= 3de,\\
l &= -3df+2c^2e,\\
m &= 9cd^2+3def-c^2e^2,\\
n &= -6cde-e^2f,\\
o &= 9d^2e+ce^3,\\
p &= -9d^2f+12c^2de+ce^2f,\\
q &= -54cd^3-27d^2ef+18c^2de^2+ce^3f,\\
r &= 9cd^2e+3de^2f-c^2e^3,\\
s &= -27d^3f+54c^2d^2e+9cde^2f-2c^3e^3,\\
t &= -81d^4f-6d^2e^3+216c^2d^3e+54cd^2e^2f-24c^3de^3-c^2e^4f,\\
u &= -27d^5-18cd^3e^2-4d^2e^3f+c^2de^4,\\
v &= -81d^4ef-6d^2e^4+216c^2d^3e^2+54cd^2e^3f-24c^3de^4-1c^2e^5f,\\
w &\ \text{(not calculated).}
\end{aligned}$$

These values are very convenient for the verification of syzygies, &c.: take, for instance, the before-mentioned relation $\delta v=-6dt-6mr+nq$, that is, if $V=(V_0, V_1 \between x, y)$, then $V_1=-6dt-6mr+nq$: calculating the three products on the right hand side, observing that f^2 when it occurs is to be replaced by its value $-d-4c^3$, and taking their sum, the figures are as follows:—

	$-6dt$	$-6mr$	$+nq$	*Sum*
d^5f	$+486$			$+486$
d^3e^3	$+36$	$+54$	-27	$+63$
c^2d^4e	-1296	-486	$+324$	-1458
cd^3e^2f	-324	-324	$+216$	-432
cde^5			$+1$	$+1$
$c^3d^2e^3$	$+144$	$+324$	-216	$+252$
c^2de^4f	$+6$	$+36$	-24	$+18$
c^4e^5		-6	$+4$	-2

where the last column is, in fact, what V_1 becomes on writing therein $a=1$, $b=0$. The verification would not of course apply to terms which contain b; thus (13.3) a derived syzygy is $jr=bt+mo$; and the foregoing values give, as they should do, $jr=mo$: we might for the verification of most of the terms in b use values $a, b, c, d, e, f^2 =1, b, 0, d, e, -d$: the only failure would be for terms containing bc.

Table No 97 (Covariants of A, in the af- or standard forms: W is not given).

The several covariants are—

A=(	1	0	$c+10$	$f+10$	a^2b+5 $c^2\ -15$	a^2e+1 $cf\ -2$	$)(x, y)^5$
	0.0	1.3	2.6	3.9	4.12	5.15	

B=a^2(	$b+1$	$e+1$	$ad\ -3$ $bc\ -1$	$)(x, y)^2$
	2.2	3.5	4.8	

C=(	$c+1$	$f+1$	a^2b+3 $c^2\ -15$	a^2e+1 $cf\ -10$	$a^3d\ +6$ a^2bc-3 $c^3\ +15$	a^2bf-3 ,, $ce+3$ a^0cf+3	$a^4b^2\ -1$ $a^3cd\ +2$ a^2bc^2+4 ,, $ef\ +1$ $a^0c^4\ -1$	$)(x, y)^6$
	2.6	3.9	4.12	5.15	6.18	7.21	8.24	

D=a^2(	$ad+1$	$bf-1$ $cc+1$	a^2b^2-1 $acd\ +3$ a^0bc^2+4 ,, $cf\ +1$	$adf\ +1$ a^0bcf+1 ,, c^2e-1	$)(x, y)^3$
	4.8	5.11	6.14	7.17	

Table No. 97 (continued).

$E=a^2($

$e+1$	$ad\ -\ 6$ a^0bc-10	$bf-12$ $ce+\ 2$	$a^2b^2\ -\ 8$ $acd\ -36$ a^0bc^2+12 $,,\ ef\ -\ 2$	$a^2be\ -\ 5$ $adf\ -24$ $a^0bcf-\ 4$ $,,\ c^2e+\ 2$	$a^3bd-\ 6$ $a^2b^2c-\ 2$ $,,\ e^2\ -\ 1$ $ac^2d\ +18$ $a^0bc^3+\ 6$ $,,\ cef+\ 2$
3.5	4.8	5.11	6.14	7.17	8.20

$\between(x,\ y)^5$

$F=($

$f+1$	$a^2b\ +\ 2$ a^0c^2-18	$a^2e\ +\ 1$ a^0cf-36	$a^3d\ +\ 34$ $a^2bc-\ 42$ $a^0c^3\ +168$	$a^2bf\ -\ 40$ $,,\ ce\ +\ 5$ a^0c^2f-126	$a^4b^2\ -\ 16$ $a^3cd\ +\ 6$ a^2bc^2+134 $,,\ cf\ -\ 5$ $a^0c^4\ -252$	$a^4be\ -\ 7$ $a^3df\ +\ 8$ $a^2bcf+\ 8$ $,,\ c^2e+55$ $a^0c^3f\ -84$	$a^5bd\ +\ 6$ a^4b^2c-22 $,,\ e^2\ -\ 1$ $a^3c^2d\ +54$ a^2bc^2+66 $,,\ cef+38$ $a^0c^5\ +72$	$a^5de\ -12$ $a^4bce\ +11$ $,,\ b^2f\ -\ 9$ $a^3cdf\ +24$ a^2bc^2f+32 $,,\ c^3e\ -45$ $a^0c^4f\ +\ 9$	$a^6b^3\ -\ 2$ $a^5bcd+\ 6$ $a^4b^2c^2\ +12$ $,,\ bef+\ 3$ $,,\ ce^2\ -\ 1$ $a^3c^3d\ -14$ $a^2bc^4\ -16$ $,,\ c^2ef-\ 5$ $a^0c^6\ -\ 2$
3.9	4.12	5.15	6.18	7.21	8.24	9.27	10.30	11.33	12.36

$\between(x,\ y)^9$

$G=a^4$

$abd\ +\ 12$ $a^0b^2c\ +\ 4$ $,,\ e^2\ +\ 1$ $=a^2g$
6.10

$H=a^2($

$acd\ +\ 6$ $a^0bc^2\ +\ 4$ $,,\ ef\ +\ 1$ $=a^2h$	$a^2be\ +\ 2$ $adf\ +\ 12$ $a^0bcf\ -\ 8$ $,,\ c^2e\ -\ 8$	$a^2b^2c\ +\ 4$ $,,\ e^2\ +\ 1$ $ac^2d\ -\ 36$ $a^0bc^3\ -\ 24$ $,,\ cef\ -\ 6$	$a^3de\ +\ 2$ $a^2b^2f\ +\ 4$ $,,\ bce\ -\ 6$ $acdf\ -\ 12$ $a^0bc^2f-\ 8$ $,,\ c^3e\ +\ 8$	$a^4b^3\ +\ 2$ $,,\ d^2\ +\ 6$ $a^3bcd\ -\ 2$ $a^2b^2c^2-\ 8$ $,,\ bef\ -\ 3$ $,,\ ce^2\ +\ 1$ $ac^3d\ +\ 6$ $a^0bc^4\ +\ 4$ $,,\ c^2ef+\ 1$
6.14	7.17	8.20	9.23	10.26

$\between(x,\ y)^4$

$I=a^2($

$bf-1$ $ce+1$ $=ai$	$a^2b^2\ -2$ $acd\ -6$ a^0bc^2+8 $,,\ ef\ -2$	$adf\ -15$ $a^0bcf+\ 5$ $,,\ c^2e-\ 5$	a^3bd-20 ac^2d+60	$a^3de\ -\ 5$ $a^2b^2f\ +\ 5$ $,,\ bce-\ 5$ $acdf\ +30$ $a^0bc^2f+\ 5$ $,,\ c^3e\ -\ 5$	$a^4b^3\ +\ 2$ $,,\ d^2\ -12$ $a^3bcd-\ 2$ $a^2b^2c^2-\ 6$ $,,\ ce^2\ -\ 2$ $ac^3d\ -30$ $a^0bc^4\ -\ 8$ $,,\ c^2ef-\ 2$	$a^4b^2e\ +\ 1$ $a^3bdf+\ 3$ $,,\ cde-\ 5$ $a^2b^2cf+\ 1$ $,,\ bc^2e-\ 5$ $,,\ e^2f\ -\ 1$ $ac^2df\ -\ 3$ $a^0bc^3f-\ 1$ $,,\ c^4e\ +\ 1$
5.11	6.14	7.17	8.20	9.23	10.26	11.29

$\between(x,\ y)^6$

Table No. 97 (continued).

$J=a^4($	$a^2b^3 + 1$ $,,\ d^2 + 9$ $a^0b^2c^2 - 4$ $,,\ bef - 2$ $,,\ ce^2 + 1$ $=a^3j$	$a^2b^2e - 1$ $abdf - 6$ $,,\ cde + 6$ $a^0b^2cf - 4$ $,,\ bc^2e + 8$ $,,\ e^2f + 1$	$)\!(x, y)^1$
	8.16	9.19	

$K=a^4($	$ade + 3$ $a^0b^2f + 2$ $,,\ bce - 2$ $=a^2k$	$a^2b^3 + 4$ $,,\ d^2 - 18$ $abcd - 18$ $a^0b^2c^2 - 16$ $,,\ bef - 5$ $,,\ ce^2 + 1$	$a^2b^2e + 1$ $abdf + 6$ $acde - 15$ $a^0b^2cf - 2$ $,,\ bc^2e - 2$ $,,\ e^2f - 1$	$a^3b^2d + 6$ $a^2b^3c + 2$ $,,\ cd^2 - 18$ $abc^2d - 30$ $adef - 9$ $a^0b^2c^3 - 8$ $,,\ bcef - 5$ $,,\ c^2e^2 + 3$	$)\!(x, y)^3$
	7.13	8.16	9.19	10.22	

$L=a^2($	$adf - 3$ $a^0bcf - 2$ $,,\ c^2e + 2$ $=a^2l$	$a^3bd - 3$ $a^2b^2c - 7$ $ac^2d + 42$ $a^0bc^3 + 28$ $,,\ cef + 7$	$a^3de - 12$ $a^2b^2f - 9$ $,,\ bce + 9$ $acdf + 63$ $a^0bc^2f + 42$ $,,\ c^3e - 42$	$a^4b^3 - 6$ $,,\ d^2 - 39$ $a^3bcd + 40$ $a^2b^2c^2 + 59$ $,,\ bef + 7$ $,,\ ce^2 - 1$ $ac^3d - 210$ $a^0bc^4 - 140$ $,,\ c^2ef - 35$	$a^4b^2e - 1$ $a^3bdf + 39$ $,,\ cde - 14$ $a^2b^2cf + 16$ $,,\ bc^2e - 12$ $,,\ e^2f + 1$ $ac^2df - 105$ $a^0bc^3f - 70$ $,,\ c^4e + 70$	$a^5b^2d + 15$ $a^4b^3c - 9$ $,,\ cd^2 + 18$ $a^3bc^2d - 33$ $,,\ def - 3$ $a^2b^2c^3 + 15$ $,,\ bcef + 21$ $,,\ c^2e^2 - 12$ $ac^4d + 126$ $a^0bc^5 + 84$ $,,\ c^3ef + 21$	$a^5bde - 7$ $a^4b^3f - 7$ $,,\ b^2ce + 14$ $,,\ d^2f + 12$ $a^3bcdf + 23$ $,,\ c^2de - 26$ $a^2b^2c^2f + 25$ $,,\ bc^3e - 53$ $,,\ ce^2f - 7$ $ac^3df + 21$ $a^0bc^4f + 14$ $,,\ c^5e - 14$	$a^6b^4 - 2$ $,,\ bd^2 + 3$ $a^5b^2cd + 10$ $,,\ de^2 + 2$ $a^4b^3c^2 + 13$ $,,\ b^2ef + 4$ $,,\ bce^2 - 2$ $,,\ c^2d^2 - 15$ $a^3bc^3d - 28$ $,,\ cdef - 7$ $a^2b^2c^4 - 19$ $,,\ bc^2ef - 10$ $,,\ c^3e^2 + 5$ $ac^5d - 6$ $a^0bc^6 - 4$ $,,\ c^4ef - 1$	$)\!(x, y)^7$
	7.17	8.20	9.23	10.26	11.29	12.32	13.35	14.38	

$M=a^4($	$a^3b^2d - 2$ $a^2b^3c - 1$ $,,\ cd^2 + 9$ $abc^2d + 12$ $,,\ def + 3$ $a^0b^2c^3 + 4$ $,,\ bcef + 2$ $,,\ c^2e^2 - 1$ $=a^4m$	$a^3bde - 1$ $a^2b^3f - 1$ $,,\ b^2ce + 2$ $,,\ d^2f + 9$ $abcdf + 12$ $,,\ c^2de - 12$ $a^0b^2c^2f + 4$ $,,\ bc^3e - 8$ $,,\ ce^2f - 1$	$a^4b^4 - 1$ $,,\ bd^2 + 3$ $a^3b^2cd + 6$ $,,\ de^2 + 1$ $a^2b^3c^2 + 5$ $,,\ b^2ef + 2$ $,,\ bce^2 - 1$ $,,\ c^2d^2 - 9$ $abc^3d - 12$ $,,\ cdef - 3$ $a^0b^2c^4 - 4$ $,,\ bc^2ef - 2$ $,,\ c^3e^2 + 1$	$)\!(x, y)^2$
	10.22	11.25	12.28	

Table No. 97 (continued).

$\mathrm{N}=a^4($

a^2b^2e $+1$	a^2b^3c $+4$	a^3bde $+12$	a^4b^4 $+4$	a^4b^3e $+2$
$a\,bdf$ $+6$	,, cd^2 $+36$	a^2b^3f $+6$	,, bd^2 $+12$	,, d^2e $+9$
,, cde -6	$a^0b^2c^3$ -16	,, b^2ce -6	a^3b^2cd $+8$	a^3b^2df $+6$
a^0b^2cf $+4$	,, $bcef$ -8	,, d^2f $+54$	,, de^2 $+4$	,, $bcde$ -10
,, bc^2e -8	,, c^2e^2 $+4$	$abcdf$ $+36$	$a^2b^3c^2$ -12	a^2b^3cf -2
,, e^2f -1		,, c^2de -36	,, b^2ef -4	,, b^2c^2e -11
			,, c^2d^2 -108	,, be^2f -3
			abc^3d -96	,, cd^2f -18
			,, $cdef$ -24	,, ce^3 $+1$
			$a^0b^2c^4$ -16	abc^2df -18
			,, bc^2ef -8	c^3de $+18$
			,, c^3e^2 $+4$	$a^0b^2c^3f$ -4
				bc^4e $+8$
$=a^3n$				ce^2f $+1$
9.19	10.22	11.25	12.28	13.31

$)(x, y)^4$

$\mathrm{O}=a^6($

a^2b^3e $+3$	a^3b^3d -6
,, d^2e $+9$	,, d^3 -54
ab^2df $+12$	a^2b^4c -2
,, $bcde$ -12	,, b^2e^2 $+1$
a^0b^3cf $+8$	,, bcd^2 -18
,, b^2c^2e -20	ab^2c^2d $+24$
,, be^2f -4	,, $bdef$ $+18$
,, ce^3 $+1$	,, cde^2 -12
	$a^0b^3c^3$ $+8$
	,, b^2cef -8
	,, bc^2e^2 -10
$=a^4o$	,, e^3f -1
11.21	12.24

$)(x, y)^1$

$\mathrm{P}=a^4($

a^3bde -2	a^4b^4 -2	a^4b^3e -5	a^5b^3d -8	a^5b^2de $+6$	a^6b^5 $+2$
a^2b^3f -1	,, bd^2 $+12$	,, d^2e -9	,, d^3 -72	a^4b^4f $+5$	,, b^2d^2 $+12$
,, d^2f -9	a^3b^2cd -10	a^3b^2df -24	a^4b^4c $+4$	,, b^3ce -13	a^5b^3cd -2
$a\,bcdf$ -12	,, de^2 -5	,, $bcde$ $+44$	,, b^2e^2 -1	,, bd^2f $+21$	,, bde^2 -3
,, c^2de $+12$	$a^2b^3c^2$ -2	a^2b^3cf -6	,, bcd^2 -24	,, cd^2e -21	,, cd^3 -36
$a^0b^2c^2f$ -4	,, b^2e^2f -1	,, b^2c^2e $+36$	a^3b^2cd $+52$	a^3b^2cdf -4	$a^4b^4c^2$ -12
,, bc^3e $+8$	,, bce^2 $+3$	,, be^2f $+6$	,, $bdef$ $+10$	,, bc^2de $+10$	,, b^3ef -5
,, ce^2f $+1$	,, c^2d^2 $+90$	,, cd^2f $+90$	,, cde^2 $+8$	,, de^2f $+4$	,, b^2ce^2 $+4$
	abc^3d $+120$	,, ce^3 -1	$a^2b^3c^3$ $+4$	$a^2b^3c^2f$ -17	,, bc^2d^2 -66
	,, $cdef$ $+30$	abc^2df $+120$	,, b^2cef -2	,, b^2c^3e $+44$	,, d^2ef -18
	$a^0b^2c^4$ $+40$	,, c^3de -120	,, bc^2e^2 $+2$	,, bce^2f $+13$	$a^3b^2c^3d$ -10
	,, bc^2ef $+20$	$a^0b^2c^3f$ $+40$	,, c^3d^2 -180	,, c^2d^2f -45	,, $bcdef$ -4
	,, c^3e^2 -10	,, bc^4e -80	,, e^3f $+1$	,, c^2e^3 -5	,, c^2de^2 $+1$
		,, c^2e^2f -10	$a\,bc^4d$ -240	abc^3df -60	$a^2b^3c^4$ $+14$
			,, c^2def -60	,, c^4de $+60$	,, b^2c^2ef $+11$
			$a^0b^2c^5$ -80	$a^0b^2c^4f$ -20	,, bc^3e^2 -11
			,, bc^3ef -40	,, bc^5e $+40$	,, c^4d^2 $+18$
			,, c^4e^2 $+20$	,, c^3e^2f -5	,, ce^3f -1
					abc^5d $+26$
					,, c^3def $+6$
					$a^0b^2c^6$ $+8$
					,, bc^4ef $+4$
$=a^4p$					,, c^5e^2 -2
11.25	12.28	13.31	14.34	15.37	16.40

$)(x, y)^5$

Table No. 97 (continued).

$\mathrm{Q}=a^6($

a^4b^5	−	2
,, b^2d^2	+	18
a^3b^3cd	+	22
,, bde^2	+	3
,, cd^3	−	54
$a^2b^4c^2$	+	12
,, b^3ef	+	5
,, b^2ce^2	−	4
,, bc^2d^2	−	108
,, d^2ef	−	27
$a\ b^2c^3d$	−	72
,, $bcdef$	−	36
,, c^2de^2	+	18
$a^0b^3c^4$	−	16
,, b^2c^2ef	−	12
,, bc^3e^2	+	12
,, ce^3f	+	1
$=a^6q$		
14.30		

$\mathrm{R}=a^6($

a^3bde	−	1	a^4b^5	+	2	a^4b^4e	+	1
a^2b^4f	+	1	,, b^2d^2	+	6	a^3b^3df	+	3
,, b^3ce	−	3	a^3b^3cd	−	2	,, b^2cde	−	11
,, bd^2f	−	9	,, bde^2	−	2	,, d^3f	−	27
,, cd^2e	+	9	,, cd^3	−	54	,, de^3	−	1
$a\ b^2cdf$	−	12	$a^2b^4c^2$	−	8	a^2b^4cf	+	1
,, bc^2de	+	24	,, b^3ef	−	4	,, b^3c^2e	−	7
,, de^2f	+	3	,, b^2ce^2	+	2	,, b^2e^2f	−	2
$a^0b^3c^2f$	−	4	,, bc^2d^2	−	72	,, bcd^2f	−	45
,, b^2c^3e	+	12	,, d^2ef	−	18	,, bce^3	+	1
,, bce^2f	+	3	ab^3c^3d	−	24	,, c^2d^2e	+	45
,, c^2e^3	−	1	,, $bcdef$	−	12	$a\ b^2c^2df$	−	24
			,, c^2de^2	+	6	,, bc^3de	+	48
						a^0b^3cf	−	4
						,, b^2c^4e	+	12
						,, bc^2e^2f	+	3
						,, c^3e^3	−	1
$=a^5r$								
13.27			14.30			15.33		

$)(x, y)^2$

$\mathrm{S}=a^6($

a^4bd^2e	+	9	a^5b^4d	+	15	a^5b^3de	−	6	a^6b^6	−	2
a^3b^3df	+	7	,, bd^3	−	27	,, d^3e	−	27	,, b^3d^2	+	9
,, b^2cde	−	12	a^4b^5c	+	3	a^4b^5f	−	3	,, d^4	−	27
,, d^3f	−	27	,, b^2cd^2	−	99	,, b^4ce	+	9	a^5b^4cd	+	18
a^2b^4cf	+	2	,, d^2e^2	−	18	,, b^2d^2f	−	9	,, b^2de^2	+	6
,, b^3c^2e	−	6	$a^3b^3c^2d$	−	114	,, bcd^2e	−	18	,, bcd^3	−	54
,, bcd^2f	−	54	,, b^2def	−	33	a^3b^3cdf	−	9	$a^4b^5c^2$	+	15
,, c^2d^2e	+	54	,, $bcde^2$	+	12	,, b^2c^2de	+	24	,, b^4ef	+	6
$a\ b^2c^2df$	−	36	,, c^2d^3	+	162	,, bde^2f	+	3	,, $b^3c^2d^2$	−	36
,, bc^3de	+	72	$a^2b^4c^2$	−	24	,, cd^3f	+	81	,, b^3ce^2	−	6
,, cde^2f	+	9	,, b^3cef	−	9	,, cde^3	−	3	,, $b^2c^2d^2$	−	27
$a^0b^3c^3f$	−	8	,, $b^2c^2e^2$	+	9	$a^2b^4c^2f$	+	6	,, bd^2ef	−	9
,, b^2c^4e	+	24	,, bc^3d^2	+	324	,, b^3c^3e	−	18	,, cd^2e^2	−	9
,, bc^2e^2f	+	6	,, cd^2ef	+	69	,, b^2ce^2f	−	9	$a^3b^3c^3d$	−	54
,, c^3e^3	−	2	$a\ b^2c^4d$	+	216	,, bc^2d^2f	+	162	,, b^2cdef	−	27
			,, bc^2def	+	120	,, bc^2e^3	+	3	,, bc^2de^2	+	3
			,, c^3de^2	−	54	,, c^3d^2e	−	162	,, c^3d^3	−	54
			$a^0b^3c^5$	+	48	$a\ b^2c^3df$	+	108	,, de^3f	−	2
			,, b^2c^3ef	+	36	,, bc^4de	−	216	$a^2b^4c^4$	−	24
			,, bc^4e^2	−	36	,, c^2de^2f	−	27	,, b^3ce^2f	−	21
			,, c^2e^3f	−	3	$a^0b^3c^4f$	+	24	,, $b^2c^3e^2$	+	21
						,, b^2c^5e	−	72	,, bc^4d^2	−	108
						,, bc^3e^2f	−	18	,, bce^3f	+	2
						,, c^4e^3	+	6	,, c^2d^2ef	−	27
									$a\ b^2c^5d$	−	72
									,, bc^3def	−	36
									,, c^4de^2	+	18
									$a^0b^3c^6$	−	16
									,, b^2c^4ef	−	12
									,, bc^5e^2	+	12
$=a^6s$									,, c^3e^3f	+	1
15.33			16.36			17.39			18.42		

$)(x, y)^3$

Table No. 97 (continued).

$T=($

a^5b^4de	$+$	7	a^6b^7	$+$	2
,, bd^3e	$+$	27	,, b^4d^2	$+$	6
a^4b^6f	$+$	1	a^5b^5cd	$-$	10
,, b^5ce	$-$	2	,, b^3de^2	$-$	8
,, b^3d^2f	$+$	24	,, b^2cd^2	$-$	54
,, b^2cd^2e	$-$	54	,, d^3e^2	$-$	27
,, d^4f	$-$	81	$a^4b^6c^2$	$-$	14
,, d^2e^3	$-$	6	,, b^5ef	$-$	7
a^3b^4cdf	$+$	16	,, b^4ce^2	$+$	9
,, b^3c^2de	$-$	76	,, $b^3c^2d^2$	$-$	84
,, b^2de^2f	$-$	12	,, b^2d^2ef	$-$	27
,, bcd^3f	$-$	216	,, bcd^2e^2	$+$	9
,, $bcde^3$	$+$	5	,, c^2d^4	$+$	162
,, c^2d^3e	$+$	216	$a^3b^4c^3d$	$-$	8
$a^2b^4c^3e$	$-$	8	,, b^3cdef	$+$	4
,, $b^2c^2d^2f$	$-$	216	,, $b^2c^2de^2$	$+$	18
,, $b^2c^2e^3$	$+$	2	,, bc^3d^3	$+$	432
,, bc^3d^2e	$+$	432	,, bde^3f	$+$	3
,, cd^2e^2f	$+$	54	,, cd^3ef	$+$	108
$a\ b^3c^3df$	$-$	96	,, cde^4	$-$	1
,, b^2c^4de	$+$	288	$a^2b^5c^4$	$+$	16
,, bc^2de^2f	$+$	72	,, b^4c^2ef	$+$	20
,, c^3de^3	$-$	24	,, $b^3c^3e^2$	$-$	24
$a^0b^4c^4f$	$-$	16	,, $b^2c^4d^2$	$+$	432
,, b^3c^5e	$+$	64	,, b^2ce^3f	$-$	5
,, $b^2c^3e^2f$	$+$	24	,, bc^2d^2ef	$+$	216
,, bc^4e^3	$-$	16	,, bc^2e^4	$+$	1
,, c^2e^4f	$-$	1	,, $c^3d^2e^2$	$-$	108
			$a\ b^3c^5d$	$+$	192
			,, b^2c^3def	$+$	144
			,, bc^4de^2	$-$	144
			,, c^2de^3f	$-$	12
			$a^0b^4c^6$	$+$	32
			,, b^3c^4ef	$+$	32
			,, $b^2c^5e^2$	$-$	48
			,, bc^3e^3f	$-$	8
$=a^8t$			,, c^4e^4	$+$	2
19.41			20.44		

$)(x, y)^1$

$U=$

a^5b^6d	$-$	3
,, b^3d^3	$+$	14
,, d^5	$-$	27
a^4b^7c	$-$	1
,, b^4cd^2	$+$	34
,, $b^2d^2e^2$	$+$	11
,, bcd^4	$-$	81
$a^3b^5c^2d$	$+$	32
,, b^4def	$+$	10
,, b^3cde^2	$-$	6
,, $b^2c^2d^3$	$-$	144
,, bd^3ef	$-$	18
,, cd^3e^2	$-$	18
$a^2b^6c^3$	$+$	8
,, b^5cef	$+$	4
,, $b^4c^2e^2$	$-$	6
,, $b^3c^3d^2$	$-$	152
,, b^2cd^2ef	$-$	60
,, $bc^2d^2e^2$	$+$	6
,, d^2e^3f	$-$	4
$a\ b^4c^4d$	$-$	80
,, b^3c^2def	$-$	56
,, $b^2c^3de^2$	$+$	48
,, $bcde^3f$	$+$	2
,, c^2de^4	$+$	1
$a^0b^5c^5$	$-$	16
,, b^4c^3ef	$+$	16
,, $b^3c^4e^2$	$+$	24
,, $b^2c^2e^3f$	$+$	4
,, bc^3e^4	$+$	1
$=a^9u$		
21·45		

Table No. 97 (concluded).

$\mathrm{V}=a^{10}($

a^6b^8	$-$	4	a^6b^7e	$-$	2
,, b^5d^2	$-$	12	,, b^4d^2e	$-$	48
a^5b^6cd	$+$	20	,, bd^4e	$-$	162
,, b^4de^2	$+$	23	a^5b^6df	$-$	6
,, b^3cd^3	$+$	108	,, b^5cde	$+$	8
,, bd^3e^2	$+$	81	,, b^3d^3f	$-$	144
$a^4b^7c^2$	$+$	28	,, b^3de^3	$+$	8
,, b^6ef	$+$	15	,, b^2cd^3e	$+$	324
,, b^5ce^2	$-$	20	,, d^5f	$+$	486
,, $b^4c^2d^2$	$+$	168	,, d^3e^3	$+$	63
,, b^3d^2ef	$+$	78	a^4b^7cf	$-$	2
,, $b^2cd^2e^2$	$-$	72	,, b^6c^2e	$+$	18
,, bc^2d^4	$-$	324	,, b^5e^2f	$+$	7
,, d^4ef	$-$	81	,, b^4cd^2f	$-$	144
,, d^2e^4	$-$	6	,, b^4ce^3	$-$	9
$a^3b^5c^3d$	$+$	16	,, $b^3c^2d^2e$	$+$	648
,, b^4cdef	$+$	8	,, $b^2d^2e^2f$	$+$	99
,, $b^3c^2de^2$	$-$	112	,, bcd^4f	$+$	1458
,, $b^2c^3d^3$	$-$	864	,, bcd^2e^3	$-$	27
,, b^2de^3f	$-$	18	,, c^2d^4e	$-$	1458
,, bcd^3ef	$-$	432	$a^3b^5c^2df$	$-$	32
,, $bcde^4$	$+$	7	,, b^4c^3de	$+$	208
,, $c^2d^3e^2$	$+$	216	,, b^3cde^2f	$+$	20
$a^2b^6c^4$	$-$	32	,, $b^2c^2d^3f$	$+$	1728
,, b^5c^2ef	$-$	40	,, $b^2c^2de^3$	$-$	40
,, $b^4c^3e^2$	$+$	40	,, bc^3d^3e	$-$	3456
,, $b^3c^4d^2$	$-$	864	,, bde^4f	$-$	3
,, b^3ce^3f	$+$	10	,, cd^3e^2f	$-$	432
,, $b^2c^2d^2ef$	$-$	648	,, cde^5	$+$	1
,, $bc^3d^2e^2$	$+$	648	$a^2b^4c^2e^2f$	$-$	20
,, cd^2e^3f	$+$	54	,, $b^3c^3d^2f$	$+$	1008
$a\ b^4c^5d$	$-$	384	,, $b^3c^3e^3$	$+$	20
,, b^3c^3def	$-$	384	,, $b^2c^4d^2e$	$-$	3024
,, $b^2c^4de^2$	$+$	576	,, b^2ce^4f	$+$	5
,, bc^2de^3f	$+$	96	,, $bc^2d^2e^2f$	$-$	756
,, c^3de^4	$-$	24	,, bc^2e^5	$-$	1
$a^0b^5c^6$	$-$	64	,, $c^3d^2e^3$	$+$	252
,, b^4c^4ef	$-$	80	$a\ b^4c^4df$	$+$	288
,, $b^3c^5e^2$	$+$	160	,, b^3c^5de	$-$	1152
,, $b^2c^3e^3f$	$+$	40	,, $b^2c^3de^2f$	$-$	432
,, bc^4e^4	$-$	20	,, bc^4de^3	$+$	288
,, c^2e^5f	$-$	1	,, c^2de^4f	$+$	18
			a^0b^5cf	$+$	32
			,, b^4c^6e	$-$	160
			,, $b^3c^4e^2f$	$-$	80
			,, $b^2c^5e^3$	$+$	80
			,, bc^3e^4f	$+$	10
$=a^9v$			,, c^4e^5	$-$	2
22.46			23.49		

$\big)\!\big(x, y)^2$

Table No. 98. Covariants of A, divided and (except as to a few coefficients) segregate.

A and B as given in Table 97 were divided and segregate.

C was divided but not segregate: the divided and segregate form is

$C = ($

$c + 1$	$f + 3$	$a^2b + 3$ $a^0c^2 - 15$	$a^2e + 1$ $a^0cf - 10$	$a^3d + 6$ $a^0bc - 3$ $,,\ c^3 + 15$	$a^3i + 3$ $a^0c^2f + 3$	$a^4b^2 - 1$ $,,\ h + 1$ $a^3cd - 4$ $a^0c^4 - 1$
2.6	3.9	4.12	5.15	6.18	7.21	8.24

$\big)\!\big(x, y)^6$

D divided and segregate is

$D = a^3($

			÷3
$d + 1$	$i + 1$	$ab^2 - 1$ $,,h + 1$ $a^0cd - 3$	$al - 1$ $a^0ci - 1$
3.3	4.6	5.9	6.12

$\big)\!\big(x, y)^3$,

an integer non-segregate form of the fractional coefficient is

$ci - 1$ $df + 1$

E was divided but not segregate: the divided and segregate form is

$E = ($

$e + 1$	$ad - 6$ $a^0bc - 10$	$ai + 12$ $a^0ce - 10$	$a^2b^2 - 8$ $,,\ h - 2$ $acd - 24$ $a^0bc^2 + 20$	$a^2be - 5$ $,,\ l + 8$ $aci - 12$ $a^0c^2e + 5$	$a^4g - 1$ $a^3bd + 6$ $a^2b^2c + 2$ $,,\ ch + 2$ $ac^2d + 6$ $a^0bc^3 - 2$
3.5	4.8	5.11	6.14	7.17	8.20

$\big)\!\big(x, y)^5$.

F was divided but not segregate: the divided and segregate form is

$F = ($

						÷3			
$f + 1$	$a^2b + 2$ $a^0c^2 - 18$	$a^2e + 1$ $a^0cf - 36$	$a^3d + 34$ $a^2bc - 42$ $a^0c^3 + 168$	$a^3i + 40$ $a^2ce - 35$ $a^0c^2f + 126$	$a^4b^2 - 16$ $,,\ h - 5$ $a^3cd + 46$ $a^2bc^2 + 155$ $a^0c^4 + 252$	$a^4be - 21$ $a^3l - 8$ $,,\ ci - 16$ $a^2c^2e + 189$ $a^0c^3f - 252$	$a^6g - 1$ $a^5bd + 18$ $a^4b^2c - 18$ $,,\ ch + 38$ $a^3c^2d - 174$ $a^2bc^2 - 86$ $a^0c^5 + 72$	$a^6k - 4$ $a^5bi + 1$ $a^4bce + 2$ $,,\ cl - 8$ $a^3c^2i - 16$ $a^2c^3e - 13$ $a^0c^4f + 9$	$a^6b^3 - 2$ $,,\ bh + 3$ $,,\ cg - 1$ $a^4b^2c^2 + 4$ $,,\ c^2h - 5$ $a^3c^3d + 16$ $a^2bc^4 + 4$ $a^0c^6 - 2$
3.9	4.12	5.15	6.18	7.21	8.24	9.27	10.30	11.33	12.36

$\big)\!\big(x, y)^9$,

where for an integer non-segregate value of the fractional coefficient, see the original form of F.

Table No. 98 (continued).

G as an invariant was divided and segregate, $G=a^6 \underset{4.0}{y}$

H divided and segregate is

				÷3	÷3	
$H=a^4($	h + 1	be +2 l −4	a^2g + 1 abd −12 a^0ch − 6	a^2k + 2 abi − 8 a^0bce − 6 ,, cl +12	a^3j + 2 a^2b^3 + 4 ,, bh − 5 ,, cg + 1 $abcd$ +12 a^0c^2h + 3	$)(x, y)^4$,
	4.4	5.7	6.10	7.13	8.16	

where the fractional coefficients are =

ade + 2 a^0b^2f + 4 ,, bce − 6 ,, cl + 4	a^2b^3 + 2 ,, d^2 + 6 $abcd$ − 2 $a^0b^2c^2$ − 8 ,, bef − 3 ,, c^2h + 1 ,, ce^2 + 1

I divided and segregate is

					÷3	÷3	÷3	
$I=a^3($	i + 1	ab^2 − 2 ,, h + 2 a^0cd −18	ad + 5 a^0ci −15	a^2bd −20 a^0c^2d +60	a^3k − 5 a^2bi −25 acl −30 a^0c^2i +45	a^4j − 4 a^3b^3 +10 ,, bh − 8 ,, cg + 4 a^2bcd − 6 abc^2 −18 ,, c^2h − 6 a^0c^3d −54	a^3b^2e + 3 ,, bl + 9 ,, ck − 5 ,, fg − 3 a^2bci − 8 ab^2cf − 9 ,, bc^2e −15 ,, c^2l + 3 a^0c^3i − 3	$)(x, y)^6$,
	4.6	5.9	6.12	7.15	8.18	9.21	10.24	

where the fractional coefficients are=

a^2de − 5 ab^2f + 5 ,, bce − 5 a^0c^2i − 5 ,, cdf +30	a^3b^3 + 2 ,, d^2 −12 a^2bcd − 2 ab^2c^2 − 6 ,, c^2h − 2 ,, ce^2 − 2 a^0c^3d −18	a^3b^2e + 1 a^2bdf + 3 ,, cde − 5 ab^2cf + 1 ,, bc^2e − 5 ,, e^2f − 1 a^0c^3i + 1 ,, c^2df − 3

Table No. 98 (continued).

J divided and segregate is

$$J = a^7\,(j,\ -n \between x,\ y)^1.$$

5.6 6.4

K divided and segregate is

$K = a^6($

$k + 1$	$aj - 2$	$an + 1$	$a^2m - 3$
	$a^0b^3 + 6$	$a^0ck - 3$	$acj + 1$
	,, $bh - 9$		$a^0b^3c - 2$
	,, $cg + 3$		,, $bch + 3$
			,, $c^2g - 1$
5.3	6.6	7.9	8.12

$\between x,\ y)^3.$

L divided and (as to first six coefficients) segregate is

$L = a^4($

			÷3	÷3			
$l + 1$	$abd - 3$	$a^2k - 4$	$a^3j - 13$	$a^3n - 3$	$a^4m - 1$	$a^3bde - 7$	$a^4b^4 - 2$
	$a^0b^2c - 7$	$abi + 1$	$a^2b^3 - 5$	$a^2bl - 45$	$a^3b^2d + 13$	$a^2b^3f - 7$	,, $bd^2 + 3$
	,, $ch + 7$	$a^0cl - 21$	,, $bh - 5$	,, $ck - 20$	,, $cj + 3$	,, $b^2ce + 14$	$a^3b^2cd + 10$
			,, $cg + 10$	$abci - 10$	$a^2b^3c - 13$	,, $d^2f + 12$	,, $de^2 + 2$
			$abcd + 30$	$a^0c^2l + 105$	,, $bch + 29$	$abcdf + 23$	$a^2b^3c^2 + 13$
			$a^0b^2c^2 + 105$		,, $c^2g - 16$	,, $c^2de - 24$	,, $b^2ef + 4$
			,, $ch - 105$		$abc^2d - 3$	$a^0b^2cf + 25$	,, $bce^2 - 2$
					$a^0b^2c^3 - 21$	,, $bc^3e - 55$	,, $c^2d^2 - 15$
					,, $c^3h + 21$	,, $c^3l - 3$	$abc^3d - 28$
						,, $c^2fh + 2$	,, $cdef - 7$
						,, $ce^2f - 7$	$a^0b^2c^4 - 19$
							,, $bc^2ef - 10$
							,, $c^4h - 1$
							,, $c^3e^2 + 5$
5.7	6.10	7.13	8.16	9.19	10.22	11.25	12.28

$\between x,\ y)^7$

where the fractional coefficients are =

$a^2b^3 - 6$	$a^2b^2e - 1$
,, $d^2 + 3$	$abdf + 39$
$abcd + 26$	,, $cde - 22$
$a^0b^2c^2 + 31$	$a^0b^2cf + 16$
,, $bef + 7$	,, $bc^2e - 4$
,, $c^2h - 7$	,, $c^2l + 19$
,, $ce^2 - 1$	,, $cfh - 8$
,, $fl - 14$	,, $e^2f + 1$

the last two coefficients have not been reduced to the segregate form.

M divided and segregate is

$M = a^8($

$m + 1$	$bk - 1$	$abj - 1$
	$p - 1$	$adg + 1$
		,,$^0cm - 1$
6.2	7.5	8.8

$\between x,\ y)^2.$

Table No. 98 (continued).

N divided and segregate is

$N=a^7($	$n\ +\ 1$	$cj\ +\ 4$	$ap\ -\ 6$ $cn\ -\ 6$	$a^2bj\ -\ 4$ $,,\ dg\ +\ 4$ $ab^4\ +\ 8$ $,,\ b^2h\ -\ 12$ $,,\ bcg\ +\ 4$ $,,\ cm\ -\ 8$ $a^0c^2j\ -\ 4$	$a^3o\ +\ 1$ $a^2bn\ -\ 1$ $acp\ +\ 2$ $a^0c^2n\ +\ 1$	$)(x,\ y)^4.$
	6.4	7.7	8.10	9.13	10.16	

O divided and segregate is

$O=a^{10}($	$o\ +\ 1$	$b^2g\ +\ 1$ $bm\ +\ 6$ $df\ -\ 6$ $gh\ -\ 1$	$)(x,\ y)^1.$
	7.1	8.4	

P divided and (as to first three coefficients) segregate is

$P=a^8($	$p\ +\ 1$	$abj\ +\ 8$ $,,\ dg\ -\ 5$ $a^0b^4\ -14$ $,,\ b^2h\ +15$ $,,\ bcg\ -\ 5$ $,,\ cm\ +10$	$a^2o\ +\ 7$ $abn\ -\ 2$ $a^0cp\ -14$	$a^2bm\ +\ 8$ $,,\ dj\ +\ 3$ $ab^3d\ +\ 9$ $,,\ bcj\ +13$ $,,\ bdh\ -\ 9$ $,,\ cdg\ -12$ $,,\ d^3\ -81$ $a^0b^4c\ -\ 5$ $,,\ b^2ch\ -\ 8$ $,,\ b^2e^2\ -\ 1$ $,,\ bc^2g\ -\ 1$ $,,\ bcd^2\ -\ 9$ $,,\ bel\ -\ 1$ $,,\ bfk\ +\ 1$ $,,\ c^2m\ -12$ $,,\ dei\ +\ 9$ $,,\ e^2h\ +\ 1$ $,,\ fp\ -\ 2$	$a^2b^2k\ -\ 3$ $,,\ bp\ -\ 3$ $,,\ em\ +\ 2$ $ab^2de\ +\ 3$ $,,\ bdl\ +\ 3$ $,,\ bfj\ +\ 8$ $,,\ d^2i\ -27$ $,,\ deh\ -\ 3$ $,,\ dfg\ -\ 8$ $a^0b^3ce\ -\ 7$ $,,\ b^2cl\ +\ 1$ $,,\ bc^2k\ +\ 9$ $,,\ bcdi\ -\ 9$ $,,\ bceh\ +10$ $,,\ c^2eg\ -\ 3$ $,,\ c^2p\ +\ 9$ $,,\ cd^2e\ +18$ $,,\ cfm\ +\ 4$ $,,\ efk\ +\ 3$	$a^3b^2j\ +\ 2$ $,,\ bdg\ -\ 2$ $a^2bcm\ +\ 2$ $,,\ bek\ -\ 1$ $,,\ ep\ -\ 1$ $ab^3cd\ +24$ $,,\ bc^2j\ -\ 6$ $,,\ bcdh\ -33$ $,,\ c^2dg\ +15$ $,,\ cd^3\ -54$ $,,\ dfk\ -\ 9$ $a^0b^4c^3\ +\ 8$ $,,\ b^2c^2h\ -11$ $,,\ bc^3g\ +\ 3$ $,,\ bc^2d^2\ -18$ $,,\ bcfk\ -\ 1$ $,,\ c^3m\ -\ 6$ $,,\ cfp\ +\ 2$	$)(x,\ y)^5,$
	7.5	8.8	9.11	10.14	11.17	10.20	

the last three coefficients have not been reduced to the segregate form.

Q as an invariant was divided and segregate, $Q=a^{12}\underset{8.0}{q}$

Table No. 98 (continued).

R divided and segregate is

		÷2	÷3	
$R=a^{11}($	r + 1	aq + 1	abo − 1	$)(x, y)^2$
		a^0b^2j + 6	,, gk − 1	
		,, bdg − 5	,, s + 3	
		,, hj − 1	a^0cr − 3	
	8.2	9.5	10.8	

where the fractional coefficients are =

b^2j + 2	bdk + 3
bdg − 2	bej + 1
dm − 6	cr + 1
	deg − 1
	dp + 3

S divided and (as to the first three coefficients) segregate is

				÷ 2	
$S=a^{12}($	s + 1	agj − 2	abr − 1	a^2bq + 4	$)(x, y)^3,$
		a^0b^3g + 2	,, do − 1	ab^3j + 4	
		,, b^2m + 3	a^0cs − 3	,, b^2dg − 4	
		,, bdj + 21		,, bdm − 31	
		,, bgh − 4		,, d^2j − 3	
		,, cg^2 + 2		a^0b^6 + 4	
		,, cq − 3		,, b^3d^2 + 16	
				,, bd^2h − 24	
				,, den + 4	
				,, fs − 1	
	9.3	10.6	11.9	12.12	

but the last coefficient is neither segregate nor integer.

T divided and segregate is

		÷2	
$T=a^{16}($	t + 1	bgm + 1	$)(x, y)^1,$
		bj^2 + 4	
		dgj − 3	
		hq − 1	
	11.1	12.4	

where the fractional coefficient is =

b^2q − 1
hq + 1
m^2 + 6

Table No. 98 (concluded).

U as an invariant was divided and segregate, $\mathrm{U}=a^{18}\quad u$
12.0

V divided and segregate is

$\mathrm{V}=a^{19}($

					÷6
v	+	1	bgr	−	5
			bjo	−	10
			gjk	+	5
			js	−	12
			nq	−	9
13.1			14.4		

where the fractional coefficient is =

dt	−	6
mr	−	6
nq	+	1

W as an invariant was divided and segregate, $\mathrm{W}=a^{27}\quad w$
18.0

Derivatives.—Article Nos. 382 to 384, and Tables Nos. 99 and 100.

382. I call to mind that any two covariants a, b, the same or different, give rise to a set of derivatives $(a, b)^1$, $(a, b)^2$, $(a, b)^3$, &c., or, as I propose to write them, $ab1$, $ab2$, $ab3$, &c., viz.:

$$\begin{aligned}
ab1&=d_x\,a\,.\,d_y\,b-\quad d_ya\,.\,d_xb\\
ab2&=d_x^2a\,.\,d_y^2b-2d_x\,d_ya\,.\,d_xd_y\,b+\quad d_y^2a\,.\quad d_x^2b\\
ab3&=d_x^3a\,.\,d_y^3b-3d_x^2d_ya\,.\,d_xd_y^2b+3d_xd_y^2a\,.\,d_x^2d_yb-d_y^3a\,.\,d_x^3b
\end{aligned}$$
&c.

or, as these are symbolically written, $ab1=\overline{12}a_1b_2$, $ab2=\overline{12}^2a_1b_2$, $ab3=\overline{12}^3a_1b_2$, &c.; where $12=\xi_1\eta_2-\xi_2\eta_1$, $=\frac{d}{dx_1}\frac{d}{dy_2}-\frac{d}{dx_2}\frac{d}{dy_1}$, the differentiations $\frac{d}{dx_1}$, $\frac{d}{dy_1}$ applying to the a_1 and the $\frac{d}{dx_2}$, $\frac{d}{dy_2}$ applying to the b_2, but the suffixes being ultimately omitted: hence if θ be the index of derivation, the derivative is thus a linear function of the differential coefficients of the order θ of the two covariants a and b respectively: and we have the general property that any such derivative, if not identically vanishing, is a covariant. If the a and the b are one and the same covariant, then obviously every odd derivative is $=0$; so that in this case the only derivatives to be considered are the even derivatives $aa2$, $aa4$, &c.: moreover, if the index of derivation θ exceeds the order of either of the component covariants, then also the derivative is $=0$: in particular neither of the covariants must be an invariant. The degree of the derivative is evidently equal to the sum of the degrees of the component covariants; the order is equal to the sum of the orders less twice the index of derivation.

383. It was by means of the theory of derivatives that GORDAN proved (for a binary quantic of any order) that the number of covariants was finite, and in the particular case of the quintic, established the system of the 23 covariants. Starting from the quantic itself a, then the system of derivatives $aa2$, $aa4$, &c., must include among itself all the covariants of the second degree, and if the entire system of these is, suppose, b, c, then the derivatives $ab1$, $ab2$, $ab3$, &c., $ac1$, $ac2$, &c., must include among them all the covariants of the third degree, and so on for the higher degrees; and in this way limiting by general reasoning the number of the independent covariants of each degree obtained by the successive steps, the foregoing conclusion is arrived at. But returning to the quintic, and supposing the system of the 23 covariants established, then knowing the deg-order of a derivative we know that it must be a linear function of the segregates of that deg-order; and we thus confirm, *à posteriori*, the results of the derivation theory. I annex the following Table No. 99, which shows all the derivatives which present themselves, and for each of them the covariants as well congregate as segregate of the same deg-order: the congregates are distinguished each by two prefixed dots, . . bf, &c. No further explanation of the arrangement is, I think, required. We see from the table in what manner the different covariants present themselves in connexion with the derivation-theory. Thus starting with the quintic itself a, we have the two derivatives $aa4$, $aa2$, which are in fact the covariants of the second degree (deg-orders 2.2 and 2.6 respectively) b and c. For the third degree we have the derivatives $ab2$, $ab1$, $ac5$, $ac4$, $ac3$, $ac2$, $ac1$: the deg-order of $ac5$ is 3.1, and there being no covariants of this deg-order, $ac5$ must, it is clear, vanish identically: $ab2$ and $ac4$ are each of them of the deg-order 3.3, but for this deg-order we have only the covariant d, and hence $ab2$ and $ac4$ must be each of them a numerical multiple of d; similarly, deg-order 3.5, $ab1$ and $ac3$ must be each of them a numerical multiple of e; deg-order 3.7, $ac2$ must be a numerical multiple of ab; and deg-order 3.9, $ac1$ must be a numerical multiple of f: the 7 derivatives, which *primâ facie* might give, each of them, a covariant of the third degree, thus give in fact only the 3 covariants d, e, f; and in order to show according to the theory of derivations that this is so, it is necessary to prove—1°, that $ac5=0$; 2°, that $ac4$ and $ab2$ differ only by a numerical factor; 3°, that $ab1$ and $ac3$ differ only by a numerical factor; 4°, that $ac2$ is a numerical multiple of ab: which being so we have the 3 new covariants. The table shows that

for degrees	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15,	16,	17,	18,	19,	20,	21,	22,	23,	24
No. of derivatives=	2,	7,	19,	29,	41,	46,	52,	46,	44,	35,	26,	19,	17,	12,	13,	6,	6,	3,	3,	1,	1,	0,	1

so that the whole number of derivatives is 429, giving the 22 covariants $b, c \ldots w$.

While it is very remarkable that (by general reasoning, as already mentioned, and with a very small amount of calculation) Gordan should have been able in effect to show this, the great excess of the number of derivatives over that of the covariants seems a reason why the derivations ought not to be made a basis of the theory.

It is to be remarked that we may consider derivatives *pq*1, *pq*2, &c., where *p*, *q* instead of being simple covariants are powers or products of covariants; but that these may be made to depend upon the derivatives formed with the simple covariants. (As to this see my paper "On the Derivatives of Three Binary Quantics," Quart. Math. Journal, t. xv. (1877) pp. 157–168.)

Table No. 99 (Index Table of Derivatives).

Deg.	2			
Ord.	0	2	4	6
		b		c
aa		4		2

2 derivs.

Deg.	3				
Ord.	1	3	5	7	9
		d	e	ab	f
ab		2	1		
ac	5	4	3	2	1

7 derivs.

Deg.	4						
Ord.	0	2	4	6	8	10	12
	g		b^2 h	i	ad bc	ae	a^2b c^2
ad		3	2	1			
ae	5	4	3	2	1		
af			5	4	3	2	1
bb	2						
bc			2	1			
cc	6		4		2		

19 derivs.

Deg.	5						
Ord.	1	3	5	7	9	11	13
	j	k	ag bd	be l	ab^2 ah cd	ai $..bf$ ce	a^2d abc
ah	4	3	2	1			
ai	5	4	3	2	1		
bd	2	1					
be		2	1				
bf				2	1		
cd		3	2	1			
ce	5	4	3	2	1		
cf		6	5	4	3	2	1

29 derivs.

Table No. 99 (continued).

Deg.	6							
Ord.	0	2	4	6	8	10	12	14
		bg m	n	aj b^3 bh cg . . d^2	ak bi . . de	a^2g abd b^2c ch . . e^2	abe al ci . . df	a^2b^2 a^2h acd bc^2 . . ef
aj			1					
ak		3	2	1				
al		5	4	3	2	1		
bh		2	1					
bi			2	1				
ch		4	3	2	1			
ci	6	5	4	3	2	1		
dd		2						
de		3	2	1				
df				3	2	1		
ee		4		2				
ef			5	4	3	2	1	
ff		8		6		4		2

41 derivs.

Deg.	7						
Ord.	1	3	5	7	9	11	13
	o	bj dg	bk eg p	abg am b^2d cj . . dh	an . . b^2e bl ck . . di . . eh fg	a^2j ab^3 abh acg . . ad^2 bcd . . ei	a^2k abi . . ade . . b^2f bce cl . . fh
am		2	1				
an	4	3	2	1			
bj	1						
bk	2	1					
bl			2	1			
cj			1				
ck		3	2	1			
cl	6	5	4	3	2	1	
dh	3	2	1				
di		3	2	1			
eh	4	3	2	1			
ei	5	4	3	2	1		
fh			4	3	2	1	
fi		6	5	4	3	2	1

46 derivs.

Table No. 99 (continued).

Deg.	8							
Ord.	0	2	4	6	8	10	12	14
	g^2 q	r	b^2g bm dj gh	as bn . . dk . . ej gi	abj adg b^4 b^2h bcg . . bd^2 cm . . ek . . h^2	abk aeg ap b^2i . . bde cn . . dl . . fj . . hi	a^2bg a^2m ab^2d acj . . adh b^3c bch . . bc^2 c^2g . . cd^2 . . el . . fk . . i^2	a^2n . . ab^2e abl ack . . adi . . ach afg bci . . bdf . . cde
ao			1					
ap	5	4	3	2	1			
bm	2	1						
bn		2	1					
cm			2	1				
cn		4	3	2	1			
dj		1						
dk	3	2	1					
dl			3	2	1			
ej			1					
ek		3	2	1				
el		5	4	3	2	1		
fj					1			
fk				3	2	1		
fl		7	6	5	4	3	2	1
hh	4		2					
hi		4	3	2	1			
ii	6		4		2			

52 derivs.

Table No. 99 (continued).

Deg.	9					
Ord.	1	3	5	7	9	11
	gj	bo gk s	ag^2 aq b^2j bdg . . dm hj	ar . . b^2k beg bp co . . dn . . em gl . . hk . . ij	ab^2g abm adj agh b^3d bcj . . bdh cdg . . d^3 . . en . . ik	a^2o abn . . adk . . aej agi . b^3e b^2l bck . . bdi . . beh . . bfg ceg cp . . d^2e . . fm . . hl
ar		2	1			
bo	1					
bp		2	1			
co			1			
cp	5	4	3	2	1	
dm	2	1				
dn	3	2	1			
em		2	1			
en	4	3	2	1		
fm				2	1	
fn			4	3	2	1
hj		1				
hk	3	2	1			
hl		4	3	2	1	
ij			1			
ik		3	2	1		
il	6	5	4	3	2	1

46 derivs.

Table No. 99 (continued).

Deg.	10						
Ord.	0	2	4	6	8	10	12
		bg^2 bq gm j^2	br . . do gn jk	agj b^3g b^2m bdj bgh cg^2 cq . . d^2g . . eo . . hm . . k^2	abo agk as b^2n . . bdk . . bej bgi cr . . deg . . dp . . hn . . im . . jl	a^2g^2 a^2q ab^2j $abdg$ . . adm ahj . . b^5 b^3h b^2cg . . b^2d^2 bcm . . bek . . bh^2 cdj cgh . . d^2h . . e^2g . . ep . . fo . . in . . kl	a^2r . . ab^2k $abeg$ abp aco . . adn . . aem agl . . ahk . . aij b^3i . . b^2de bcn . . bdl . . bfj . . bhi . . cdk . . cej cgi . . d^2i . . deh . . dfg
as		3	2	1			
br	2	1					
cr			2	1			
do		1					
dp		3	2	1			
eo			1				
ep	5	4	3	2	1		
fo					1		
fp			5	4	3	2	1
hm		2	1				
hn	4	3	2	1			
im			2	1			
in		4	3	2	1		
jk		1					
jl				1			
kk		2					
kl			3	2	1		
ll		6		4		2	

44 derivs.

Table No. 99 (continued).

Deg.	11				
Ord.	1	3	5	7	9
	go t	bgj dg^2 dq jm	b^2o bgk bs . . dr eg^2 eq gp . . ho . . jn . . km	abg^2 abq agm aj^2 b^3j b^2dg . . bdm bhj cgj . . d^2j . . dgh . . er . . io . . kn	abr . . ado agn ajk . . b^3k . . b^2eg b^2p bco . . bdn . . bem bgl . . bhk . . bij cjk cs . . d^2k . . dej . . dgi . . egh fg^2 fq . . hp . . lm
bs	2	1			
cr		3	2	1	
dr	2	1			
er		2	1		
fr				2	1
ho		1			
hp	4	3	2	1	
io			1		
ip	5	4	3	2	1
jm	1				
jn		1			
km	2	1			
kn	3	2	1		
lm	2	1			
ln		4	3	2	1

35 derivs.

Table No. 99 (continued).

Deg.	12					
Ord.	0	2	4	6	8	10
	g^3	gr	b^2g^2	ago	$abgj$	ab^2o
	g^9	jo	b^2q	at	. . adg^2	$abgk$
	u		bgm	b^2r	adq	abs
			bj^2	. . bdo	ajm	. . adr
			dgj	bgn	b^4g	aeg^2
			g^2h	bjk	b^3m	aeq
			hq	. . dgk	b^2dj	agp
			. . ko	. . ds	b^2gh	. . aho
			. . m^2	. . egj	bcg^2	. . ajn
				g^2i	bcq	. . akm
				. . hr	. . bd^2g	b^3n
				iq	. . beo	. . b^2dk
				. . jp	. . bhm	. . b^2ej
				. . mn	. . bk^2	. . b^2gi
					cgm	bcr
					cj^2	. . $bdeg$
					. . d^2m	. . bdp
					. . dhj	. . bhn
					. . egk	. . bim
					. . es	. . bjl
					. . gh^2	. . cdo
					. . ir	cgn
					. . kp	cjk
					. . lo	. . d^2n
					. . n^2	. . dem
						. . dgl
						. . dhk
						. . dij
						. . ehj
						. . fgj
						. . ghi
at			1			
ds	3	2	1			
es		3	2	1		
fs				3	2	1
hr		2	1			
ir			2	1		
jo	1					
jp			1			
ko		1				
kp		3	2	1		
lo				1		
lp		5	4	3	2	1

26 derivs.

Table No. 99 (continued).

Deg.	13			
Ord.	1	3	5	7
	g^2j jq v	bgo bt g^2k gs . . jr kq . . mo	ag^3 agq au b^2gj bdg^2 bdq bjm . . dgm . . dj^2 ghj . . kr . . no	agr ajo . . b^3o . . b^2gk b^2s . . bdr beg^2 beq bgp . . bho . . bjn . . bkm cgo ct . . d^2o . . dgn . . djk . . egm . . ej^2 g^2l . . ghk . . gij . . hs . . lq . . mp
bt	1			
ct			1	
hs	3	2	1	
is		3	2	1
jr	1			
kr		1		
lr				1
mo	1			
mp		2	1	
no		1		
np	4	3	2	1

19 derivs.

Table No. 99 (continued).

Deg.	14						15			
Ord.		0	2	4	6	8		1	3	5
			bg^3 bgq bu g^2m gj^2 mq . . o^2	bqr bjo . . dgo . . dt g^2n gjk js . . mr nq	. . ag^2j ajq av b^3g^2 b^3q b^2gm b^2j^2 $bdgj$ bg^2h bhq . . bko . . bm^2 cg^3 cgq cu . . d^2g^2 . . d^2q . . djm . . ego . . et . . ghm . . gk^2 . . hj^2 . . ks . . nr . . op	$abgo$ abt ag^2k ags . . ajr akq . . amo b^3r . . b^2do . . b^2gn b^2jk . . $bdgk$ . . bds . . $begj$ bg^2i . . bhr biq . . bjp . . bmn cgr cjo . . d^2r . . deg^2 . . deq . . dgp . . dho . . djn . . dkm . . ejm . . ghn . . gim . . gjl . . hjk . . ij^2		g^2o gt oq	bg^2j bjq bv dg^3 dgq . . du gjm . . j^3 . . or	. . b^2go b^2t bg^2k bgs . . bjr bkq . . bmo . . dgr . . djo eg^3 egq . . eu g^2p . . gho . . gjn . . gkm . . ht . . j^2k . . ms pq
	dt			1			bv	1		
	et				1		cv			1
	ft					1	ht		1	
	js		1				it			1
	ks	3	2	1			ms	2	1	
	ls			3	2	1	ns	3	2	1
	mr	2	1				or	1		
	nr		2	1			pr		2	1
	op			1						
	pp		4		2					

17 derivs.

12 derivs.

Table No. 99 (continued).

Deg.	16					
Ord.	0	2	4	6	8	Col. 8 concl.
	g^4	g^2r	b^2g^3	ag^2o	. . abg^2j	. . ekq
	g^2q	gjo	b^2gq	agt	$abjq$	. . cmo
	gu	jt	b^2u	aoq	abv	. . g^2h^2
	q^2	qr	bg^2m	. . b^2gr	. . adg^3	. . gir
			bgj^2	b^2jo	$adgq$	. . gkp
			bmq	. . $bdgo$	. . adu	. . glo
			. . bo^2	. . bdt	$agjm$	. . gn^2
			dg^2j	bg^2n	. . aj^3	. . h^2q
			djq	$bgjk$	. . aor	. . hko
			. . dv	bjs	b^4g^2	. . hm^2
			g^3h	. . bmr	b^4q	. . ijo
			ghq	. . bnq	b^3gm	. . jkn
			. . gko	. . dg^2k	b^3j^2	. . k^2m
			. . gm^2	. . dgs	b^2dgj	. . lt
			. . hu	. . djr	b^2g^2h	. . ps
			. . j^2m	. . dkq	b^2hq	
			. . kt	. . dmo	. . b^2ko	
			. . os	. . eg^2j	. . b^2m^2	
			. . r^2	. . egj	bcg^3	
				. . ev	$bcgq$	
				g^3i	bcu	
				. . ghr	. . bd^2g^2	
				giq	. . bd^2q	
				. . gjp	. . $bdjm$	
				. . gmn	. . $bego$	
				. . hjo	. . bet	
				. . iu	. . $bghm$	
				. . j^2n	. . bgk^2	
				. . jkm	. . bhj^2	
					. . bks	
					. . bur	
					. . bop	
					cg^2m	
					cgj^2	
					cmq	
					. . co^2	
					. . d^2gm	
					. . d^2j^2	
					. . $dghj$	
					. . dkr	
					. . dno	
					. . eg^2k	
					. . egs	
					. . ejr	
dv		1				
ev			1			
fv					1	
jt	1					
kt		1				
lt				1		
op			1			
os		1				
pp		4		2		
ps		3	2	1		

13 derivs.

Table No. 99 (continued).

Deg.	17		
Ord.	1	3	5
	g^3j	bg^2o	ag^4
	gjq	bgt	ag^2q
	gv	boq	agu
	ju	g^3k	aq^2
		g^2s	b^2g^2j
		. . gjr	b^2jq
		gkq	b^2v
		. . gmo	bdg^3
		. . j^2o	$bdgq$
		ku	bdu
		. . mt	$bgjm$
		qs	. . bj^3
			. . bor
			. . dg^2m
			. . dgj^2
			. . dmq
			. . do^2
			g^2hj
			. . gkr
			. . gno
			hjq
			. . hv
			. . jko
			. . jm^2
			. . nt
			. . rs
hv		1	
iv			1
mt	1		
nt		1	
rs	2	1	

6 derivs.

Deg.	18			
Ord.	0	2	4	6
	w	bg^4	bg^2r	. . ag^3j
		bg^2q	. . $bgjo$	$agjq$
		bgu	bjt	agv
		bq^2	bqr	aju
		g^3m	. . dg^2o	b^3g^3
		g^2j^2	. . dgt	b^3gq
		gmq	. . doq	b^3u
		. . go^2	g^3n	b^2g^2m
		j^2q	g^2jk	b^2gj^2
		. . jv	gjs	b^2mq
		mu	. . gmr	. . b^2o^2
		. . ot	gnq	bdg^2j
			. . j^2r	$bdjq$
			jkq	. . bdv
			. . jmo	bg^3h
			. . kv	$bghq$
			nu	. . $bgko$
				. . bgm^2
				bhu
				. . bj^2m
				. . bkt
				. . bos
				. . br^2
				cg^4
				cg^2q
				cgu
				cq^2
				. . d^2g^3
				. . d^2gq
				. . d^2u
				. . $dgjm$
				. . dj^3
				. . dor
				. . eg^2o
				. . egt
				. . eoq
				. . g^2hm
				. . g^2k^2
				. . ghj^2
				. . gks
				. . gnr
				. . gop
				. . hmq
				. . ho^2
				. . jkr
				. . jno
				. . k^2q
				. . kmo
				. . m^3
				. . pt
				. . s^2
jv	1			
kv		1		
lv				1
ot	1			
pt			1	
ss		2		

6 derivs.

Deg.	19	
Ord.	1	3
	g^3o	bg^3j
	g^2t	$bgjq$
	gqo	bgv
	ou	bju
	qt	dg^4
		dg^2q
		. . djt
		dq^2
		g^2jm
		. . gj^3
		. . gor
		jmq
		. . jo^2
		. . mv
		. . rt
mv	1	
nv		1
rt	1	

3 derivs.

Table No. 99 (concluded).

Deg.	20			21	22		24
Ord.	0	2	4	1	0	2	0
	g^5 g^3q g^2u gq^2 qu	bw g^3r g^2jo . . gjt gqr . . joq . . ov ru	b^2g^4 b^2g^2q b^2gu b^2q^2 bg^3m bg^2j^2 $bgmq$ . . bgo^2 bj^2q . . bjv bmu . . bot dg^3q $dgjq$ . . dgv dju g^4h g^2hq . . g^2ko . . g^2m^2 ghu . . gj^2m . . gkt . . gos . . gr^2 hq^2 . . j^4 . . jor . . koq . . m^2q . . mo^2 . . st	g^4j g^2jq g^2v gju jq^2 qv	gw	bg^5 bg^3q bg^2u bgq^2 bqu g^4m g^3j^2 g^2mq . . g^2o^2 gj^2q . . gjv gmu . . got j^2u mq^2 . . o^2q . . t^2	g^6 g^4q g^3u g^2q^2 gqu q^3 u^2
ro				1			
tv							1
sv						1	
ov	1						
pv			1				
sv		1					
	3 derivs.			1 deriv.	1 deriv.		1 deriv.

384. The Canonical form (using the divided expressions, Table No. 98) is peculiarly convenient for the calculation of the derivatives. Some attention is required in regard to the numerical determination: it will be observed that A is given in the standard form $(A_0, A_1, A_2, A_3, A_4, A_5 \between x, y)^5$, while the other covariants are given in the denumerate forms $B=(B_0, B_1, B_2 \between x, y)^2$ &c.: these must be converted into the other form $B=(B_0, \tfrac{1}{2}B_1, B_2 \between x, y)^2$, $C=(C_0, \tfrac{1}{6}C_1, \tfrac{1}{15}C_2, \tfrac{1}{20}C_3, \tfrac{1}{15}C_4, \tfrac{1}{6}C_5, C_6 \between x, y)^6$, &c., the numerical coefficients being of course the reciprocals of the binomial coefficients. We thus have, for instance, the leading coefficients,

$$l.c. \text{ of } AC2 = A_0.\tfrac{1}{15}C_2 - 2.\ A_1.\tfrac{1}{6}C_1 + A_2.C_0, \quad \text{but}$$

$$,,\quad ,,\quad BC2 = B_0.\tfrac{1}{15}C_2 - 2.\tfrac{1}{2}B_1.\tfrac{1}{6}C_1 + B_2.C_0.$$

Moreover, as regards the covariants AA2, AA4, &c., we take what are properly the half-values,

$$
\begin{array}{lll}
l.c. \text{ of } \mathrm{AA}2 = \mathrm{A}_0\mathrm{A}_2 - \mathrm{A}_1{}^2 & & (\text{instead of } \mathrm{A}_0\mathrm{A}_2 - 2\mathrm{A}_1\mathrm{A}_1 + \mathrm{A}_2\mathrm{A}_0), \\
\text{,, ,, } \mathrm{AA}4 = \mathrm{A}_0\mathrm{A}_4 - 4\mathrm{A}_1\mathrm{A}^3 + 3\mathrm{A}_2{}^2 & & (\text{instead of } \mathrm{A}_0\mathrm{A}_4 - 4\mathrm{A}_1\mathrm{A}_3 + 6\mathrm{A}_2\mathrm{A}_2 - 4\mathrm{A}_3\mathrm{A}_1 - \mathrm{A}_4\mathrm{A}_0), \\
\qquad \&\text{c.,} & &
\end{array}
$$

and similarly

$$
\begin{array}{l}
l.c. \text{ of } \mathrm{BB}2 = \mathrm{B}_0\mathrm{B}_2 - (\tfrac{1}{2}\mathrm{B}_1)^2 \\
\text{,, ,, } \mathrm{CC}2 = \mathrm{C}_0 . \tfrac{1}{15}\mathrm{C}_2 - (\tfrac{1}{6}\mathrm{C}_1)^2 \\
\qquad \&\text{c.}
\end{array}
$$

Any one of these leading coefficients, for instance $l.c.$ of AC2, is equal to the corresponding covariant derivative, multiplied, it may be, by a power of a: the index of this power being at once found by comparing the deg-orders, these in fact differing by a multiple of 1.5 the deg-order of a. Thus

$$
\begin{array}{lll}
aa2,\ \mathrm{A}_0\mathrm{A}_2 - \ \mathrm{A}_1{}^2 & \text{deg-orders are } 2.6,\ 2.6 & \text{or } aa2 = \mathrm{A}_0\mathrm{A}_2 - \mathrm{A}_1{}^2, \\
aa4,\ \mathrm{A}_0\mathrm{A}_4 - 4\mathrm{A}_1\mathrm{A}_3 + 3\mathrm{A}_2{}^2, & \text{deg-orders are } 2{\cdot}2,\ 4{\cdot}12 & \text{or } aa4 = \dfrac{1}{a^2}(\mathrm{A}_0\mathrm{A}_4 - 4\mathrm{A}_1\mathrm{A}_3 + 3\mathrm{A}_2{}^2);
\end{array}
$$

we have in fact

$$
\begin{array}{rlcl}
\mathrm{A}_0\mathrm{A}_2 - \mathrm{A}_1{}^2 = & 1.c - 0^2 = c & : & \text{and } aa2 = c, \\
\mathrm{A}_0\mathrm{A}_4 - 4\mathrm{A}_1\mathrm{A}_3 + 3\mathrm{A}_2{}^2 = & 1\ .\ (a^2b - 3c^2) - 4\ .\ 0\ .\ f + 3\ .\ c^2, = a^2b & : & \text{and } aa4 = b.
\end{array}
$$

As another instance, and for the purpose of showing how the calculation is actually effected, consider the derivative $ch2$, which is to be calculated from the leading coefficient of CH2, $= \mathrm{C}_0 . \tfrac{1}{6}\mathrm{H}_2 - 2 . \tfrac{1}{6}\mathrm{C}_1 . \tfrac{1}{4}\mathrm{H}_1 + \tfrac{1}{15}\mathrm{C}_2 . \mathrm{H}_0$: this is

$$
\begin{array}{rl}
= & c(\tfrac{1}{6}a^2g - 2abd - ch) \\
& -2\ .\ \tfrac{1}{2}f\,(\tfrac{1}{2}be - l) \\
& +\ (\tfrac{1}{5}a^2b - c^2)h
\end{array}
$$

=column next written down; but this column contains congregate terms which have to be replaced by their segregate values (see Table No. 96, deg-order 8·16); and we thus obtain

	a^3j	a^2b^3	a^2bh	a^2cg	$abcd$	b^2c^2	c^2h
$\tfrac{1}{5}a^2bh$			$+\tfrac{1}{5}$				
$+\tfrac{1}{6}a^2cg$				$+\tfrac{1}{6}$			
$-2abcd$					-2		
$-\tfrac{1}{2}bef$			$-\tfrac{1}{2}$		$+3$	$+2$	
$-2c^2h$							-2
$+fl$	$\tfrac{1}{3}$	$-\tfrac{1}{3}$	$+\tfrac{2}{3}$	$-\tfrac{1}{3}$	-1	-2	$+2$
$=$	$\tfrac{1}{3}$	$-\tfrac{1}{3}$	$+\tfrac{11}{30}$	$-\tfrac{1}{6}$	0	0	0

viz., the terms other than those divisible by a^2 all disappear: we may either abbreviate the calculation by omitting them *ab initio*, or retain them for the sake of the verification afforded by their disappearance. The factor a^2 divides out, and the final result is

$$ch2 = \tfrac{1}{3}aj - \tfrac{1}{3}b^3 + \tfrac{11}{30}bh - \tfrac{1}{6}cg,$$

which is the proper segregate expression of the derivative $ch2$: of course, we have deg-order CH2=8.16, deg-order $ch2$=6.6, and the difference is 2.10, the double of 1.5, so that the factor a^2 is as it ought to be.

Table 100 (The Derivatives up to the Sixth Order).

Degree 2.

2.2	b
$aa4$	$+1$

2.6	c
$aa2$	$+1$

Degree 3.

3.1	
$ac5$	0

3.3	d
$ab2$	-3
$ac4$	$+\frac{12}{5}$

3.5	e
$ab1$	$+\frac{1}{2}$
$ac3$	$+\frac{1}{20}$

3.7	ab
$ac2$	$+\frac{1}{5}$

3.9	f
$ac1$	$+\frac{1}{2}$

Degree 4.

4.0	g
$ae5$	-2
$bb2$	$-\frac{1}{4}$
$cc6$	$-\frac{1}{40}$

4.2	
$ad3$	0
$ae4$	0

4.4	b^2	h
$ad2$	$-\frac{1}{3}$	$+\frac{1}{3}$
$ae3$	$-\frac{4}{5}$	$-\frac{6}{5}$
$af5$	$+\frac{62}{63}$	$-\frac{83}{63}$
$bc2$	$+\frac{1}{5}$	$-\frac{1}{2}$
$cc4$	$+\frac{3}{25}$	$-\frac{1}{10}$

4.6	i
$ad1$	$+\frac{1}{3}$
$ae2$	$+\frac{6}{5}$
$af4$	$+\frac{13}{63}$
$bc1$	$+\frac{1}{2}$

4.8	ad	bc
$ae1$	$-\frac{6}{5}$	-2
$af3$	$+\frac{59}{42}$	$-\frac{5}{6}$
$cc2$	$+\frac{1}{4}$	$-\frac{1}{20}$

4.10	ae
$af2$	$+1$

4.12	a^2b	c^2
$af1$	$+\frac{2}{9}$	-2

Table No. 100 (concluded).

Degree 5.

5.1	j
ah 4	$+2$
ai 5	$-\frac{4}{3}$
bd 2	$-\frac{1}{3}$
ce 5	$-\frac{8}{5}$

5.3	k
ah 3	$+\frac{1}{2}$
ai 4	$+\frac{1}{3}$
bd 1	$-\frac{1}{6}$
be 2	$-\frac{3}{5}$
cd 3	$+\frac{3}{20}$
ce 4	$+\frac{2}{25}$
cf 6	$-\frac{19}{42}$

5.5	ag	bd
ah 2	$+\frac{1}{6}$	-2
ai 3	$+0$	-2
be 1	$-\frac{1}{2}$	$+\frac{24}{5}$
cd 2	0	$-\frac{2}{15}$
ce 3	$-\frac{1}{20}$	$-\frac{48}{5}$
df 5	$-\frac{1}{72}$	$+\frac{8}{63}$

5.7	be	l
ah 1	-2	-4
ai 2	0	$+\frac{1}{3}$
bf 2	$-\frac{7}{36}$	$+1$
cd 1	0	$+\frac{1}{6}$
ce 2	$\frac{1}{5}$	$-\frac{2}{5}$
df 4	$-\frac{1}{90}$	$+\frac{43}{315}$

5.9	ab^2	ah	cd
ai 1	$-\frac{1}{3}$	$+\frac{1}{3}$	$+3$
bf 1	$+\frac{2}{9}$	$-\frac{1}{2}$	$+3$
ce 1	0	$-\frac{1}{2}$	$+\frac{9}{5}$
df 3	$+\frac{2}{15}$	$-\frac{11}{120}$	$-\frac{439}{420}$

5.11	ai	ce
df 2	$+\frac{1}{45}$	$+\frac{1}{180}$

5.13	a^2d	abc
df 1	$+\frac{1}{2}$	$-\frac{5}{8}$

Degree 6.

6.0	
ci 6	0

6.2	bg	m
ak 3	0	-4
al 5	0	$+\frac{20}{7}$
bh 2	$-\frac{1}{3}$	-2
ch 4	$\frac{1}{10}$	$-\frac{2}{5}$
ci 5	0	$+\frac{2}{3}$
dd 2	0	$+\frac{1}{9}$
de 3	0	$-\frac{4}{5}$
ee 4	-1	$-\frac{48}{25}$
ff 8	$-\frac{5}{324}$	$-\frac{68}{567}$

6.4	n
aj 1	-1
ak 2	$+\frac{1}{3}$
al 4	$-\frac{1}{35}$
bh 1	$+\frac{1}{2}$
bi 2	$+\frac{1}{3}$
ch 3	$+\frac{3}{10}$
ci 4	$+\frac{1}{15}$
de 2	$-\frac{1}{3}$
ef 5	$-\frac{64}{63}$

6.6	aj	b^3	bh	cg
a 1	$-\frac{2}{3}$	$+2$	-3	$+1$
al 3	$-\frac{16}{35}$	$+\frac{2}{7}$	$-\frac{5}{7}$	$+\frac{3}{7}$
bi 1	$-\frac{1}{3}$	$+\frac{5}{6}$	$-\frac{1}{2}$	0
ch 2	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{11}{36}$	$-\frac{1}{6}$
ci 3	$-\frac{1}{6}$	$-\frac{1}{30}$	$+\frac{11}{3}$	$+\frac{7}{60}$
de 1	$-\frac{2}{15}$	$+\frac{2}{15}$	$+\frac{1}{15}$	$-\frac{1}{5}$
df 3	$+\frac{59}{378}$	$-\frac{143}{378}$	$+\frac{425}{756}$	$-\frac{139}{756}$
ee 2	$-\frac{4}{25}$	$+\frac{4}{25}$	$-\frac{38}{25}$	$+\frac{9}{25}$
ef 4	$+\frac{236}{315}$	$-\frac{4}{105}$	$+\frac{10}{63}$	$-\frac{71}{105}$
ff 6	$-\frac{1529}{7938}$	$+\frac{2873}{7938}$	$+\frac{3533}{15876}$	$-\frac{5591}{31752}$

6.8	ak	bi
al 2	$-\frac{4}{21}$	$+\frac{1}{21}$
ch 1	$+\frac{1}{6}$	$+\frac{1}{3}$
ci 2	$+\frac{1}{9}$	$+\frac{4}{45}$
df 2	$-\frac{11}{108}$	$-\frac{1}{54}$
ef 3	$+\frac{32}{315}$	$+\frac{64}{315}$

6.10	a^2g	abd	b^2c	ch
al 1	0	$-\frac{3}{7}$	-1	$+1$
ci 1	0	$-\frac{1}{2}$	$+\frac{1}{6}$	$-\frac{1}{6}$
df 1	0	$-\frac{1}{9}$	$+\frac{1}{3}$	$-\frac{1}{3}$
ef 2	$+\frac{1}{36}$	$+\frac{7}{5}$	$-\frac{19}{45}$	$-\frac{4}{5}$
ff 4	$+\frac{1}{432}$	$-\frac{53}{756}$	$+\frac{89}{756}$	$-\frac{8}{63}$

6.12	abe	al	ci
ef 1	$\frac{2}{9}$	$-\frac{2}{5}$	$-\frac{6}{5}$

6.14	a^2b^2	a^2h	acd	bc^2
ff 2	$-\frac{4}{81}$	$+\frac{1}{36}$	$+\frac{5}{6}$	$-\frac{2}{9}$

which is complete to the sixth degree. I had calculated the derivatives up to the tenth degree, but the results were not in the segregate form.

On the form of the Numerical Generating Functions: the N.G.F. of a Sextic. Article Nos. 385, 386.

385. It is to be remarked that the R.G.F. is derived not from the fraction in its least terms, which is algebraically the most simple form of the N.G.F., but from a form which contains common factors in the numerator and denominator: thus for the quadric, the cubic, and the quartic, writing down the two forms (identical in the case of the quadric) these are—

Quadric

$$\text{N.G.F.} = \frac{1}{1-ax^2 .\ 1-a^2}$$

Cubic

$$\text{N.G.F.} = \frac{1-ax+a^2x^2}{1-a^4 .\ 1-ax^3 .\ 1-ax} \qquad = \frac{1-a^6x^6}{1-a^4 .\ 1-ax^3 .\ 1-a^2x^2 .\ 1-a^3x^3 .}$$

Quartic

$$\text{N.G.F.} = \frac{1-ax^2+a^2x^4}{1-a^2 .\ 1-a^3 .\ 1-ax^4 .\ 1-ax^2} \qquad = \frac{1-a^6x^{12}}{1-a^2 .\ 1-a^3 .\ 1-ax^4 .\ 1-a^2x^2 .\ 1-a^3x^6 .}$$

For the quintic the two forms are, N.G.F. =

(1			$-a^6$			$+a^{12})x^0$
$+(-1$		$+a^4$	$+2a^6$			$-a^{12})ax^1$
$+($	$+a^2$			$-a^8$		$+a^{10})x^2$
$+(-1$		$+a^4$	$+a^6$	$+a^8$	$-a^{10}$	$-a^{12})ax^3$
$+(+1$	$+a^2$	$-a^4$	$-a^6$	$-a^8$		$+a^{12})a^2x^4$
$+($	$-a^2$	$+a^4$			$-a^{10}$	$)a^3x^5$
$+(+1$			$-2a^6$	$-a^8$		$+a^{12})a^2x^6$
$+(-1$			$+a^6$			$-a^{12})a^3x^7$

divided by

$$1-a^4 .\ 1-a^6 .\ 1-a^8 .\ 1-ax^5 .\ 1-ax^3 .\ 1-ax$$

and

$($ 1									$+a^{18})x^0$
$($		a^4	$+a^6$		$+a^{10}$	$+a^{12}$			$)ax$
$($		a^4	$+a^6$	$+a^8$	$+a^{10}$		$+a^{14}$		$-a^{18})a^2x^2$
$($ 1	$+a^2$	$+a^4$		$+a^8$					$)a^3x^3$
$($ 1	$+a^2$	$+a^4$	$+a^6$		$+a^{10}$		$-a^{14}$		$)a^4x^4$
$($ 1		$+a^4$	$+a^6$					$-a^{16}$	$)a^3x^5$
$($	a^2					$-a^{12}$	$-a^{14}$		$)a^2x^6$
$($		a^4		$-a^8$		$-a^{12}$	$-a^{14}$	$-a^{16}$	$-a^{18})ax^7$
$($					$-a^{10}$	$-a^{12}$	$-a^{14}$	$-a^{16}$	$-a^{18})a^2x^8$
$($		$-a^4$		$-a^8$	$-a^{10}$	$-a^{12}$	$-a^{14}$		$)a^3x^9$
$($			$-a^6$	$-a^8$		$-a^{12}$	$-a^{14}$		$)a^4x^{10}$
$(-1$									$-a^{18})a^5x^{11}$

divided by

$$1-a^4.\ 1-a^8.\ 1-a^{12}.\ 1-ax^5.\ 1-a^2x^2.\ 1-a^2x^6$$

this last being in fact equivalent to that used for the determination of the R.G.F.

386. For the sextic the forms are, N.G.F. =

$($ 1	$+a$		$-a^3$	$-a^4$	$-a^5$		$+a^7$	$+a^8)x^0$
$(-1$	$-a$	$+a^2$	$+2a^3$	$+2a^4$	$+a^5$		$-a^7$	$-a^8)ax^2$
$(-1$		$+a^2$	$+a^3$	$+a^4$	$+a^5$		$-a^7$	$-a^8)ax^4$
$($ 1	$+a$		$-a^3$	$-a^4$	$-a^5$	$-a^6$		$+a^8)a^2x^6$
$($ 1	$+a$		$-a^3$	$-2a^4$	$-2a^5$	$-a^6$	$+a^7$	$+a^8)a^2x^8$
$(-1$	$-a$		$+a^3$	$+a^4$	$+a^5$		$-a^7$	$-a^8)a^3x^{10}$

divided by

$$1+a.\ 1-a^2.\ 1-a^3.\ 1-a^4.\ 1-a^5.\ 1-ax^6.\ 1-ax^4.\ 1-ax^2$$

and

1															$+a^{15}$	$)x^0$
$+(\ 1$		$+a^2$		$+a^4$	$+a^5$		$+a^7$		$+a^9$							$)a^3x^2$
$+($		$+a^2$	$+a^3$	$+a^4$	$+a^5$	$+a^6$	$+a^7$	$+a^8$	$+a^9$		$+a^{11}$					$)a^2x^4$
$+(\ 1$	$+a$		$+2a^3$		$+a^5$	$+a^6$		$+a^8$					$-a^{13}$			$)a^3x^6$
$+($	$+a^{\cdot 5}$		$+a^{2\cdot 5}$		$+a^{4\cdot 5}$						$-a^{10\cdot 5}$		$-a^{12\cdot 5}$		$-a^{14\cdot 5}$	$)a^{2\cdot 5}x^8$
$+($		$+a^2$					$-a^7$		$-a^9$	$-a^{10}$		$-2a^{12}$		$-a^{14}$	$-a^{15}$	$)a^2x^{10}$
$+($				$-a^4$		$-a^6$	$-a^7$	$-a^8$	$-a^9$	$-a^{10}$	$-a^{11}$	$-a^{12}$	$-a^{13}$			$)a^3x^{12}$
$+($						$-a^6$		$-a^8$		$-a^{10}$	$-a^{11}$		$-a^{13}$		$-a^{15}$	$)a^2x^{14}$
$+(-1$															$-a^{15}$	$)a^5x^{16}$

divided by

$$1-a^2 . 1-a^4 . 1-a^6 . 1-a^{10} . 1-ax^6 . 1-a^2x^4 . 1-a^2x^8$$

where observe that in the middle term, although for symmetry $a^{\cdot 5}$ $(=\sqrt{a})$ has been introduced into the expression, the coefficient is really rational, viz., the term is $(a^3+a^5+a^7-a^{13}-a^{15}-a^{17})x^8$. The second form or one equivalent to it is due to SYLVESTER: I do not know whether he divided out the common factors so as to obtain the first form. I assume that it would be possible from this second form to obtain a R.G.F., and thence to establish for the 26 covariants of the sextic a theory such as has been given for the 23 covariants of the quintic, but I have not entered upon this question.

Table 93 *bis* (The covariant S, adopted form $=-(\mathrm{D}, \mathrm{M})$).

In this Table, a, b, c, d, e, f denote, as in the tables of former memoirs, the coefficients of the quintic form $(a, b, c, d, e, f \between x, y)^5$.

S=(

$a^3b^0c^3f^3$	− 2	$a^3b^0c^2df^3$	− 3	$a^3b^0cd^2f^3$	+ 3	$a^3b^0d^3f^3$	+ 2
c^2def^2	+ 15	$c^2e^2f^2$	+ 3	cde^2f^2	− 6	$d^2e^2f^2$	− 6
c^2e^3f	− 9	cd^2ef^2	+ 24	ce^4f	+ 3	de^4f	+ 6
cd^3f^2	− 9	cde^3f	− 42	d^3ef^2	− 3	e^6	− 2
cd^2e^2f	− 6	ce^5	+ 18	d^2e^3f	+ 6	$a^2b\,cd^2f^3$	− 15
cde^4	+ 9	d^4f^2	− 18	de^5	− 3	cde^2f^2	+ 30
d^4ef	+ 9	d^3e^2f	+ 33	$a^2b^2d^2f^3$	− 3	ce^4f	− 15
d^3e^3	− 7	d^2e^4	− 15	de^2f^2	+ 6	d^3ef^2	+ 15
$a^2b^2c^2f^3$	+ 6	$a^2b^2cdf^3$	+ 6	e^4f	− 3	d^2e^3f	-- 30
$cdef^2$	− 30	ce^2f^2	− 6	,, $b\,c^2df^3$	− 24	de^5	+ 15
ce^3f	+ 18	d^2ef^2	− 24	$c^2e^2f^2$	+ 24	,, $b^0c^3df^3$	+ 9
d^3f^2	+ 9	de^3f	+ 42	cd^2ef^2	+ 78	$c^3e^2f^2$	− 9
d^2e^2f	+ 6	e^5	− 18	cde^3f	−108	$c^2d^2ef^2$	− 21
de^4	− 9	,, $b\,c^3f^3$	+ 3	ce^5	+ 30	c^2de^3f	+ 15
,, $b\,c^3ef^2$	− 15	c^2def^2	− 78	d^4f^2	− 24	c^2e^5	+ 6
$c^2d^2f^2$	+ 21	c^2e^3f	+ 69	d^3e^2f	+ 24	cd^4f^2	+. 3
c^2de^2f	− 6	cd^3f^2	+ 93	,, $b^0c^4f^3$	+ 18	cd^3e^2f	+ 21
c^2e^4	+ 18	cd^2e^2f	− 51	c^3def^2	− 93	cd^2e^4	− 24
cd^3ef	+ 30	cde^4	− 33	c^3e^3f	+ 21	d^5ef	− 9
cd^2e^3	− 51	d^4ef	− 57	$c^2d^3f^2$	+ 36	d^4e^3	+ 9
d^5f	− 36	d^3e^3	+ 54	$c^2d^2e^2f$	+123	$a\,b^3d^2f^3$	+ 9
d^4e^2	+ 39	,, $b^0c^4ef^2$	+ 24	c^2de^4	− 51	de^2f^2	− 18
,, $b^0c^4df^2$	− 3	$c^3d^2f^2$	− 36	cd^4ef	−111	e^4f	+ 9
c^4e^2f	+ 45	c^3de^2f	− 9	cd^3e^3	+ 39	,, $b^2c^2df^3$	+ 6
c^3d^2ef	− 84	c^3e^4	− 54	d^6f	+ 27	$c^2e^2f^2$	− 6
c^3de^3	− 63	c^2d^3ef	+ 24	d^5e^2	− 9	cd^2ef^2	+ 6
c^2d^4f	+ 45	$c^2d^2e^3$	+129	$a\,b^3cdf^3$	+ 42	cde^3f	− 24
$c^2d^3e^2$	+150	cd^5f	+ 9	ce^2f^2	− 42	ce^5	+ 18
cd^5e	−117	cd^4e^2	−114	d^2ef^2	− 69	d^4f^2	− 45
d^7	+ 27	d^6e	+ 27	de^3f	+ 96	d^3e^2f	+ 96
$a^1b^4cf^3$	− 6	$a^1b^4df^3$	− 3	e^5	− 27	d^2e^4	− 51
def^2	+ 15	e^2f^2	+ 3	,, $b^2c^3f^3$	− 33	,, $b\,c^4f^3$	− 9
e^3f	− 9	,, $b^3c^2f^3$	− 6	c^2def^2	+ 51	c^3def^2	− 30
,, $b^3c^2ef^2$	+ 30	$cdef^2$	+108	c^2e^3f	+ 48	c^3e^3f	+ 66
cd^2f^2	− 15	ce^3f	− 96	cd^3f^2	+ 9	$c^2d^3f^2$	+ 84
cde^2f	+ 24	d^3f^2	− 21	cd^2e^2f	−147	$c^2d^2e^2f$	− 36
ce^4	− 45	d^2e^2f	− 48	cde^4	+ 39	c^2de^4	−102
d^3ef	− 66	de^4	+ 63	d^4ef	+ 78	cd^4ef	−174
d^2e^3	+ 72	,, $b^2c^3ef^2$	− 24	d^3e^3	− 45	cd^3e^3	+210
,, $b^2c^3df^2$	− 21	c^2d^2f	−123	,, $b\,c^4ef^2$	+ 57	d^6f	+ 63
c^3e^2f	− 96	c^2de^2f	+147	$c^3d^2f^2$	− 24	d^5e^2	− 72
c^2d^2ef	+ 36	c^2e^4	+ 66	c^3de^2f	− 78	,, $b^0c^5ef^2$	+ 36
c^2de^3	+213	cd^3ef	+ 78	c^3e^4	− 60	$c^4d^2f^2$	− 45
cd^4f	+120	cd^2e^3	−186	c^2d^3ef	+ 36	c^4de^2f	−120
cd^3e^2	−303	d^5f	+ 51	$c^2d^2e^3$	+108	c^4e^4	− 6
d^5e	+ 51	d^4e^2	− 9	cd^5f	− 24	c^3d^3ef	+204
,, $b\,c^5f^2$	+ 9	,, $b\,c^4df^2$	+111	cd^4e^2	− 6	$c^3d^2e^3$	+120
c^4def	+174	c^4e^2f	− 78	d^6e	− 9	c^2d^5f	− 66
c^4e^3	− 36	c^3d^2ef	− 36	,, $b^0c^5df^2$	− 9	$c^2d^4e^2$	−240
c^3d^3f	−204	c^3de^3	− 54	c^5e^2f	− 51	cd^6e	+144
$c^3d^2e^2$	−174	c^2d^4f	− 96	c^4d^2ef	+ 96	d^8	− 27
c^2d^4e	+330	$c^2d^3e^2$	+150	c^4de^3	+111	$a^0b^4cdf^3$	− 9
cd^6	− 99	cd^5e	+ 30	c^3d^4f	− 27	ce^2f^2	+ 9

$\between x, y)^3$

(continued on next page.)

(continued from last page.)

$a^1b^0c^6ef$	-63	$a^1b\,d^7$	-27	$a^1b^0c^3d^3e^2$	-234	$a^0b^4d^2ef^2$	-18
c^5d^2f	$+66$	,, $b^0c^6f^2$	-27	c^2d^5e	$+141$	de^3f	$+45$
c^5de^2	$+99$	c^5def	$+24$	cd^7	-27	e^5	-27
c^4d^3e	-147	c^5e^3	$+54$	$a^0b^5df^3$	-18	,, $b^3c^3f^3$	$+7$
c^3d^5	$+45$	c^4d^3f	$+27$	e^2f^2	$+18$	c^2def^2	$+51$
$a^0b^6f^3$	$+2$	$c^4d^2e^2$	-93	,, $b^4c^2f^3$	$+15$	c^2e^3f	-72
b^5cef^2	-15	c^3d^4e	$+6$	$cdef^2$	$+33$	cd^3f^2	$+63$
d^2f^2	-6	c^2d^6	$+9$	ce^3f	-63	cd^2e^2f	-213
de^2f	-18	$a^0b^5cf^3$	$+3$	d^3f^2	$+54$	cde^4	$+171$
e^4	$+27$	def^2	-30	d^2e^2f	-66	d^4ef	$+36$
,, $b^4c^2df^2$	$+24$	e^3f	$+27$	de^4	$+27$	d^3e^3	-43
c^2e^2f	$+51$	,, $b^4cd^2f^2$	$+51$	,, $b^3c^3ef^2$	-54	,, $b^2c^4ef^2$	-39
cd^2ef	$+102$	cde^2f	-39	$c^2d^2f^2$	-129	$c^3d^2f^2$	-150
cde^3	-171	ce^4	-27	c^2de^2f	$+186$	c^3de^2f	$+303$
d^4f	$+6$	d^3ef	$+60$	c^2e^4	$+45$	c^3e^4	-18
d^3e^2	$+18$	d^2e^3	-45	cd^3ef	$+54$	c^2d^3ef	$+174$
,, $b^3c^4f^2$	-9	,, $b^3c^3df^2$	-39	cd^2e^3	-96	$c^2d^2e^3$	-345
c^3def	-210	c^3e^2f	$+45$	d^5f	-54	cd^5f	-99
c^3e^3	$+43$	c^2d^2ef	-108	d^4e^2	$+48$	cd^4e^2	$+192$
c^2d^3f	-120	c^2de^3	$+96$	,, $b^2c^4df^2$	$+114$	d^6e	-18
$c^2d^2e^2$	$+345$	cd^4f	-111	c^4e^2f	$+9$	,, $b\,c^5df^2$	$+117$
cd^4e	-87	cd^3e^2	$+147$	c^3d^2ef	-150	c^5e^2f	-51
d^6	-2	d^5e	-30	c^3de^3	-147	c^4d^2ef	-330
,, b^2c^5ef	$+72$	,, $b^2c^5f^2$	$+9$	c^2d^4f	$+93$	c^4de^3	$+87$
c^4d^2f	$+240$	c^4def	$+6$	$c^2d^3e^2$	$+150$	c^3d^4f	$+147$
c^4de^2	-192	c^4e^3	-48	cd^5e	-87	$c^3d^3e^2$	$+186$
c^3d^3e	-186	c^3d^3f	$+234$	d^7	$+18$	c^2d^5e	-201
c^2d^5	$+96$	$c^3d^2e^2$	-150	,, $b\,c^6f^2$	-27	cd^7	$+45$
,, $b\,c^6df$	-144	c^2d^4e	-108	c^5def	-30	,, $b^0c^7f^2$	-27
c^6e^2	$+18$	cd^6	$+57$	c^5e^3	$+30$	c^6def	$+99$
c^5d^2e	$+201$	,, $b\,c^6ef$	$+9$	c^4d^3f	-6	c^6e^3	$+2$
c^4d^4	-87	c^5d^2f	-141	$c^4d^2e^2$	$+108$	c^5d^3f	-45
,, b^0c^8f	$+27$	c^5de^2	$+87$	c^3d^4e	-96	$c^5d^2e^2$	-96
c^7de	-45	c^4d^3e	$+96$	c^2d^6	$+21$	c^4d^4e	$+87$
c^6d^3	$+20$	c^3d^5	-51	,, b^0c^7ef	$+27$	c^3d^6	-20
		,, b^0c^7df	$+27$	c^6d^2f	-9		
		c^7e^2	-18	c^6de^2	-57		
		c^6d^2e	-21	c^5d^3e	$+51$		
		c^5d^4	$+12$	c^4d^5	-12		

I remark that I calculated the first two coefficients S_0, S_1, and deduced the other two S_2 from S_1, and S_3 from S_0, by reversing the order of the letters (or which is the same thing, interchanging a and f, b and e, c and d) and reversing also the signs of the numerical coefficients. This process for S_2, S_3 is to a very great extent a verification of the values of S_0, S_1. For, as presently mentioned, the terms of S_0 form subdivisions such that in each subdivision the sum of the numerical coefficients is $=0$: in passing by the reversal process to the value of S_3, the terms are distributed into an entirely new set of subdivisions, and then in each of these subdivisions the sum of the numerical coefficients is found to be $=0$; and the like as regards S_1 and S_3.

If in the expressions for S_0, S_1, S_2, S_3 we first write $d=e=f=1$, thus in effect combining the numerical coefficients for the terms which contain the same powers in a, b, c, we find

$$\begin{aligned}
\mathrm{S}_0 = \; & a^3(-2c^3+6c^2-6c+2) \\
& +a^2\{b^2(6c^2-12c-6)+b(-15c^3+33c^2-21c+3) \\
& \qquad +b^0(42c^4-147c^3+195c^2-117c+27)\} \\
& +a\ \{b^4.\ 0+b^3(30c^2-36c+6)+b^2(-117c^3+249c^2-183c+51) \\
& \qquad +b(9c^5+138c^4-378c^3+330c^2-99c)+b^0(-63c^6+165c^5-147c^4+45c^3)\} \\
& +a^0.\{b^6.\ 2+b^5(-15c+3)+b^4(75c^2-69c+24)+b^3(-9c^4-167c^3+225c^2-87c-2) \\
& \qquad +b^2(72c^5+48c^4-186c^3+96c^2)+b(-126c^6+201c^5-87c^4) \\
& \qquad +b^0(27c^8-45c^7+20c^6)\}
\end{aligned}$$

which for $c=1$ becomes

$$=2b^6-12b^5+30b^4-40b^3+30b^2-12b+2, \text{ that is } 2(b-1)^6.$$

and for $b=1$, becomes $=0$.

$$\begin{aligned}
\mathrm{S}_2 = \; & a^3(0c^2+0c+0) \\
& +a^2\{b^2(0c+0)+b(3c^3-9c^2+9c-3)+b^0(24c^4-99c^3+153c^2-105c+27)\} \\
& +a\ \{b^4.\ 0+b^3(-6c^2+12c-6)+b^2(-24c^3+90c^2-108c+42) \\
& \qquad +b(33c^4-90c^3+54c^2+30c-27)+b^0.(-27c^6+78c^5-66c^4+6c^3+9c^2)\} \\
& +a^0\{b^5(3c-3)+b^4(-15c+15)+b^3(6c^3-12c^2+36c-30) \\
& \qquad +b^2(9c^5-42c^4+84c^3-108c^2+57c)+b(9c^6-54c^5+96c^4-51c^3) \\
& \qquad +b^0(9c^7-9c^6)\}
\end{aligned}$$

which for $c=1$ becomes $=0$.

$$\begin{aligned}
\mathrm{S}_3 = \; & a^3(0c+0) \\
& +a^2\{b^2.\ 0+b(0c^2+0c+0)+b^0(18c^4-72c^3+108c^2-72c+18)\} \\
& +a\ \{b^3(0c+0)+b^2(-33c^3+99c^2-99c+33)+b(57c^4-162c^3+144c^2-30c-9) \\
& \qquad +b^0(-60c^5+207c^4-261c^3+141c^2-27c)\} \\
& +a^0\{b^5.\ 0+b^4(15c^2-30c+15)+b^3(-54c^3+102c^2-42c-6) \\
& \qquad +b^2(123c^4-297c^3+243c^2-87c+18)+b(-27c^6+102c^4-96c^3+21c^2) \\
& \qquad +b^0(27c^7-66c^6+51c^5-12c^4)\}
\end{aligned}$$

which for $c=1$ becomes $=0$.

$$
\begin{aligned}
S_4 = {} & a^3 .\ 0 \\
& +a^2\{b(0c+0)+b^0(0c^3+0c^2+0c+0)\} \\
& +a\ \{b^3 .\ 0+b^2(0c^2+0c+0)+b(-9c^4+36c^3-54c^2+36c-9) \\
& \qquad +b^0(36c^5-171c^4+324c^3-306c^2+144c-27)\} \\
& +a^0\{b^4(0c+0)+b^3(7c^3-21c^2+21c-7)+b^2(-39c^4+135c^3-171c^2+93c-18) \\
& \qquad +b(66c^5-243c^4+333c^3-201c^2+45c) \\
& \qquad +b^0(-27c^7+101c^6-141c^5+87c^4-20c^3)\}
\end{aligned}
$$

which for $c=1$ becomes $=0$.

It follows that for $c=d=e=f=1$, the value of the covariant S is $=2(b-1)^6x^3$, which might be easily verified.

XX. *On the Classification of Loci.*

By W. K. Clifford, *F.R.S., Professor of Applied Mathematics in University College, London.*

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Part I.—CURVES.

By a *curve* we mean a continuous one-dimensional aggregate of any sort of elements, and therefore not merely a curve in the ordinary geometrical sense, but also a singly infinite system of curves, surfaces, complexes, &c., such that one condition is sufficient to determine a finite number of them. The elements may be regarded as determined by k coordinates; and then, if these be connected by $k-1$ equations of any order, the curve is either the whole aggregate of common solutions of these equations, or, when this breaks up into algebraically distinct parts, the curve is one of these parts. It is thus convenient to employ still further the language of geometry, and to speak of such a curve as the complete or partial intersection of $k-1$ loci in flat space of k dimensions, or, as we shall sometimes say, in a k-flat. If a certain number, say h, of the equations are linear, it is evidently possible by a linear transformation to make these equations equate h of the coordinates to zero; it is then convenient to leave these coordinates out of consideration altogether, and only to regard the remaining $k-h-1$ equations between $k-h$ coordinates. In this case the curve will, therefore, be regarded as a curve in flat space of $k-h$ dimensions. And, in general, when we speak of a curve as in flat space of k dimensions, we mean that it cannot exist in flat space of $k-1$ dimensions.

The whole aggregate of linear complexes may be regarded as constituting a space of five dimensions, in which the *special* complexes, or straight lines, constitute a quadric locus. A ruled surface, or scroll, will be thus regarded as a curve lying in a quadric locus in a flat space of five dimensions. If, however, the generators of the scroll all belong to the same linear complex, the scroll must be regarded as a curve lying in a quadric locus in a flat space of *four* dimensions. And if, further, the scroll has two linear directrices, so that the generators belong to a linear congruence, then the scroll may be regarded as a curve lying on an ordinary quadric surface in three dimensions. Thus, for example, quartic scrolls having two linear directrices correspond either to quadri-quadric curves of deficiency 1 (that is, they are *elliptic* curves whose coordinates may be expressed as elliptic functions of one variable), or to the curves of deficiency 0 which are the partial intersections of a quadric and a cubic surface (that is, they are unicursal curves).

This view of ruled surfaces is made excellent use of by Voss, "Zur Theorie der windschiefen Flächen," Math. Annalen, vol. viii. p. 54.

Similar considerations apply to surfaces. By a *surface* we shall mean, in general, a continuous two-dimensional aggregate (which may also be called a *two-spread* or *two-way locus*) of any elements whatever, curves, surfaces, complexes, &c., defined by the whole or a portion of the system of solutions of $k-2$ equations among k coordinates. We shall assume that none of these equations are linear, and then shall speak of the surface as in a flat space of k dimensions. We shall in certain cases go further, and speak of an h-spread or h-way locus, viz., a locus determined by the whole or an algebraically separate portion of the system of solutions of $k-h$ equations among k coordinates; if none of these equations are linear, the h-way locus will be said to be in k dimensions. The general point of view is that of Professor CAYLEY, "On the Curves which satisfy given Conditions," Phil. Trans., Vol. 158 (1868), pp. 75–144; the methods of enumeration are those of Dr. SALMON, 'Solid Geometry,' p. 261.

Theorem A. *Every proper curve of the* n*th order is in a flat space of* n *dimensions or less.* For through $n+1$ points of it we can draw a flat space of n dimensions, which must therefore contain the curve, since it meets it in a number of points greater than its order.

Thus, for example, there is no curve of the second order, in space of any number of dimensions, except a plane conic. If, therefore, a system of curves, in a plane or on any surface, is such that two curves of the system can be drawn through an arbitrary point, then the coordinates of a varying curve of the system may be represented by $x_i+2\theta y_i+\theta^2 z_i$ $(i=1, 2, 3 \ldots k)$, and the envelope of the system is, in the case of plane curves, a curve having the equation $\sqrt{U}+\sqrt{V}+\sqrt{W}=0$, where U, V, W are three curves of the system; in the case of curves on a surface, it is the intersection of the surface with another having an equation of that form.*

[* Professor HENRICI has kindly written for me the following notes in elucidation of this argument:—

"In the first sentence of the paper it is stated that by a *curve* is meant any one-dimensional aggregate of any sort of elements. The definitions given are algebraical, but the reasoning later on becomes more and more geometrical.

"In this note the connexion between the algebraical definition and the geometrical reasoning will be shown in the case where the elements are plane curves of order n.

"If we suppose a curve given by its equation in point coordinates we may take the coefficients as homogeneous coordinates of the curve.

"As there are $\frac{n(n+3)}{2}$ ratios of these coefficients, it follows that all curves of order n in a plane constitute a $\frac{n(n+3)}{2}$ spread, and this will be a *flat* spread as no relation has been supposed between the coordinates.

"To determine in this spread a k-flat, $k<\frac{n(n+3)}{2}$, we have to assume a sufficient number of equations between the coordinates, or denoting by $n_1, n_2, \ldots$ curves of order n we may write down the equation of

To particularise still further, a system of conics having the characteristic $\mu=2$ must always have quadruple contact with a quartic curve; and the different species may be enumerated by studying the successive degeneration of the curve, ending with the fundamental system $\nu=1$, when it breaks up into four straight lines.

So again, there is no quadric scroll, in any number of dimensions, except the ordinary quadric surface which is in flat space of three dimensions.

A curve of the third order must be either the known skew cubic in three dimensions, or a plane cubic. Hence, if a system of curves be such that three of them can be drawn through an arbitrary point, the equation of any curve of the system is of one of the two forms—

$$\mathrm{U}+3\mathrm{V}t\quad+3\mathrm{W}t^2+\mathrm{X}t^3=0,$$
$$\mathrm{U}+\ \mathrm{V}\,\mathrm{sn}^2u+2\mathrm{W}\,\mathrm{sn}\,u\,\mathrm{cn}\,u\,\mathrm{dn}\,u=0,$$

where t, u are parameters. Hence it is easy to write down the equations to the envelopes in the two cases, and to enumerate the distinct species.

---

one element in the k-flat in the form $a_1u_1+a_2u_2+\ \ldots\ +a_{k+1}u_{k+1}=0$, and take the k ratios of the a as the coordinates of a variable curve.

"For $k=2$ we get a *net* as the flat space of two dimensions or as the *plane* in this space, and for $k=1$ a pencil corresponding to the *line*.

"If, on the other hand, we assume in the k-flat $k-1$ equations between the coordinates a, there remains a singly infinite number of curves, that is according to Professor CLIFFORD a *curve* (with curves as elements), according to the usual nomenclature a series of curves.

"To determine the order of this curve we have to find the number of elements on it which satisfy a linear relation between the coordinates. In our case the condition that a curve shall pass through a given point gives such a relation, and the number of curves through a point is the *order* in question.

"Hence, if we wish to extend a theorem relating to a curve (in the ordinary sense with points as elements, but in any number of dimensions) to a proposition relating to a series of curves, or if we wish to illustrate in a plane a theorem relating to a curve in more dimensions than three, we have instead of a point on the curve to take a curve in the series, and to replace the order of the curve by the index of the series.

"The theorem that every curve of order two is a *plane* curve becomes thus—the curves in a series of index 2 belong to a *net*.

"Further, the coordinates of a point on a conic may be represented as expressions of the second degree in a variable parameter, say by $x_i+2\theta y_i+\theta^2 z_i$; where $i=1, 2, 3$, if the coordinates are taken in the plane of the conic, but if they are taken in space we have to take $i=1, 2, 3, 4$, and so on for more dimensions. The locus of these points, that is, the conic, is then given by an equation of the form

$$\sqrt{\mathrm{U}}+\sqrt{\mathrm{V}}+\sqrt{\mathrm{W}}=0$$

where U, V, W are three of the points.

"If we apply this to our series we obtain the results stated in the text, viz., the coordinates of any curve of a series such that two curves pass through a given point are of the form quoted, and the equation of the envelope is of the form

$$\sqrt{\mathrm{U}}+\sqrt{\mathrm{V}}+\sqrt{\mathrm{W}}=0,$$

U, V, W being three of the curves.

"Similarly, if the series is such that three pass through any point, then the series may be considered as a 'curve' of order three, and the statements made in the text follow at once from the known properties about cubic curves, which are either unicursal (twisted, or plane nodal, cubics) or they are plane curves of deficiency one."—January, 1879.]

A cubic scroll must be of the nature of the skew cubic, because it is a curve (with complexes for elements) which is obliged to lie on a quadric locus (that of the special complexes, or straight lines).

Theorem B. *A curve of order* n *in flat space of* k *dimensions (and no less) may be represented, point for point, on a curve of order* n−k+2 *in a plane.*

The proposition is obvious when $k=3$. The cone standing on a curve of order n (in ordinary space of three dimensions), and having its vertex at a point of the curve, is of order $n-1$; if then we cut this cone by a plane, we have the tortuous curve represented, point for point, on a plane curve of order $n-1$.

Now this process is applicable in general. Starting with an arbitrary point, P, of a curve in any number of dimensions, let us join this point to all the other points of the curve; we shall thus get a *cone* of order $n-1$. For any flat locus of $k-1$ dimensions drawn through the point P must meet the curve in n points, of which P is one; and therefore it must meet the cone in $n-1$ lines. Hence, if we cut this cone by such a flat $(k-1)$way locus *not* passing through P, we shall get a curve of order $n-1$ in flat space of $k-1$ dimensions, which is a point-for-point representation of the original curve. By continuing this process we may go on diminishing the order of the curve and the number of dimensions by equal quantities, until we have subtracted $k-2$ from each; when we are left with a curve of order $n-k+2$ in a plane.

The reduction may, however, be effected in one step. A flat $(k-2)$way locus may be drawn through $k-1$ arbitrary points. Suppose it to contain $k-2$ consecutive points of the curve at P, and another variable point, Q, of the curve. Such a locus will meet an arbitrary plane in one point, R. As Q then moves about on the curve, R will trace out on the plane a curve which corresponds to it, point for point. But this curve is of order $n-k+2$, for a flat $(k-1)$way locus, passing through $k-2$ consecutive points of the original curve at P, will meet that curve in $n-k+2$ other points, and therefore will meet also the locus of R in $n-k+2$ points. This locus is, therefore, of order $n-k+2$, as was to be proved.

The fixed points through which the variable $(k-2)$way locus passes need not all be united at P, but they may be any $k-2$ arbitrary points on the curve.

We will now consider some examples of this remark.

1. *Unicursal curve of order* n *in* n-*dimensional space.*

A curve of order n *in flat space of* n *dimensions (and no less) is always unicursal.*—We may prove this independently by considering a variable $(n-1)$flat which passes through $n-1$ fixed points on the curve. Its equation will be of the form $A+tA'=0$, where t is a variable parameter, and it will meet the curve in one other point, which is thus associated with a value of t.

The equations to such a curve may always be written in the form—

$$0=\begin{Vmatrix} A, B, C \ldots K \\ B, C, D \ldots L \end{Vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the A, B, C . . . K, L are linear functions of the coordinates, and the number of columns is $=n$. For the $n+1$ homogeneous coordinates are proportional to rational integral functions of t of the n^{th} order. Solving these $n+1$ equations for $1, t, t^2 \ldots t^n$ we find

$$1, t, t^2 \ldots t^n = A, B, C \ldots L,$$

which is equivalent to the system written down above.

The more general system of equations—

$$0=\begin{Vmatrix} A, B \ldots K \\ A', B' \ldots K' \end{Vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the A . . . K, A′ . . . K′ are linear functions as before, may always and easily be reduced to the former, for they are got by eliminating t from the n equations.

$$\begin{aligned} &A+tA'=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) \\ &B+tB'=0, \\ &\vdots \\ &K+tK'=0. \end{aligned}$$

We may, however, solve these equations for the ratios of the coordinates, which will thus be expressed as rational functions of t of the n^{th} order. Solving these for $1, t, t^2 \ldots t^n$ we come back to the previous system.

The equations (3) exhibit the curve as the locus of the intersection of corresponding elements in n projective pencils.

The equation to the $(n-1)$flat which passes through the n points whose parameters are $t_1, t_2 \ldots t_n$, is easily seen to be—

$$0=\begin{vmatrix} A, & B, & C, \ldots & L \\ 1, & t_1, & t_1^2, \ldots & t_1^n \\ 1, & t_2, & t_2^2, \ldots & t_2^n \\ \vdots & & & \\ 1, & t_n, & t_n^2, \ldots & t_n^n \end{vmatrix}$$

But this equation is manifestly divisible by the coefficient of L, which is the product of the differences of all the t. If we write—

$$\begin{aligned} &\Sigma_1=t_1+t_2+t_3+ \ldots +t_n, \\ &\Sigma_2=t_1t_2+t_1t_3+t_2t_3+ \ldots +t_{n-1}t_n, \\ &\text{etc.}=\text{etc.} \\ &\Sigma_n=t_1t_2 \ldots t_n, \end{aligned}$$

then the equation is

$$0=L-K\Sigma_1+\ldots\pm B\Sigma_{n-1}\mp A\Sigma_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If we omit the suffixes of the t in this formula we obtain the equation to the osculant $(n-1)$flat at the point t. Namely (beginning at the other end), it is—

$$0=\mathrm{A}t^n-n\mathrm{B}t^{n-1}+\tfrac{1}{2}n(n-1)\mathrm{C}t^{n-2}-\ldots\pm n\mathrm{K}t\mp\mathrm{L} \quad \ldots\ldots \quad (5)$$

and we see at once that *the class of such a curve is always equal to its order.*

We thus obtain a very useful representation (*Abbildung*) of the points of the n-dimensional space by means of groups of n points on such a unicursal curve, namely, each point in the space is represented by the points of contact of the n osculant $(n-1)$flats which pass through it. The use of such a representation of ordinary three-dimensional space by means of a skew cubic was pointed out by Dr. HIRST, and the corresponding representation of a plane by means of a conic has been used by M. DARBOUX ('Sur une classe remarquable de courbes et de surfaces algébriques,' Paris, 1873, Note II., p. 183), and by me ("On the Transformation of Elliptic Functions," Proc. Lond. Math. Soc., vol. vii. (1875), pp. 25–38 and 225–233). It may be worth while to mention that an extension to all space of the theory of the in-and-circumscribed polygon may be obtained by this means.

A curve of this kind determines also a dualistic correspondence in the space of n dimensions. Through every point may be drawn n osculant $(n-1)$flats, and through their points of contact another $(n-1)$flat, which shall be called the *polar* of the point. If the point moves along a straight line its polar will pass through a fixed $(n-2)$flat, the *polar* of the line. And generally if the point lies in any k flat the polar will pass through a fixed $(n-k-1)$flat.

When $n=2$ we have the ordinary system of polar reciprocals in regard to a plane conic. When $n=3$ we have that system in regard to a skew cubic which is described by SCHRÖTER, 'Crelle,' vol. lxv. p. 39. These two systems are typical respectively of the cases in which n is even and odd. When n is even, the relation between the coordinates of two points, which expresses that each lies in the polar of the other, is a symmetrical one; consequently those points which lie in their own polars are points on a certain quadric locus, and the system is merely that of the poles and polars in regard to this quadric locus upon which the curve lies. The equation to this locus is at once obtained by equating to zero the quadrinvariant of the form $(1, t)^n$ which occurs in the equation (5) of the osculant $(n-1)$flat, namely, it is

$$0=\mathrm{AL}-n\mathrm{BK}+\tfrac{1}{2}n(n-1)\mathrm{CH}-\text{etc.} \quad \ldots\ldots \quad (6)$$

To prove this, observe that if in the equation (5) we substitute the coordinates of any point p, the values of t which satisfy the equation are the parameters of the points of contact of the osculant $(n-1)$flats which pass through the point. If $t_1, t_2, \ldots t_n$ be these values, the equation (4) represents the $(n-1)$flat which passes through the points of contact, that is to say, the polar of the point. Now if we denote by $\mathrm{A}', \mathrm{B}', \ldots$ the results of substituting the coordinates of the point p in $\mathrm{A}, \mathrm{B}, \ldots$ then we shall have—

$$\begin{aligned} A'\Sigma_1 &= nB', \qquad (7) \\ A'\Sigma_2 &= \tfrac{1}{2}n(n-1)C', \\ &\vdots \\ A'\Sigma_n &= L' \end{aligned}$$

so that, when n is even the equation of the polar is—

$$0=AL'+A'L-n(BK'+B'K)+\tfrac{1}{2}n(n-1)(C'H+C'H)-\text{etc.} \quad . \quad . \quad . \quad . \quad (8)$$

that is, it is simply the polar of the point in regard to the quadric (6).

It is to be observed that the quadric is completely determined when the curve is given. I reserve the question of the conditions to which the curve is subject when the quadric locus is given, or, say, the discussion of the problem to represent the relation of poles and polars in regard to a quadric locus (in space of an even number of dimensions) by means of a unicursal curve.

But when n is odd, the last term of equation (4) is negative, and the equation of the polar is—

$$0=AL'-A'L-n(BK'-B'K)+\tfrac{1}{2}n(n-1)(CH'-C'H)-\text{etc.} \quad . \quad . \quad . \quad (9)$$

that is, it is skew symmetrical, and *every point lies upon its polar.* It is convenient to use the term *co-flat* for $n+1$ points, which are in the same $(n-1)$flat; with this nomenclature we may say that *when* n *is odd every point is co-flat with the* n *points of contact of the osculant* (n—1)*flats, which can be drawn through it.* This will be recognised as an extension of the property of a skew cubic, that every point in space is co-planar with the points of contact of the three osculating planes which can be drawn through it.

A case of this skew symmetrical relation is given by any arbitrary state of motion of the whole space as a rigid body, the relation between two points being that the line joining them moves perpendicularly to itself. The polar of any point is an $(n-1)$flat drawn through it perpendicular to the direction of its motion. When n is even there is always one point which remains at rest, and all the polars pass through this point. Thus the general motion of a solid in an even number of dimensions always depends in this simple way on the motion in one dimension less. In an odd number of dimensions, however, every point moves in the general case; but if any point is at rest, then all the points in a certain straight line are at rest.

Besides its order and class, a curve has, in general, characteristic numbers intermediate to these, which may be called its first rank, second rank, etc. The first rank is the order of the locus traced out by straight lines through two consecutive points of the curve; the second rank, of that traced out by planes through three consecutive points; and generally the k^{th} rank is the order of the $(k+1)$wide locus traced out by k-flats through $k+1$ consecutive points. For the curve just considered these numbers are $2(n-1)$, $3(n-2)$, . . . $(k+1)(n-k)$; it is convenient to derive them from the corresponding numbers for its projection, the unicursal curve of order n in $n-1$ dimensions, to which we now proceed.

2. *Unicursal curve of order* n *in* n−1 *dimensions.*

Every curve of order n *in flat space of* n−1 *dimensions is either unicursal or elliptic.* For it may be represented point-for-point on a plane cubic.

We shall treat these two cases in succession. They are exemplified by the two species of quartics in ordinary tri-dimensional space.

The coordinates of a point on the unicursal curve are proportional to rational integral functions of a parameter t. This representation may be simplified in a manner due to ROSANES, 'Crelle,' vol. lxxv. p. 166. We have n binary quantics of order n; now these may be linearly combined in n different ways so as to produce a perfect n^{th} power. Hence the original quantics may be expressed each as a linear function of the n^{th} powers of the same n linear quantics. Thus, for example, three binary cubics may be simultaneously reduced to the forms.

$$\begin{gathered} au^3+b\,v^3+c\,w^3, \\ a'u^3+b'\,v^3+c'\,w^3, \\ a''u^3+b''v^3+c''w^3, \end{gathered}$$

where $u+v+w=0$ identically. If the x, y, z of a point in a plane are respectively proportional to these cubics, we may, by solving the equations for u^3, v^3, w^3, obtain three linear functions X, Y, Z of the coordinates, which are respectively proportional to u^3, v^3, w^3. Transforming them to the new triangle whose sides are $X=0$, $Y=0$, $Z=0$ we must have the equation of a unicursal cubic expressed in the form

$$X^{\frac{1}{3}}+Y^{\frac{1}{3}}+Z^{\frac{1}{3}}=0.$$

It is clear that the lines $X=0$, $Y=0$, $Z=0$ are tangents at the three points of inflexion.

In general, let the n quantics be

$$\begin{gathered} a_0+na_1t+\ldots+a_nt^n, \\ b_0+nb_1t+\ldots+b_nt^n, \\ \vdots \\ h_0+nh_1t+\ldots+h_nt^n, \end{gathered}$$

then the linear quantics u, v, w . . . are the factors of

$$\begin{vmatrix} t^n, & -nt^{n-1}, & \frac{1}{2}n(n-1)t^{n-2}, & \ldots t \\ a_0, & a_1, & a_2, & \ldots a_n \\ b_0, & b_1, & b_2, & \ldots b_n \\ \vdots & \vdots & \vdots & \vdots \\ h_0, & h_1, & h_2, & \ldots h_n \end{vmatrix}$$

Since there are $n-2$ identical relations between n linear quantics, the $n-2$ equations of the unicursal curve may be written in the form

$$\left\| \begin{matrix} X_1^{\frac{1}{n}}, & X_2^{\frac{1}{n}}, & \ldots & X_n^{\frac{1}{n}} \\ \alpha_1, & \alpha_2, & \ldots & \alpha_n \\ \beta_1, & \beta_2, & \ldots & \beta_n \end{matrix} \right\| =0\,;$$

it is evident that the equations $X_1, X_2, \cdots X_n=0$ represent stationary osculant $(n-2)$flats, that is to say, $(n-2)$flats which pass through n consecutive points of the curve.

The properties of this curve may be very conveniently studied by regarding it as a projection of the curve considered in the last section. If all the points of that curve be joined to a point O, not situated upon it, the joining lines will form a cone of order n; and on cutting this cone by an $(n-1)$flat we shall obtain the curve now under discussion.

The n points of superosculation, whose existence has just been proved, are then clearly the projections of the points of contact of osculant $(n-1)$flats to the full-skew curve drawn through the point O. It follows that *when* n *is odd, these* n *points of superosculation are on the same* (n$-$2)*flat;* but when n is even this is not the case, unless the point O lies on the quadric locus associated with the full-skew curve, in which case we have a special variety of the projection. Thus the three points of inflexion of a nodal cubic are in one straight line; but a unicursal skew quartic in ordinary space has not in general the property that the points of contact of its four stationary osculating planes are in one plane. The property established above for the full-skew curve shows that this will be the case if the four points form an equianharmonic system, or if the quadrinvariant of the quartic which determines them is equal to zero. And generally when n is even, the n points of superosculation will be co-flat if, and only if, the quantic in t which determines them has its quadrinvariant zero.

By using the values of the coordinates of a variable point of the curve expressed in terms of a parameter t, we may obtain an expression of this quadrinvariant and also of its product by the discriminant in terms of the roots of the quantic. Let $\alpha_1, \alpha_2, \ldots \alpha_n$ be the values of t which belong to the points of superosculation, and $x_1, x_2, \ldots x_n$ the coordinates of a variable point on the curve. Then we may write

$$x_i=(t-\alpha_i)^n, \qquad i=1, 2, \ldots n,$$

and the coordinates of the point α_i are $(\alpha_i-\alpha_1)^n, (\alpha_i-\alpha_2)^n, \ldots (\alpha_i-\alpha_n)^n$. If for shortness we write $(h\ k)$ instead of $\alpha_h-\alpha_k$, then the condition that the n points shall be co-flat is

$$0=\begin{vmatrix} 0\ , & (12)^n, & (13)^n, & \ldots & (1n)^n \\ (21)^n, & 0\ , & (23)^n, & \ldots & (2n)^n \\ (31)^n, & (32)^n, & 0\ , & \ldots & (3n)^n \\ \vdots & \vdots & \vdots & & \vdots \\ (n1)^n, & (n2)^n, & (n3)^n, & \ldots & 0 \end{vmatrix}.$$

This is obviously always satisfied if n is odd, for then the determinant is skew symmetrical, and being of odd order it necessarily vanishes. If, however, n is even, the determinant is a symmetrical function of the roots which vanishes when any two of them are equal; and consequently it must contain as a factor the product of the squares of their differences. Now the determinant is of the order $2n$ in each root, and the discriminant is of order $2(n-1)$; therefore the remaining factor is of order 2 in each root, and being a symmetrical invariant must be a function of the squares of their differences. It can therefore be no other than $\Sigma(\alpha_1-\alpha_2)^2(\alpha_3-\alpha_4)^2 \ldots (\alpha_{n-1}-\alpha_n)^2$; this is, to a factor *près*, equal to the quadrinvariant of the form $(t-\alpha_1)(t-\alpha_2)\ldots(t-\alpha_n)$.

The equation to the $(n-2)$flat passing through two consecutive points of the curve at t, and through $n-3$ other points $p\ q \ldots u$, is clearly

$$0=\begin{vmatrix} x\ dx\ p\ q \ldots y \\ 1\ \ 2\ \ 3\ 4 \ldots n \end{vmatrix}$$

where the y are current coordinates, and the determinant is expressed in umbral notation. Writing in this for x_i, $(t-\alpha_i)^n$, and for dx_i, $n(t-\alpha_i)^{n-1}dt$, we may observe that the determinant

$$\begin{vmatrix} (t-\alpha_1)^n\ , & (t-\alpha_2)^n \\ (t-\alpha_1)^{n-1}, & (t-\alpha_2)^{n-1} \end{vmatrix}=(\alpha_2-\alpha_1)(t-\alpha_1)^{n-1}(t-\alpha_2)^{n-1},$$

so that the equation is of order $2(n-1)$ in t. It thence follows that $2(n-1)$ different $(n-2)$flats may be drawn through $n-2$ arbitrary points to touch the curve; or that the developable traced out by the tangent lines is of the order $2(n-1)$.

Similarly, from the value of the determinant

$$\begin{vmatrix} (t-\alpha_1)^n\ , & (t-\alpha_2)^n\ , & \ldots & (t-\alpha_{k+1})^n \\ (t-\alpha_1)^{n-1}, & (t-\alpha_2)^{n-1}, & \ldots & (t-\alpha_{k+1})^{n-1} \\ \vdots & \vdots & & \vdots \\ (t-\alpha_1)^{n-k}, & (t-\alpha_2)^{n-k}, & \ldots & (t-\alpha_{k+1})^{n-k} \end{vmatrix}$$

which is equal to the product of the differences of $\alpha_1, \alpha_2, \ldots \alpha_{k+1}$ multiplied by

$$\{(t-\alpha_1)(t-\alpha_2)\ldots(t-\alpha_{k+1})\}^{n-k},$$

we may conclude that the number of $(n-2)$flats which can be drawn through k consecutive points of the curve and through $n-k$ other arbitrary points is $(k+1)(n-k)$; or that the k-wide locus which is traced out by $(k-1)$flats passing through k consecutive points is of the order $(k+1)(n-k)$. For the equation of an $(n-2)$flat passing through k consecutive points is clearly

$$0=\left|\begin{array}{llllllll} x & dx & d^2x & \ldots & d^{k-1}x & p & q \ldots & y \\ 1 & 2 & 3 & \ldots & k, & & \ldots & n \end{array}\right|$$

where we must substitute for the x_i, dx_i, d^2x_i, etc., the descending powers of $t-a_i$ beginning at the n^{th}. Making k equal to $n-1$ we obtain the equation of the osculant $(n-2)$ flat at any point of the curve; it is

$$0=\frac{\mathrm{P}_1y_1}{(t-a_1)^2}+\frac{\mathrm{P}_2y_2}{(t-a_2)^2}+\ldots+\frac{\mathrm{P}_ny_n}{(t-a_n)^2},$$

where P_i = product of the differences of all the a except a_i. Thus the class of the curve is $2(n-1)$.

3. *Unicursal curve of order* n *in* n—k *dimensions.*

The characteristic numbers belonging to this curve may at once be obtained by regarding it as a projection of the full-skew curve. The number of ranks is $n-k-2$, and the numerical values of them are respectively $2(n-1)$, $3(n-2)$, ... $k(n-k+1)$; the class is $(k+1)(n-k)$; and the number of points of superosculation is $(k+2)(n-k-1)$. For example, the unicursal quintic in three dimensions is of rank $2.4, =8$, and of class $3.3, =9$, and it has $4.2, =8$ superosculant planes.

Convenient forms of the equations may be got by eliminating some of the variables from the equations of the full-skew curve; but care must be taken to select these variables so that the resulting system is sufficiently general.

4. *Elliptic (or bicursal) curve of order* n *in* n—1 *dimensions.*

We have proved already that a curve of order n in $n-1$ dimensions can be represented, point for point, on a plane cubic. If, therefore, it is not unicursal, its coordinates can be expressed in terms of elliptic functions of a single parameter. Now, it follows from the investigations of CLEBSCH, 'Crelle,' vol. lxiv. (1864), pp. 210–270, that if n points of the curve are co-flat, the sum of their parameters will differ from a certain constant by a sum of integer multiples of the two periods of the elliptic function. Let the periods be ω and ω', then if t_1, t_2, ... t_n are the parameters of the points,

$$t_1+t_2+\ldots+t_n=c+a\omega+b\omega',$$

where c is a constant, and a, b are integers. To find the points of superosculation, we must suppose the n points to become identical, or the t, still satisfying this equation, to become equal. We thus obtain

$$nt=c+a\omega+b\omega$$

$$t=\frac{c}{n}+\frac{a}{n}\omega+\frac{b}{n}\omega',$$

and values of t, representing distinct points, will be got by giving to the numbers a, b the values 0, 1, . . . $n-1$ independently. Hence *there are* n^2 *points of superosculation.*

Thus a plane cubic has nine inflexional tangents, and a quadri-quadric curve has sixteen superosculant planes.

Propositions hold good in general in regard to the grouping of these points, which are analogous to those which relate to the inflexions of a cubic. Thus, *an* $(n-2)$*flat drawn through* $n-1$ *of them will always pass either through another besides, or through the tangent line at one of the* $n-1$. This is obvious from the values already given for the parameters of points of superosculation.

Through any point of the curve can be drawn $(n-1)^2$ osculant $(n-2)$flats. This is proved in the same way as the preceding proposition, which is, in fact, the projection of it; for if through the given point we draw a cone containing the curve, and cut it by an $(n-2)$flat, the section will be an elliptic curve of order $n-1$ in $n-2$ dimensions, and the projections of the points whose osculant $(n-2)$flats pass through the given point will be points of superosculation on the projected curve. Hence, also, the lines joining the given point to the points of contact are grouped in respect of co-flatness in the same way as the points of superosculation in the curve of next lower order.

More generally, through k given points of the curve there can be drawn $(n-k)^2$ $(n-2)$flats which have $(n-k)$ pointic contact with the curve. If u_1, u_2, . . . u_k are the parameters of the k given points, those of the required points are given by

$$u=\frac{1}{n-k}(u_1+u_2+\ \ldots\ u_k+a\omega+b\omega'),$$

where the integers a, b may take independently the values 0, 1, . . . $n-k-1$.

From these results we may now determine the various ranks and the class of the curve. Suppose that we know the number of $(n-2)$flats which can be drawn through $n-2$ arbitrary points in space—or, which is the same thing, through an arbitrary $(n-3)$flat P—to touch a certain curve. Then, if the arbitrary $(n-3)$flat meets the curve in any point, *two* of these will coincide at that point. For taking an $(n-4)$flat in the $(n-3)$flat, and joining it to all the points of the curve by $(n-3)$flats, we may cut this figure by a plane or 2 flat. Every $(n-3)$flat will cut this plane in a single point. The problem is then reduced to drawing tangents from a point (viz., the intersection of P by the plane) to a plane curve; and we know that when this point lies on the curve, two of the tangents coincide at it.

In general, a certain number of $(n-2)$flats can be drawn through an arbitrary $(n-k-1)$flat to have k-pointic contact with a given curve; this number is, in fact, the $(k-1)^{\text{th}}$ rank of the curve. If the arbitrary $(n-k-1)$flat meets the curve at any point, then k of these $(n-2)$flats coincide at that point. For we may project the whole figure from an $(n-k-2)$flat lying in the $(n-k-1)$flat on to a k flat. The problem is then reduced to drawing $(k-1)$flats through a given point to have k-pointic contact with a curve in k dimensions. Now we know, from the example of the full-skew curve, that, when the point lies on the curve, k of these coincide at the point.

If the arbitrary $(n-k-1)$flat meet the curve in more points than one, k of the osculants will coincide at each of them; and this result is not affected by the union of the points into one. In particular, if it meet the curve in $n-k$ coincident points, the number of osculants which there coincide is $k(n-k)$.

Applying now these general considerations to the elliptic curve, we find at once that the $(k-1)^{\text{th}}$ rank of it is nk. For we must add to the number $k(n-k)$, just obtained, the number, k^2, given by the theory of elliptic functions for the $(n-2)$flats drawn through $n-k$ consecutive points of the curve to have k-pointic contact elsewhere. In particular, the class of the curve is $n(n-1)$; we have observed already that the number of superosculants is n^2.

Thus, a plane cubic is of order 3, class 6, and has 9 inflexions; a quadri-quadric is of order 4, rank 8, class 12, and has 16 superosculant planes. We learn, moreover, that a quintic curve in four dimensions, when not unicursal, is of first rank 10, second rank 15, class 20, with 25 points of superosculation. Hence a quintic in three dimensions, with five apparent dps., is of rank 10, class 15, and has 20 superosculant planes; this follows by projection from the former case.

A curve of this kind, viz., an elliptic curve of order n in an $(n-1)$flat has its coordinates $x_1, x_2 \ldots x_n$ determined by the equations

$$x_1, x_2, \ldots x_n = 1, t, t', t^2, tt' \ldots$$

(the last term on the right being $t^{(\frac{1}{2}n-1)}t'$ or else $t^{\frac{1}{2}n}$, according as n is odd or even), where $t = \operatorname{sn}^2(u + i\text{K}')$ and

$$t' = \frac{dt}{du} = 2\operatorname{sn}(u+i\text{K}')\operatorname{cn}(u+i\text{K}')\operatorname{dn}(u+i\text{K}') = \sqrt{2t(1-t)(1-k^2t)}.$$

[If n is even, we may write $t = \operatorname{sn}^2 u$ instead of $\operatorname{sn}^2(u+i\text{K}')$.] The condition for n points $u_1, u_2 \ldots u_n$ to be co-flat is then

$$u_1 + u_2 + \ldots + u_n = 0.$$

See LINDEMANN; CLEBSCH'S 'Lectures on Geometry,' vol. ii.

[*Theory of Derived Points on an Elliptic (or Bicursal) Curve.*

SYLVESTER'S theory of derived points on a plane cubic is as follows :—Starting from any given point on the curve, we may construct its *tangential*, or point where the tangent at the original point meets the curve again; similarly we may construct the tangential of the tangential, or second tangential, and so on. By joining any two non-consecutive points on this series, we can find their *residual*, the point where the joining line meets the curve again. In this way we obtain an infinite group of points derived from (and including) the original point, such that the line joining any two of them is either tangent at one of these or passes through a third point of the group. It is to be observed that all points on the curve uniquely derived from the given point by any geometrical process (*e.g.*, the point where the conic of five-pointic contact meets the curve again, the point where cubics of eight-pointic contact meet the curve again, &c.) are included in the group.

The coordinates of any derived point may be expressed rationally in terms of the coordinates of the original point, and the order of the functions to which they are proportional is always a square number. Thus the three coordinates of the tangential are proportional to quartic functions of the coordinates of the original. If the square root of the order of these functions be called the order of the derived point, then we have the theorem that when three derived points are in a straight line, the order ot one of them is equal to the sum of the orders of the other two. It is observed that there is no derived point whose order is divisible by 3. By help of this observation it is easy to make out a scheme of the orders; for when we join two points, the order of their residual must be the sum or the difference of the orders of the points, and one or the other of these is always divisible by 3.

This theory is really a geometric representation of the multiplication of elliptic functions. The coordinates x_1, x_2, x_3 of any point on the cubic curve may by proper choice of axes be made proportional to elliptic functions of a parameter u, so that

$$x_1 : x_2 : x_3 = 1 : \operatorname{sn}^2(u+i\mathrm{K}') : \operatorname{sn}(u+i\mathrm{K}')\operatorname{cn}(u+i\mathrm{K}')\operatorname{dn}(u+i\mathrm{K}').$$

This being so, if u, v, w are parameters of three points in a straight line, we shall have $u+v+w=0$. If v be the tangential of u, the three points u, u, v are in a straight line, and $2u+v=0$, or $v=-2u$. Hence the series of tangentials has for parameters

$$u,\ -2u,\ +4u,\ -8u,\ \&c.:$$

and in general the parameter of any derived point is of the form nu, where n is a positive or negative integer. The number n, taken positively, coincides with what was called the order of the derived point. For the elliptic functions of nu are of the order n^2 in the elliptic functions of u.

In this way all points of the theory are explained, excepting the fact that no derived point has its order divisible by 3.

Moreover, we see at once that the theory can be extended to other curves of deficiency 1; as, for example, the quadri-quadric curve. Starting with any point on this curve, we may find the point where the osculating plane at that point meets the curve again; then repeat the process with the point so found, and so on. The plane joining any three of these points will meet the curve in another derived point, or else touch it at one of the three points. The plane drawn through one derived point to touch the curve at another derived point will meet it again in a derived point, *or* touch at the first point, *or* osculate at the second. The coordinates of any derived point are of the order n^2 in those of the original point, where $\pm n$ may be called the order of the derived point. In this case the order of no derived point is divisible by 2.

I was desirous of finding a similar representation of the multiplication of hyper-elliptic and Abelian functions; and therefore sought for cases in which derived elements might be found on *curves* (in the sense explained in the beginning of this paper) of deficiency greater than 1. For this purpose I considered scrolls. Taking an arbitrary generator on a quartic scroll having two linear directrices, we may draw a one-sheeted hyperboloid through three consecutive generators at that place; this will meet the quartic scroll in one other generator, which is thus uniquely derived from the given one. Similarly on a quintic scroll contained in a linear complex, the two tractors of four consecutive generators meet the scroll in two other points lying on a generator. And on a sextic scroll not contained in a linear complex, the linear complex having five-line contact at a given generator (containing five consecutive generators) will contain one other generator of the scroll. In these three cases, then, from any three, four, or five generators we may uniquely derive a fourth, fifth, or sixth generator respectively; and the whole theory of derived elements may be applied to the generators of these scrolls.

Unfortunately, however, each of the scrolls considered is at most of deficiency 1, so that we merely get more illustrations of the multiplication of elliptic functions. And it may be shown, in general, that a *curve* on which such a theory of derived points is possible, is at most of deficiency 1.

Suppose that it has no singular points, and that $k-1$ points on it being given, there is uniquely determined one other point. If this is effected (as in the above examples) by drawing a flat space through the $k-1$ points, which meets the curve in one other point, then it must be of the order k. Moreover, it must be in a flat space of so many dimensions that the flat of one dimension less is determined by $k-1$ points. Now a $(k-2)$flat is determined by $k-1$ points; therefore, the curve is in a $(k-1)$flat.

Thus the impossibility of extending the theory of derivation to curves of deficiency greater than unity is equivalent to the proposition that a curve of order k in $k-1$ dimensions is at most of deficiency 1. This failure was the starting point of the present paper.

It remains to explain why, in the group of numbers expressing the orders of the derived points, only certain forms present themselves. Let that number which, with $k-1$ other numbers, makes up zero, be called the *residual* of those numbers; it is, in fact, their sum taken negatively. Then the process of forming the group is to start from unity, and add the residual of every $k-1$ numbers of the group, repetitions being allowed. I say that by this process we shall only get numbers of the form $mk+1$. For let m_1k+1, m_2k+1, &c. be $k-1$ such numbers, then their residual is $-(m_1+m_2+\ldots)k-k-1$, which is a number of the same form. Now as unity, with which we start, is of this form, it follows that all the numbers of the group must be of the form $mk+1$.—January, 1879.]

CURVES OF DEFICIENCY p.

Theorems relating to Abelian Functions.

It will be convenient to put together shortly those propositions relating to the application of Abelian functions to curves which will be wanted in the sequel.

The aggregate of the real and imaginary points on a curve constitutes a two-way spread, or surface, which may be transformed, by stretching without tearing, into the surface of a body with p holes in it. On this surface there are $2p$ distinct closed curves which cannot without breaking be shrunk into a point, namely, one round each hole, and one through each hole. Any other irreducible circuit must be made up of combinations of these.

If any rational function of the coordinates be integrated from one point to another along the curve-spread, the value of the integral will depend upon the path of the integration. If the integral becomes infinite at any points, it may be altered in value by making the path go round one or more of these; but in any case it may be altered by incorporating into the path any of the $2p$ closed circuits just mentioned. It is found that there are p distinct rational functions of the coordinates whose integrals do not become infinite at any point of the curve-spread. Any other integral which is everywhere finite must be a linear combination of these. Of such linear combinations it is convenient to take a certain set as the *normal* set; they are so chosen that each of them, when integrated along a closed path which goes *round* a hole, gives zero for all the holes but one, and πi for that one; thus, the p integrals, which we may call $u_1, u_2, u_3, \ldots u_p$, are associated one by one with the p holes $1, 2, \ldots p$. If they are integrated along a closed curve passing *through* the hole h, let the values be called a_{h1}, a_{h2}, a_{hp}; then it is found that $a_{hk}=a_{kh}$, or the integral of u_h through the hole k is equal to the integral of u_k through the hole h.

If we now take all the integrals from a point x to a point y along the same path, and if $u_1, u_2, \ldots u_p$ are the set of values for one such path, and $U_1, U_2, \ldots U_p$ for another path, then we must have

$$
\begin{array}{l}
\mathrm{U}_1=u_1+m_1\pi i+q_1a_{11}+q_2a_{12}+\ldots+q_pa_{1p},\\
\mathrm{U}_2=u_2+m_2\pi i+q_2a_2+q_2a_{22}+\ldots+q_pa_{2p},\\
\vdots\\
\mathrm{U}_p=u_p+m_p\pi i+q_1a_{p1}+q_2a_{p2}+\ldots+q_pa_{pp},
\end{array}
$$

where the numbers m, q are integers; namely, m_h is the additional number of times the new path has gone round the hole h, and q_h is the additional number of times it has gone through that hole. We shall write these equations thus

$$\mathrm{U}_1, \mathrm{U}_2, \ldots \mathrm{U}_\nu \equiv u_1, u_2 \ldots u_p \ (\text{mod. } \pi i, a),$$

and shall say that the quantities U are *congruent* to the quantities u in respect of the periods πi, a.

Suppose now that the curve has no actual nodes, and that a locus of any order intersects it in the points $x_1, x_2, \ldots x_m$. Then, if another locus of the same order intersects it in the points $y_1, y_2, \ldots y_m$, and we take any one of the integrals, say u, from x_1 to y_1, from x_2 to y_2, ... from x_m to y_m, the sum of these results will be congruent to zero. That is to say

$$\Sigma\int_x^y du_h \equiv 0.$$

Here the Σ refers to the suffixes of the x and y, not to h. There are p such equations. This is ABEL's Theorem.

When the curve is in a k-flat and of the order n, we shall use this theorem chiefly for its n intersections with a $(k-1)$flat. If we regard the lower limits of the integrals (the points x) as fixed, the integrals for any point y may be regarded as parameters belonging to that point, and then ABEL's Theorem gives us p equations between the parameters of n points which lie on a $(k-1)$flat. The truth of these equations is *necessary* to the points lying on a $(k-1)$flat, but it may not be sufficient. Thus in a bicircular quartic curve, $p=1$, we have one equation to express that four points are in a straight line, and if the points are collinear the equation is true. But it does not follow from the equation that the points are collinear; in fact, the equation holds equally good if the points are in a circle.

If the sums of the parameters of p points are given, that is, if we have the p equations—

$$\left(\int^{x_1}+\int^{x}+\ldots+\int^{x_p}\right)du_h=v_h \qquad \left[h=1,\ 2, \ldots p\right],$$

the v_h being arbitrary constants, and the lower limits of the integrals being supposed constant; then the upper limits $x_1, x_2, \ldots x_p$ may be expressed in terms of the quantities v_k—namely, they are the roots of an equation of degree p whose coefficients are products of ϑ-functions of the v. If

$$\phi(m_1, m_2, \ldots m_p)=\Sigma m_h m_k a_{hk}+2\Sigma m_h v_h,$$

then

$$\vartheta(v_1, v_2, \ldots v_p) = \Sigma^p e^{\phi(m)},$$

the Σ^p indicating that each of the p integers $m_1, m_2, \ldots m_p$ is to take all integral values positive and negative. When the lower limits are so chosen that the sum of the parameters is zero for the complete intersection by any locus, this ϑ-function has remarkable properties. If we sum each of the parameters for any $p-1$ points on the curve, the ϑ-function whose arguments are these sums will vanish. That is if

$$v_h = \left(\int^{x_1} + \int^{x_2} + \ldots \int^{x_{p-1}}\right) du_h,$$

then $\vartheta(v_1 v_2 \ldots v_p) = 0$. If these sums are taken for any $p-2$ points, not only will the ϑ vanish, but also its differential coefficient in regard to any one of the points. And generally, if we take for the v the sums of the parameters for $p-r$ points, the ϑ and its first $r-1$ differential coefficients in regard to any of the points will vanish.

Conversely, if the p quantities v are such that $\vartheta(v)$ and its first $r-1$ differential coefficients vanish, then it is possible to find $p-r$ points x such that

$$\left(\int^{x_1} + \int^{x_2} + \ldots + \int^{x_{p-r}}\right) du_h = v_h.$$

Although here the number of equations is greater by r than the number of unknown quantities, yet it is possible to satisfy them all in virtue of the relations existing between them.

Relation between the Order and Deficiency of a Curve.

We shall now apply these theorems to the study of curves existing in k dimensions, of the order n and deficiency p. A $(k-1)$flat cuts such a curve in n points, such that the sum of each of the p parameters, for the n points, is zero. But a $(k-1)$flat is determined by k points; so that, k arbitrary points being chosen on the curve, it is always possible to find $n-k$ other points, so that the sum of each parameter for the whole n points shall be zero. Let then $-v_1, -v_2, \ldots -v_p$ be the sums of the parameters for the given k points; then to find the remaining $n-k$ points we have the p equations

$$\left(\int^{x_1} + \int^{x_2} + \ldots + \int^{x_{n-k}}\right) du_h = v_h.$$

If p is not greater than $n-k$, we know that these equations can be solved, although the solution may be indeterminate. But if $p > n-k$, the equations cannot be solved unless certain conditions are satisfied by the v. Let $p-n+k=r$, then r conditions must be satisfied; namely the v must be sums of the parameters of not more than $p-r$ (or $n-k$) points. But they are sums of the parameters of k points; therefore k is not greater than $n-k$, or $2k$ is not greater than n. We have proved then that

If $p > n-k$, *then* $2k$ *is not greater than* n.

Conversely, *if* $k > \frac{1}{2}n$, p *is at most equal to* $n-k$.

We may also state the proposition in this way. *A curve of order* n *and deficiency* p, *not greater than* $\frac{1}{2}n$, *can at most exist in* $n-p$ *dimensions.*

[It appears, therefore, that the theorems at the beginning of the paper may be extended, and that in n dimensions we have the curve of order n which is unicursal, the curve of order $n+1$, and deficiency at most 1, of order $n+2$, and deficiency at most 2, and so on till we come to the order $2n$, which is the first case of exception, and may have deficiency $n+1$. This curve is the natural geometric representation of the general Abelian functions, its multiple tangent flats playing the same part as the double tangents of the quartic curve in RIEMANN's beautiful paper on the case $p=3$. H. WEBER has noticed that in four dimensions this curve is the complete intersection of three quadric loci.—January, 1879.]

XXI. *On the Osteology of* Polyodon folium.

By Thomas William Bridge, *B.A., Scholar of Trinity College, and Demonstrator of Comparative Anatomy in the University of Cambridge.*

Communicated by Alfred Newton, *M.A., F.R.S., Professor of Zoology and Comparative Anatomy in the University of Cambridge.*

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[Plates 55–57.]

This Ganoid appears to have been first described, under the name of *Squalus spatula*, by Maduit in the 'Journal de Physique' for 1774, pp. 384–386, plate 2, fig. 1. Subsequently, several zoologists described and figured the external appearance of *Polyodon* under the various synonyms of *Polyodon* (Lacépède), *Spatularia reticulata* (Shaw), and *Planirostra* (Lesueur). The systematic position of the genus appears to have changed as often as its name. Originally regarded as an Elasmobranch, and subsequently as a Teleostean, it was reserved for H. Müller, in 1846, to elucidate its real affinities and to indicate its marked distinctness from the other families of Ganoids by establishing the suborder Chondrostei for its reception.

The osteology and myology of *Polyodon* have received comparatively scanty attention. The following are the chief memoirs relating to its anatomical structure with which I am acquainted :—

I. 'Spatulariarum Anatomiam descripsit Tabulaque illustravit Albertus Wagner. Berolini, 1848.

II. The male and female urino-generative organs are described by Hyrtl. ('Geschlechts-u. Harnwerk bei den Ganoiden.' Denkschriften der K. Akad., Wien. Vol. iii.)

III. Owen gives a brief description of the structure of the skull in the 'Osteological Catalogue of the Museum of the Royal College of Surgeons, London.' Vol. i.

IV. Huxley also briefly describes the cranium in his 'Lectures on the Vertebrate Skull,' Lecture XI., p. 202, and in his 'Anatomy of Vertebrated Animals,' pp. 139–140.

V. Traquair ('The Ganoid Fishes of the British Carboniferous Formations,' Part I. —*Palæoniscidæ*) refers to the structure of the quadrato-pterygoid cartilage in *Polyodon*, and also supplies some interesting facts relating to the arrangement of the fin rays, and the disposition of the fulcra and rhomboidal scales; and also discusses the relationship of *Polyodon* to the Palæoniscidæ.

VI. Burt G. Wilder ("Notes on the North American Ganoids: *Amia, Lepidosteus, Acipenser*, and *Polyodon*." Proc. Am. Ass. for Advancement of Science, 1875 and 1876) has described the brain of this genus.

VII. Gegenbaur has described and figured the shoulder-girdle and pectoral fin in his 'Untersuchungen zur vergleichenden Anatomie der Wirbelthiere.' Parts I. and II. 1864.

VIII. Thacher ("Ventral Fins of Ganoids," Trans. Connecticut Academy, vol. iv., 1877) describes and figures the structure of the ventral fins of *Polyodon*.

In so far as the above-mentioned memoirs refer to the structure of the cranium in *Polyodon*, I have ventured in this paper to supplement them by giving a more complete description of the skull accompanied by figures. I have also attempted to compare in detail the skull of *Polyodon* with the skulls of Teleostei, Elasmobranchs, Amphibia, and with that of its nearest living ally, the Sturgeon; and in addition, I have endeavoured, roughly and tentatively, to point out the bearing of the facts elucidated on the phylogenetic relationship of *Polyodon* to the Elasmobranchs on the one hand, and to the remaining Ganoids and to the Amphibia on the other.

The destination of my specimen, which belonged to the Museum of Comparative Anatomy of the University of Cambridge, prevented me from investigating its histological structure, and particularly the minute anatomy of the vertebral column.

Of the various papers and memoirs on the structure of the skull in the Ichthyopsida with which I am acquainted, I must acknowledge my especial indebtedness to Professor Huxley's articles on the "Amphibia" (Encyc. Britannica, vol. i.), and on *Ceratodus fosteri* (Proc. Zool. Soc., 1876); to Dr. Traquair's previously cited memoir on the Palæoniscidæ; and also to Professor W. K. Parker, not only for the numerous memoirs with which he has enriched this department of anatomical science, but for many suggestions, and for the information which he has given me about results as yet unpublished.

My thanks are also due to my friend Mr. A. C. Haddon, Scholar of Christ's College, Cambridge, for the drawings from which the annexed plates were taken.

Membrane bones of the Cranium. (Plate 55, figs. 1, 2, 3, and Plate 56, fig. 4.)

The cranial membrane bones of *Polyodon* are thin, narrow, and much elongated, studded with small pointed Ganoid tubercles which radiate from the comparatively thick central portion of each splint towards the thinner and almost fibrous extremities. Their sutural margins are so closely interwoven that it is almost impossible to separate contiguous splints from one another, and in their regular and symmetrical arrangement they present a striking contrast to the dermal plates of the Sturgeon. A pair of mesially opposed splints (Plate 55, fig. 1, b^1) overlie the supraoccipital region, and extend backwards for a considerable distance dorsad of the coalesced anterior vetebræ, terminating behind in a pointed process. Though constricted in the centre, they expand at their extremities both in front and behind, and are connected on each side with short transverse processes from the inner edges of the lateral splints, c^1 and c^2. Anteriorly, they are suturally united to a second pair of splints of much the same character (b^2), which, however, are not in contact along the median line, but are separated by the interposition of the median azygous element, a^1. The lateral splints, c^1 and c^2, are applied along the lateral margins of the chondrocranium. The former is somewhat Y-shaped, with one arm much shorter than the other. The stem is produced backwards to the level of the pointed process formed by the bones b^1, to the expanded hinder ends of which the short arm of c^1 is attached, while the long arm passes forwards and unites with c^2. From the inferior surface of c^1 a vertical lamina of bone descends, and is applied by its oblique anterior margin along the whole length of a diagonal ridge that traverses the lateral aspect of the cranio-spinal cartilage and to the decurrent flap of cartilage in which the ridge terminates behind and above. Posteriorly, each vertical plate diverges outwards and becomes separated from the cartilage of the coalesced anterior vertebræ by a mass of muscle. The next lateral splint, c^2, continuously overlies the pterotic ridge and sphenotic process, roofs in the orbit, and extends forwards to the nasal capsule. A process from its inner margin unites it with the junction of the bones, b^1 and b^2. An elongated oval fontanelle, covered in by membrane in the fresh skull, is left on each side, between the splints b^1 internally, and c^1 and c^2 externally, and through its anterior part the cranial cartilage and the parietal foramina can be seen.

A much smaller median fontanelle also exists between the splint a^1 and the hinder extremities of the splints b^2, and a second pair of oblong lateral fontanelles are limited internally and externally by the bones b^2, c^2, and c^3. Looking into either of the last-mentioned vacuities, the cranial cartilage and the orbital cavity can be seen. Continuing forwards the series c^1 and c^2, there is a third splint, c^3, which extends from the roof of the nasal sac along the lateral margin of the base of the rostrum as far as the anterior ends of the median bone a^1 and the paired lateral splints b^2. We have, therefore, at the base of the rostrum three longitudinally arranged series of splints, viz.: a^1, b^1 and b^2, and c^1, c^2, and c^3, forming a medio-dorsal, an inner lateral, and an external lateral series; and from this point the bones of each series are serially repeated along the roof of the rostrum to its anterior end, but instead of being interrupted by fontanelles, the rostral roofing splints are firmly adherent to one another by their thin overlapping squamose margins. There are three successive median splints, a^2, a^3, and a^4, which are disposed along the medio-dorsal line of the rostrum. The series b is continued along the rostrum by the paired splints, b^3, b^4, and b^5. It will be noticed that the paired bones of this series are in contact along the median line, and alternate with the azygous elements of the series a. The bone c^3 is in like manner repeated by one or two long splints applied to the supero-lateral margin of the rostral cartilage; but as these are much overlapped by the rayed marginal ossicles d^1, they are not visible until the latter have been removed.

The marginal ossicles, d^1 d^1, partially overlap the preceding splints. Their rays unite with those of the star-shaped osseous spiculæ by which the flexible lateral edges of the rostrum are strengthened and supported.

Circum-orbital Bones.—The orbit is bounded behind, below, and in front by a series of four or five small elongated orbital bones. Of these, two are postorbital, and the others suborbital and preorbital elements. They very much resemble the corresponding bones in many Siluroids (Ex. *Silurus*, *Synodontis*, &c.). The last suborbital projects backwards beyond the postorbital series as in these genera. There are no supraorbital bones. The uppermost postorbital and the preorbital are loosely attached to the sphenotic process and to the inferior margin of the nasal opening respectively.

The difficulty of correlating these investing parostoses with those which exist in other Vertebrata has been referred to by Professor HUXLEY ('Vertebrate Skull,' p. 203), and is mainly due to the fact that ossification has continuously invaded the fibrous tracts, instead of originating in distinct and definite centres. Hence in *Polyodon* one continuous splint may cover regions of the subjacent chondrocranium, which in other Ganoids and in Teleostei are invested by several distinct splints.

This difficulty is further enhanced by the want of a perfectly satisfactory scheme for the systematic classification and definite nomenclature of such superficial splints. The only attempt in this direction with which I am acquainted is that suggested by Professor PARKER in his 'Morphology of the Skull' (pp. 343–346). From the consideration of the dermal armature in the Siluroid fish *Callichthys*, he regards its cranial splints as being the serial homologues of the supero-lateral and infero-lateral plates with which its body is invested.

Thus the two supraoccipital derm bones, the single parietal, the paired frontals, and the azygous ethmoid are the serial homologues, whether azygous or paired, of the upper three-fourths of the supero-lateral body plates; while the post-temporal, supratemporal, dermo-sphenotic, the circumorbital ossicles, the lachrymals and nasals correspond to the ventral fourths of those plates, which have been segmented off, and serve for the transmission of the cranial prolongation of the lateral line mucus canal. The subdivision of the infero-lateral body plates gives rise in the skull to the opercular bones, maxillæ, jugals, præmaxillæ, branchiostegal rays, and the external mandibular splints. This scheme seems to me to be susceptible of one or two slight modifications, which were suggested to me by a careful examination of the very generalised splints that invest the posterior cranial region of the Sturgeon and the rostrum of *Polyodon*. We may, I think, with advantage classify the body splints into four series—a median dorsal, a supero-lateral, a lateral, an infero-lateral, and a median ventral series, the lateral series being devoted to the transmission of the lateral slime canal, and each element perforated by a mucous pore. This arrangement is practically that which obtains in the post-cranial armature of the Siluroid genus *Loricaria*. The median dorsal series is represented in the trunk by the median row of scales or scutes, which in *Acipenser* and in other fishes is continued along the median line of the dorsum, and in the skull by the dermo-supraoccipital and dermo-ethmoid bones. This series is rarely continuous, but is repeatedly interrupted in the trunk by the development of the dorsal fins, and in the skull by the interpolation of certain elements of the supero-lateral series. The elements of the first series are usually azygous and mesial, as in *Acipenser*, *Amia*, and Siluroids, &c.; but both in the skull and in the trunk they may be broken up into distinct pieces, as, for example, in the former when there are paired dermo-supraoccipitals and paired dermo-ethmoids, as in *Polypterus*, *Lepidosteus* and many extinct Crossopterygian Ganoids, or paired dermo-supraoccipitals only, as in the extinct Labyrinthodonta.

The supero-lateral series is represented in the skull by the parietal and frontal bones. As is the case with the elements of the first series, the parietals and frontals may, in different crania, be either single and mesial, or paired and lateral in position.

When paired parietals and frontals co-exist with either paired dermo-ethmoids or paired dermo-supraoccipitals, the typical mesial position of the latter is obscured, and they appear to form elements of the series to which the former belong, and to be homologous with them.*

The lateral or slime-canal series of scales is represented in the cranium by the post-temporal, supra-temporal, dermo-sphenotic, and nasal bones lying above the orbit, and by the postorbital, suborbital, lachrymal, and preorbital elements situated below the orbit.

The membrane bones forming the infero-lateral series in the cranium are closely related to the mandibular and hyoid arches and to the palato-quadrate arcade, and in consequence are much more subdivided than are the elements of the supero-lateral series. They include, in a linear series, the supraclavicle, operculum, præoperculum, quadrato-jugal, jugal, maxilla, and præmaxilla, forming one group of homologues; the clavicle, suboperculum, and interoperculum form a second group; and the interclavicles, the branchiostegal rays, jugular plates, and mandibular splints constitute a third group of serial homologues. On arranging these elements in accordance with the transverse segmentation into visceral arches, we have the supraclavicle, clavicle, and interclavicle, in relation with the coraco-scapular arch; the operculum, suboperculum, branchiostegal rays and jugular plates in relation with the hyoid arch, and the preoperculum, interoperculum, angular, supra-angular, dentary, quadrato-jugal, jugal, and maxilla, associated with the mandibular arch and its pterygoid outgrowth. The medio-ventral series is not so distinct as either of the others; it is possibly represented in the skull by the azygous jugular plates, or more probably by the urohyal bones of Teleostei.

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* The bones which Mr. Parker calls paired dermo-supraoccipitals in *Callichthys* seem to me to be really the first pair of supero-lateral plates, while his median parietal is really the dermo-supraoccipital. At all events the latter seems to correspond to the bone which in *Clarius* and other Siluroids is called by that name, and which is also perforated by a median foramen.

This slightly modified schema differs from the one suggested by Professor PARKER only in the recognition of a medio-dorsal and a medio-ventral series, and the consequent alteration in the elements which are to constitute homologous series. Thus I do not consider the dermo-supraoccipital, the parietals and frontals, and the dermo-ethmoid as serial homologues, but prefer to regard the first and the last as forming one series of homologues—the medio-dorsal, while the two other bones form a second series—the supero-lateral. As will be seen shortly, this arrangement is justified by the disposition of the elements of the cranial dermal armature in *Acipenser*.

We are now in a position to endeavour to ascertain how far the cranial parostoses of *Polyodon* may be expressed in terms of this schema.

HUXLEY* refers as follows to the posterior cranial splints and the fontanelles which they enclose: "In the dorsal region it presents an anterior and posterior pair of perichondrial ossifications, separated by oblong laminæ from lateral bony plates of the same character, but the homology of these with those in the roof of the Teleostean skull is not to my mind satisfactorily demonstrated." The "anterior and posterior pairs of ossifications" referred to are those marked b^1 and b^2, and the "lateral bony plates of the same character" are c^1 and c^2. According to OWEN† the two first-mentioned pairs of bones are the parietals and frontals respectively, and the two latter the mastoid and postfrontal elements.

At first sight it seemed not impossible that the splints b^1 were the homologues of the paired dermo-supraoccipitals of *Polypterus*, *Lepidosteus*, and the *Labyrinthodonta*, but as they evidently belong to the same supero-lateral series as the paired splints, b^2, which are separated by the medio-dorsal element a^1, I am inclined to agree with OWEN, and regard them as corresponding to the parietals of other vertebrata, mesially united owing to the suppression of the proper median bone—the dermo-supraoccipital. It will be noticed that the splints (b^3) forming the third pair of supero-lateral elements are also in contact along the median line of the skull, in front of the azygous element, a^1, just as the parietals are mesially opposed behind it, thus conforming to the alternating arrangement of the medio-dorsal and supero-lateral splints of the roof of the rostrum.

The next pair of supero-lateral splints (b^2) are clearly the equivalents of the frontals, while a^1, the median bone which separates them, is a dermo-ethmoid. Three serial homologues of the last-mentioned bone (a^2, a^3, a^4,) continue the median series to the end of the rostrum. In a similar manner the parietals and frontals are serially repeated by their paired homologues b^3, b^4, and b^5. The separation of the elements of the dermo-ethmoid series by the interpolation and mesial apposition of the paired supero-lateral splints has already been pointed out.

The bone c^1, the most posterior element of the lateral series, serves to attach the supraclavicle to the skull, and might, therefore, be regarded as a post-temporal splint, especially as there is no other bone which can possibly represent that element. Its

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* 'Lectures on the Vertebrate Skull,' p. 203.

† 'Osteological Catalogue of the Museum of the College of Surgeons,' vol. i. p. 83.

horizontal and descending plates would appear to correspond to the two processes for attachment to the postero-lateral angles of the skull into which the post-temporal bone is divided in *Amia*, *Lepidosteus*, and in the majority of Teleostei.

In the Sturgeon, however, Professor PARKER* has described a complete series of pectoral splints, consisting of an interclavicle, a clavicle, a supraclavicle, and a post-temporal, the latter being attached to the skull by the bone marked c^1 (fig. 1, p. 690), (supra-temporal, PARKER), which, in possessing horizontal and descending laminæ, is in every way comparable to the bone designated c^1 in *Polyodon* (Plate 55, fig. 1). That these two bones are homologues I have but little doubt, and if the bone called post-temporal in *Acipenser* is correctly so named, then we must regard both the former as being supra-temporal elements, the proper post-temporal being altogether suppressed in *Polyodon*.

Dr. TRAQUAIR,† in referring to the shoulder-girdle of *Polyodon*, calls the bone c^1 the squamosal, but as this term is now usually applied to the bone which in the higher Vertebrata results from the coalescence of the præoperculum with a supra-temporal element, it would seem better, if it represents any part of the compound squamosal, to regard c^1 as being a supra-temporal element only. I am, however, much inclined (for reasons to be mentioned in describing the shoulder-girdle) to regard the splint in question as being, in both Ganoids, the homologue of the post-temporal of Teleostei.

The splint c^2 overlies the pterotic ridge and the sphenotic process, forms a roof to the orbit, and extends forwards far enough to overlie the nasal capsules. I imagine that this splint represents the distinct supra-temporal, dermo-sphenotic, and, perhaps, the ecto-ethmoidal elements of *Acipenser*. The thickened sphenotic portion of this bone suggests that ossification first began in that region, but afterwards continuously invaded fibrous tracts in front and behind which in the Sturgeon are separately ossified from distinct centres.

The parostosis c^3 is apparently a nasal bone, but may possibly be a dermal ecto-ethmoid, and the splints which I have described as being applied to the supero-lateral margin of the cartilaginous rostrum, serially repeat these elements of the lateral line series.

The rayed marginal ossicles belong to the infero-lateral series, and the first of them is applied to the outer side of the nasal bone (c^3). It is hardly possible to correlate these ossicles with any that exist in other Fishes, but perhaps they may be regarded as corresponding to that part of the infero-lateral series which is generally represented in other Fishes by the premaxillæ. In *Acipenser* they are replaced by a series of bony plates which bound the infero-lateral margin of the rostrum.

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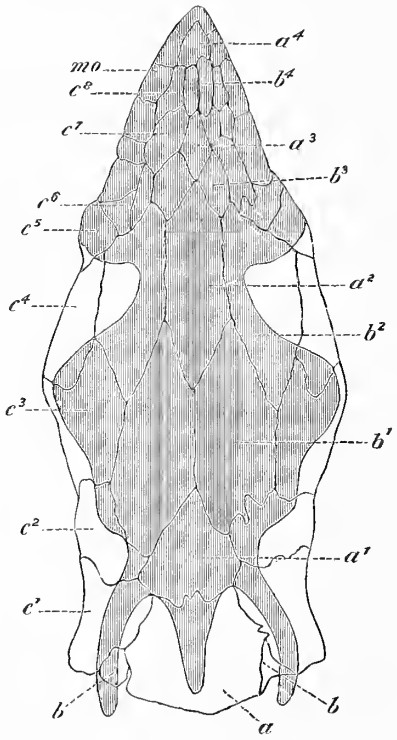
* "Monograph on the Structure and Development of the Shoulder-girdle and Sternum in the Vertebrata." Ray Society, 1878.

† "Ganoid Fishes of the British Carboniferous Formations, Part I.—Palæoniscidæ." Mem. Palæontographical Soc., 1877.

If the arrangement of the splints which invest the posterior cranial region of the Sturgeon throws light on the nature of those similarly located in *Polyodon,* it is also true that the disposition of the rostral splints in the latter Ganoid suggests the plan on which those of the former are arranged.

In no two specimens of *Acipenser* are the anterior cranial splints disposed in quite the same way, and the arrangement may even differ on opposite sides of the same skull. From an examination of several skulls, and especially of one now in the University Museum, and from which the annexed woodcut was taken, I think it may be demonstrated that the splints in both genera are arranged in accordance with a common plan.

Thus in the woodcut the splints marked a^1 to a^4 correspond to the medio-dorsal series in the typical scheme, and in *Polyodon.* The one marked a^1 is a dagger-shaped bone, and evidently represents the dermo-supraoccipital. The next mesial splint (a^2) is a dermo-ethmoid, and in this specimen is separated from the preceding bone by the mesial apposition of b^1 b^1—two bones belonging to the supero-lateral series—though in most skulls the long blade of the dermo-supraoccipital completely separates them, and suturally unites with the hinder margin of the ethmoid.



The splints a^2 to a^4 continue this series to the end of the rostrum, and, as in *Polyodon,* are more or less completely separated from one another by the interpolation of the paired elements of the next series. The scute marked a is the first of the medio-dorsal plates of the trunk. In the supero-lateral series, the paired osseous plates marked b^1 and b^2 are the parietals and frontals; and, as in *Polyodon*, the former are in contact mesially, though partially separated behind by the dermo-supraoccipital, which, as a distinct bone, does not exist in the latter Fish.

The bones b^3 and b^4 repeat the parietals and frontals along the sides of the snout.

The splint c^1 (post-temporal), the first splint of the third, or infero-lateral series, resembles the corresponding bone in *Polyodon* in developing an extensive descending osseous lamina which is applied by its anterior margin to the cartilage that forms the lateral walls of the cranio-spinal canal. The splints c^2–c^5, which are represented by a single bone (c^2) in *Polyodon*, correspond to the supra-temporal, dermo-sphenotic, supra-orbital, and ecto-ethmoid elements of the Teleostean skull. c^6 is a nasal bone, and c^7 and c^8 continue this series forwards along the rostrum.

Postorbital, suborbital, and præorbital bones surround the orbit, and the inferior margin of the rostrum is fringed by a series of five or six plate-like marginal ossicles (*m.o.*) precisely as in *Polyodon*.

Thus it will be seen that, notwithstanding the superficial differences between the cranial splints of the two genera, due to the tendency in *Polyodon* of the osseous centres to continuously invade fibrous tracts usually ossified separately, and to the irregular asymmetrical disposition and proneness to segmentation of the splints in *Acipenser*, their fundamental arrangement is much the same in both Ganoids.

The foregoing considerations seem to me to suggest an explanation of the dermal armature of *Ceratodus*, which is somewhat different from that proposed by Professor HUXLEY.* Comparing the skull of that Fish with those of *Acipenser* and *Polyodon*, it will be seen that the anterior and posterior median bones (A^1 and A^2) in the former are the equivalents of two elements in the medio-dorsal series of the latter, viz., the dermo-supraoccipital and dermo-ethmoid, while the anterior and posterior inner lateral bones (C^1 and D^2) correspond to the parietals and frontals.†

The bones marked E and D are elements of the lateral series, and probably correspond to the bones called supra-temporal in the Sturgeon, or, if coalescent, they would be the equivalent of the greater part of the bone c^2 in *Polyodon*.

But as Dr. GÜNTHER remarks the fact that all the cranial derm bones in *Ceratodus*, with the exception of the median ethmoid, lie external to the temporal muscles, throws considerable doubt on the results of any attempt to identify these bones with those which in other Vertebrata lie beneath the muscles and directly upon the cartilage of the cranial roof. The former are evidently ossifications of a more superficial fibrous tract than that in which the latter are usually formed.

If the two splints which invest the rostrum of the Pike's cranium, and are marked 2–2

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* "Contributions to Morphology. Ichthyopsida, No. 1.—On *Ceratodus fosteri*, with Observations on the Classification of Fishes." Proc. Zool. Soc., 1876.

† It will be noticed that in *Ceratodus*, as is commonly the case in *Acipenser*, the dermo-ethmoid and dermo-supraoccipital bones are suturally united.

in HUXLEY's figure,* are not paired dermo-ethmoids, corresponding to the paired endosteally formed centres which in that Teleostean represent the median ethmoid of other Fishes, it would appear that they must correspond to the paired ossifications marked b^3 in both *Polyodon* and *Acipenser*, the serial homologues of the parietals and frontals. The bones marked 1 in the same figure apparently belong to the lateral series, and would probably represent the nasals of other Fishes.

It may also be mentioned that the generalised medio-dorsal and supero-lateral cranial plates which invest the rostra of such Teleostean Fishes as *Ostracion*, *Loricaria*, &c., exhibit the same alternating arrangement which has been pointed out in *Polyodon* and *Acipenser*.

The parasphenoid (Plate 55, figs. 2 and 3; Plate 56, fig. 4) is of great length, extending from a point just in front of the nasal capsules to a considerable distance beneath the coalesced anterior vertebræ. The postcranial part of the bone expands and bifurcates in order to allow the efferent branchial vessels to reach the anterior part of the dorsal aorta, and the two slightly divergent arms receive between them the anterior part of the massive notochord and the commencement of the hæmal canal. The cranial part of the bone is slightly convex below and concave above where it is applied to the cartilage of the basis cranii. After slightly expanding between the nasal sacs the parasphenoid contracts in the orbital region, and again expanding beneath the auditory capsules, sends off on each side a short triangular basi-temporal wing (*b.tp.*) which passes upwards and outwards between the foramina for the exit of the palatine and spiracular divisions of the seventh nerve, closely applied to the cartilage that forms the thin outer wall of the short Facial canal which traverses the outer wall of the periotic capsule between those foramina, and terminates above in a point just in front of the lower end of the groove for the head of the hyomandibular. At the base of this process the parasphenoid gives off a second and smaller process, which is directed outward and backwards as in *Amia*, and to the angle between the two the pharyngo-branchial of the first branchial arch is attached. (Plate 55, fig. 3, *b.tp.*)

Just in front of its bifurcation the lower surface of the parasphenoid is marked by two osseous knobs, and the lateral margins of its anterior moiety are slightly overlapped by the adjacent cartilage. In front of the parasphenoid, and anterior to the internasal area, there is a pair of splints (fig. 2, *vo.*) which appear to represent the vomers; and wedged in between their anterior ends there is a second mesial splint (p^2) extending backwards nearly to the anterior end of the parasphenoid. If the paired splints are correctly to be regarded as vomers, then this azygous splint would appear to be the homologue of the anterior parasphenoid described by PARKER† as existing in *Rana pipiens*.

In front of the mesial splint (p^2) there is a third median splint (p^3) continuing the

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* 'The Anatomy of Vertebrated Animals,' p. 154, fig. 46.

† "Structure and Development of the Skull of the Common Frog," Phil. Trans., Vol. 161, p. 193, 1871.

series to the end of the rostrum, and, as is the case with the medio-dorsal elements, they are separated from one another by the interposition and mutual apposition of the two pairs of splints (v^2, v^3) which serially repeat the vomers along the ventral surface of the rostrum. Hence, just as the elements of the medio-dorsal and supero-lateral series—the supraoccipital and dermo-ethmoid, the parietals and frontals—are serially repeated along the dorsal surface of the rostrum, so there is a similar serial repetition of the parasphenoid and vomers along the ventral surface.

There is also on each side a ventral series of rayed marginal ossicles (*m.o.*) which overlap the vomerine series. The rays of these ossicles are connected with those of the star-shaped spicules that support the ventral layer of the flexible coriaceous margin of the rostrum. In the interspace between the cartilage of the rostrum and the upper and lower layers of its flexible margin I noticed two large nerves running forwards to the extremity of the rostrum; these nerves are probably the rostral branches of the first and second divisions of the Trigeminus.

The Chondrocranium. (Plates 56 and 57, figs. 5, 6, and 7.)

When stripped of its investing parostoses the cranium presents the appearance represented in Plate 56, fig. 5.

Its most striking feature is the enormously elongated prenasal rostrum. The skull with its rostral prolongation is about one-third of the entire length of the axial skeleton, the rostrum alone being one-fourth of the length of body, and at least twice as long as the rest of the chondrocranium.

About eight or nine of the anterior vertebræ have so completely coalesced with each other and with the skull, that the only indications of the primitive segmentation into distinct vertebræ is the serial arrangement of the obliquely disposed pairs of foramina for the exit of the anterior and posterior roots of the spinal nerves, the anterior root leaving the spinal column below and a little in front of the posterior root.

The posterior limit of the cranium seems to be approximately indicated by a prominent ridge-like elevation of the lateral walls of the cartilaginous mass formed by the coalescence of the occipital region with the adjacent vertebræ. This ridge (Plate 57, fig. 7, *a*), which is mainly an outgrowth from the cartilage of the two anterior vertebræ, first becomes evident beneath the Vagus foramen, and then passes obliquely upwards and backwards between the foramina for the exit of the anterior and posterior roots of the first spinal nerve to terminate behind the postero-external angle of the skull in a free, decurrent, wing-like process (fig. 7). The vertical lamina of the post-temporal bone is applied to the outer side of this process, and its anterior margin just overlaps the oblique ridge (fig. 3). In front of the ridge the occipital region is slightly constricted, and behind it the bulging notochord and paired foramina for the spinal nerves become visible. When the chondrocranium is viewed from above (fig. 5) the ear capsules are seen to form conspicuous lateral outgrowths terminating anteriorly in projections which reach nearly to the antorbital region. The slightly contracted orbital region is limited in front by the almost pedunculated nasal capsules.

Behind the auditory region the cranial roof becomes constricted into a band-like area, which is formed partly by the coalesced apices of the anterior neural arches, and partly[*] by the roof of the occipital region. The divergent processes ($\phi\phi$) are the posterior ends of the decurrent flaps of cartilage previously mentioned.

The floor of the occipito-periotic region, where it rests on the parasphenoid, is very narrow, but the external walls diverge outwards as they pass upwards to join the margins of the flat and comparatively broad upper surface of this part of the skull; hence a transverse section is almost triangular in shape. The width of the cranium in this part is mainly due to the massive periotic sacs, and the growth of cartilage round the semicircular canals is thick enough to prevent them affording any external indications of their position. The upper and external margin of each otic capsule is produced outwards into a prominent pterotic ridge, and forwards into a stout sphenotic process, which arches outwards, downwards, and forwards, and also forms a roof to the hinder part of the orbit. Posteriorly the pterotic ridges terminate in the rounded blunt projections which form the postero-external angles of the otic capsules.

The outer face of each capsule is traversed by a deep vertical groove with a prominent anterior lip, which extends from the pterotic ridge to a point just above the facial foramen, and serves for the articulation of the hyomandibular. This groove, limited in point by its projecting anterior border, is seen in fig. 5 (*hy.g.*), as an emargination of the pterotic ridge behind the sphenotic process. The mesial portion of the cranial roof forms an elongated band-like area, flattened, like the pterotic ridges, by the impress of the overlying membrane bones—the parietals and frontals; but on each side of it, in the roof of the otic capsule, there is an *f*-shaped groove (*f.g.*), the anterior end of which diverges outwards into the sphenotic prominence, while its hinder end terminates on the postero-lateral margin of the pterotic ridge. The groove is visible externally through the oblong fontanelle, which on each side is left between the post-temporal, dermo-sphenotic, and parietal splints (fig. 1). In the posterior part of each groove there is an infundibuliform opening (*p.f.*), which is the outer termination of a short but wide canal leading into the cranial cavity; and in the anterior part there is a much smaller foramen, which communicates by a short narrow canal with a slit-like aperture (figs. 3 and 4, *x.*) on the infero-lateral surface of the otic cartilage, between the metapterygoid ligament and the articular groove for the reception of the head of the hyomandibular. These foramina will be referred to subsequently. In front of the wing-like sphenotic prominences the chondrocranium is constricted to form the orbits. A slight backward outgrowth from the upper margin of the ethmo-trabecular region, together with the produced orbital margins of the sphenotic process and pterotic ridge, furnish a partial roof to the orbit. Previous to the removal of the dermo-sphenotic bone (c^2), which extends continuously over each

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[*] As represented in Plate 56, fig. 5, this band-like area is one-sixteenth of an inch too wide.

of these processes, this roof was a complete one. A partial floor is formed for each orbit by a rough irregular outgrowth of cartilage (*o.f.*, figs. 3 and 4) from the inferior edge of its inner wall. There is no supraorbital ridge, the junction of the inner wall of the orbit with the cranial roof being gently rounded off. The aliethmoidal, aliseptal, and subnasal outgrowths have combined to form a cup-shaped nasal capsule, supported on a short, thick neck. The thick posterior wall of each capsule is slightly produced backwards over the orbit beneath the dermo-postfrontal bone, and the lower lip of the cup, besides being somewhat more prominent than the upper, is slightly emarginate. This emargination of the subnasal outgrowth corresponds in position to the laterally placed posterior narial aperture (fig. 3, *p.n.*); the anterior nares, though situated in front of the former, are also lateral in position (*a.n.*). I could detect nothing suggestive of the existence of either a free or a coalesced antorbital or palatine process. The only possible representative of that element is the slightly produced inferior and external angle of the posterior wall of the nasal sac. The long and depressed cartilaginous prenasal rostrum is small relatively to its size when invested with its splints. The upper surface is gently convex from side to side, but its floor and sides are straight. It diminishes very gradually in height, but rapidly in width, towards its slightly expanded and flattened anterior termination. As in *Acipenser*, the rostral axis is slightly tilted upwards, so as to make a widely open angle above with the cranial axis. The spatulate appearance of its anterior end, as seen in Plate 55, figs. 1 and 2, is not due to the configuration of the axial cartilage, but to the disposition of the investing membrane bones. The lateral margins of the extreme anterior end of the rostrum are slightly flattened out, as in *Raia* and *Rhinchobatus*, and, as in these genera, are suggestive of the presence of a pair of upper labial cartilages. The roof of the rostrum is complete, with the exception of a small median fontanelle (figs. 5 and 6, *a.m.f.*) situated just in front of the nasal sacs, and evidently corresponding to the much larger anterior median fontanelle of the Skate and *Rhinchobatus*. In general appearance the rostrum of *Polyodon* is very like that of the last-named Elasmobranch, as will be seen on referring to the excellent figures of the latter given by GEGENBAUR in his 'Untersuchungen zur vergleichenden Anatomie der Wirbelthiere' (plate iii., fig. 1; plate vi., fig. 3, and plate ix., fig. 2). The rostrum of *Rhinchobatus*, however, differs from that of *Polyodon* in the presence of labial cartilages coalesced with its anterior end, in the larger size of its median fontanelle, and in the absence of a lamina perpendicularis separating the cavity of the rostrum from the cranial cavity. But, as previously suggested, it is not improbable that the winglike expansions of the anterior end of rostrum in the Ganoid may point to the presence of upper labial cartilages which have coalesced with it.

HUXLEY,[*] in describing the chondrocranium of the Sturgeon, refers to the coalescence of the anterior vertebræ with each other and with the skull. He says: " At this point there is in the cranio-spinal cartilage of both the Sturgeon and Spatularia a

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[*] 'Lectures on Vertebrate Skull,' pp. 204, 205.

great dilatation of the neural canal, which is closed above only by a membranous fontanelle." This description, in so far as it refers to *Polyodon*, is not quite accurate. There is, it is true, a large posterior median fontanelle in the Sturgeon which extends forwards for a short distance in front of the Vagus foramen, and is therefore, in part at all events, a cranial fontanelle. But in *Polyodon* I could find nothing at all comparable to this large and conspicuous fontanelle of *Acipenser*. The neurapophyses of the anterior vertebræ coalesce over the spinal cord and anteriorly blend with the cartilage of the occipital roof, and the continuity of the roof of the spinal canal seems to be uninterrupted.

We may now consider the foramina which I have already referred to as existing in the *f*-shaped groove in the roof of the periotic capsula. Each infundibuliform orifice is situated on the outer side of the common canal by which the anterior and posterior vertical semicircular canals open into the membranous vestibule, and the short but relatively wide passage into which the orifice leads passes downwards and inwards beneath the arch of the posterior canal, and opens into the cranial cavity just behind the recess in which the vestibular sac is lodged (Plates 56 and 57, figs. 5, 6, and 7, *p.f.*) These foramina at first appeared to correspond to the paired posterior fontanelles which are seen in the roof of the Frog's chondrocranium when the overlying parieto-frontal bones have been removed, or to similar vacuities existing in the roof of the Salmon's skull, which in the latter are spaces left on each side of a recurrent growth of cartilage derived from the ethmoidal region. But these fenestræ have nothing to do with the otic capsule; they are always mesially placed with regard to it, and their passages of communication with the cranial cavity do not pass beneath the arch of the posterior vertical canal, or in any way perforate the periotic cartilage.

Another alternative is to regard these infundibuliform orifices and their canals as being due, like the parietal foramina of the Elasmobranchs, to the persistence of the canals by which the primitive auditory involutions of the embryo *Polyodon* communicated with the exterior. But this view may be met by the objection that in the Selachii the parietal foramina and their canals do not pass beneath the arch of the posterior vertical canal, but are situated altogether to the mesial side of the vestibule and its canals. Moreover, in the Shark these foramina do not communicate with the cranial cavity but with the central cavity of the periotic cartilage in which the membranous vestibule is lodged. In mitigation of the force of the first objection, it may be urged that in the young *Polyodon* the cœcal outgrowth from the vestibule, which eventually becomes the posterior vertical semicircular canal, may have grown round the primitive involution from the exterior instead of having its growth limited to the outer side of that ingrowth, as appears to have been the case in the young Shark. In opposition to the second objection it may be said that if the inner wall of the periotic capsule be atrophied, as it is in all Teleostei and Ganoids, then the parietal foramina will necessarily communicate with the cranial cavity, as do the infundibuliform orifices in *Polyodon*.

Both in the young Tadpole and in the very early stages of the developmental history of the Salmon, the chondrification of the roof of the periotic capsule is incomplete in the region of the primitive involution; and it seems conceivable that in *Polyodon* this primitive fenestration has been retained in the adult, while the atrophy of the inner or cranial wall of the capsule has caused it to communicate with the interior of the cranium.

Though the last view of the nature of these fenestræ seems to me to be the more probable one, yet it is possible that neither is correct, and that they may be of secondary origin—the result of local absorption of the cartilage of the otic capsule.

But if my view be correct, then the existence of these parietal foramina in *Polyodon* is an important anatomical feature in which that Fish differs from all other Ganoids and Teleostei, and approaches the Plagiostome Elasmobranchii. Mr. PARKER informs me that he has discovered similar perforations of the otic cartilage beneath the posterior canal in the Urodele Amphibian, *Siren lacertina*, to which he attaches a like significance. This fact is an important addition to the list of anatomical characters common to the Ganoids, Elasmobranchs and Amphibia.

I may add that in *Acipenser* there is a depression in the roof of the otic cartilage external to the posterior vertical semicircular canal, and at the bottom of it there is a fat-infiltrated canal which penetrates for a short distance into the substance of the cartilage, but does not communicate with the cranial cavity. The enormous growth of the cranial cartilage, which has effectually masked other embryonic features in the cranial structure of the Sturgeon, has probably obliterated the primitive communication of these canals with the cranial cavity.

The oblique lateral ridges and decurrent wing-like processes described in *Polyodon* are also represented in the Sturgeon. Behind the auditory organs, "and separated from them by a deep lateral fossa, are two wing-like processes, which are directed outwards and obliquely backwards, and proceed not from the walls of the cranium proper but from those of the spinal column, where it joins the skull."* This ridge, however, runs from above downwards and backwards, the foramina for anterior roots lying above the ridge, and those for posterior roots below it.

The second and smaller foramen (figs. 3 and 4, *x.*) in the anterior part of the *f*-shaped groove lies at the bottom of a deep basin-shaped depression in the roof of the otic cartilage, and communicates through a short canal with an elongated slit-like opening (*x.*) on the infero-lateral face of the periotic capsule, between the articular groove for the hyomandibular, and the attachment of the metapterygoid ligament. In the fresh specimen it was seen that the slit-like inferior opening was situated in the inner wall of the spiracular passage, and that the mucous membrane of the pharynx was continued through it into the canal, but the upper part of the latter and the depression into which it opened above, as well as the *f*-shaped groove, were filled with connective tissue infiltrated with fat. It may, in fact, be said that the first visceral

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* HUXLEY, *loc. cit.*

cleft splits into two canals, a large canal opening at the spiracle, and a smaller one which perforates the otic cartilage and opens on the superior surface of the skull. The canal is quite external to the semicircular canals and vestibule, though it approaches nearest to the horizontal canal. The meaning of this spiracular diverticulum, at first sight, was not very clear. I was inclined to regard it as having some connexion with the remarkable fenestration of the infero-lateral face of the periotic capsule which occurs in the very early stages of the embryo Salmon, and which PARKER[*] has called the "primordial fenestra ovalis," from its analogy to the fenestra ovalis of the abranchiate Vertebrata.

But further consideration suggested to me that the diverticulum in question might be nothing more than a result of the rapid growth of the cartilage of the ear-sacs round the upper part of the first visceral cleft, so as to constrict it off from the rest of the cleft which remains as the spiracular canal.

On examining a fresh Sturgeon's skull, I found a diverticulum of the spiracular passage precisely similar to, but relatively smaller than that described in *Polyodon,* and, as in the latter, opening into a basin-shaped depression in the roof of the otic capsule. The inferior orifice was just in front of the hyomandibular, and behind the mandibular gill; the upper part of the canal was filled with fat.

Periotic Bones (Plates 55 and 57, figs. 3 and 7).—At first it seemed evident that the membrane bones previously described were the only ossifications developed in connexion with the chondrocranium of *Polyodon,* but after a careful examination I detected a small circular ectosteal scale, marked with concentric lines of growth overlying the posterior part of the horizontal semicircular canal. This scale must be a rudimentary representative of the pterotic (*pt.o.*). Behind the latter and immediately in front of the Vagus foramen, there was a second small ectosteal scale, oval in shape and slightly larger than the first. From its relation to the ampulla of the posterior vertical semicircular canal, this ectosteal patch must represent an opisthotic element (*op.o.*).[†] These appear to be the only representatives of the otic bones of other Fishes. A careful investigation of the epiotic, prootic, and sphenotic regions, as well as of the rest of the external surface of the chondrocranium, failed to reveal the existence of any other ectosteal or of endosteal centres.

Cranial Foramina.—Certain foramina opening on the roof of the cranium have already been noticed, but there still remain those which perforate its lateral walls and serve for the transmission of the cranial nerves and vessels. A small foramen perforates the side walls of the broad posterior part of the rostrum, at about the same distance in front of the olfactory sacs as the anterior median fenestra (figs. 3 and 6, Z). In the absence of a supra-orbital ridge there is no antorbital foramen for the Ramus ophthalmicus superficialis which would appear to pass dorsad of the neck of each nasal

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[*] "Structure and Development of the Skull in the Salmon," Phil. Trans., 1872, p. 118.

[†] The relations of the otic bones to the horizontal and posterior vertical semicircular canals is well shown in Plate 57, fig. 7.

sac. On the anterior and posterior walls of each sac there is a small foramen, which I imagine may be the equivalents of the orbital and prenasal openings of the orbito-nasal canal of Elasmobranchs. The optic foramen (II) is situated at about the middle of the inner wall of the orbit; above it there is a small aperture (*m.o.*) for the motores oculi, and behind it are two relatively large foramina for the branches of the fifth nerve.

The Facial nerve (fig. 6, *Fa.*) on leaving the cranial cavity passes obliquely backwards and outwards through the cartilage of the otic capsule, and instead of opening directly on its external surface, communicates with a short antero-posterior canal, the outer wall of which is formed by an oblique bar of cartilage strengthened externally by the basi-temporal wing of the parasphenoid. The comparatively large anterior and posterior apertures of this canal are marked VII and VII′ in fig. 3. This obliquely disposed cartilaginous bar resembles a similar process (Knorpelspange), described by GEGENBAUR as holding a similar relation to the Facial foramen in *Rhinchobatus, Trygon,* and *Pristis.* In these Elasmobranchs the anterior opening of the canal is for the exit of the Ramus palatinus, while the posterior one transmits the common stem of the pre-spiracular and post-spiracular branches of the Facial nerve; and it seems probable that the canal in *Polyodon,* so similar in all other respects, subserves a like function. Whether the foramen marked *s* in fig. 6 is for the pre-spiracular nerve, or, as is more probable, simply transmits a vessel, I am unable to say. Immediately over the pointed apex of each basi-temporal ala the slit-like inferior termination (*x.*) of the canal, whose upper extremity opens into the anterior part of the *f*-shaped groove, can be seen.

Behind the posterior opening of the Facial canal is the small aperture for the exit of the Glossopharyngeal nerve, and still more posteriorly, and a little above it, is the large oval foramen for the Vagus. The anterior and posterior roots of the spinal nerves pass out of the spinal canal through a series of pairs of obliquely-disposed foramina, the one for the anterior root being inferior to, and slightly in advance of that for the posterior root. A supra-orbital foramen passes upwards through the posterior wall of the orbit, and opens into the adjacent part of the *f*-shaped groove.

Fig. 6 represents a vertical and longitudinal section of the cranium and rostrum. "The notochord, extremely large in the spinal column, rapidly diminishes in size as it enters the skull, and becoming a mere thread, terminates behind the pituitary fossa." The hæmal canal (*h.c.*) extends forwards between the diverging arms of the hinder part of the parasphenoid, and terminates where the former meet the body of the bone; and the canal (B) which is included between the upper halves of the neurapophyses, lying dorsad of the spinal canal and quite distinct from it, opens anteriorly on the hinder part of the cranial roof. The cranial cavity is high, but much compressed laterally. It extends forwards between the orbits, and is terminated anteriorly by a lamina perpendicularis, on either side of which the Olfactory nerves diverge to the nasal capsules. The spinal canal is enormously dilated in front, and with that part of the cranial cavity which lies behind the foramina for the Vagi, forms the comparatively high but compressed portion of the cranio-spinal canal. In front of these foramina the cavity gradually diminishes in height and width towards its rounded anterior termination against the lamina perpendicularis. As in other Ganoids and in all Teleostei, the inner wall of the periotic capsule is incomplete, the recess in which the membranous vestibular sac is lodged communicating widely with the cranial cavity. The letters *a.sc.*, *p.sc.*, and *h.sc.*, mark the openings of the semicircular canals into the vestibular recess. In the middle of the outer wall of this recess there is a small fenestra (*a*) which opens into the Facial canal behind the point where the seventh nerve enters the latter. The condition of my specimen would not admit of a successful dissection of the cranial nerves, hence I am unable to say definitely what vessels and nerves were transmitted by the closely aggregated foramina which perforate the side walls of the anterior part of the cranium.

The numerals VII (fig. 6) indicate the cranial end of the passage by which the Facial nerve reaches the Facial canal; V is the internally single aperture for the exit of the Trigeminus, and in front of it the small and shallow pituitary fossa is seen to be completely floored by cartilage. The small foramen (*Ca.*) on the anterior margin of the fossa may serve for the transmission of the carotid artery, while the small apertures indicated by the letters *m.o.* and *tr.* probably transmit the motores oculi, and the trochlearis. The external opening of the last-mentioned nerve is just in front of that for the fifth nerve;[*] II is the optic foramen. All these passages pass very obliquely outwards and forwards through the cranial walls. The Olfactory nerve breaks up into a number of small filaments, each of which reaches the olfactory mucous membrane through a separate opening in the inner wall of the nasal capsule, thus giving rise to a true cribriform plate. So far as I am aware, in all other branchiate Ichthyopsida this nerve passes bodily through a single large foramen to reach the interior of the nasal sac. The cavity of the rostrum is quite distinct from the cranial cavity, and terminates at some little distance behind its anterior end, which is much depressed and solid. It communicates with the exterior by the single median fenestra in its roof (*a.m.f.*), and by two lateral foramina (*z.*). It seems to me that this singular rostral cavity may be compared to the large "meso-ethmoidal fat cavity," which Mr. PARKER[†] has described as existing in the adult Salmon, in front of the coalesced orbito-sphenoids and lamina perpendicularis. Both occupy similar morphological areas, and each has a superior median foramen situated beneath the azygous dermo-ethmoid splint. If this comparison be correct, and the meso-ethmoidal fat cavity of *Salmo* is truly homologous with the rostral cavity of *Polyodon* and the Elasmobranchs, then its origin, as the result of a secondary fenestration of the internasal cartilage, is another example of a not unfrequent phenomenon in developmental history—the appearance of particular structure as a secondary feature, though normally and in the lower types it may be a primary one.

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[*] The aperture *v* seems to transmit a vessel.

[†] PARKER, "On the Structure and Development of the Skull in the Salmon." Phil. Trans., 1872, p. 108.

From what has been said, it is apparent that in all essential features the chondrocranium of *Acipenser* resembles that of *Polyodon*. The massive growth of the cranial cartilage in the former has masked many features which are very evident in the latter. To this cause are due the obliteration of the fore part of the cranial cavity, which, consequently, appears to terminate behind the orbits where the brain also ends, though in reality it is continued as a very narrow canal, upwards and forwards, nearly as far as the internasal region, and the conversion of the rostrum into an almost solid mass of cartilage, traversed by anastomosing fat cavities. To the same cause is due also the obliteration of the passage by which it is probable the parietal depression primitively communicated with the cranial cavity. The growth of the cartilage round the margins of the parasphenoid, over the junction of the latter with the vomers, and round the tracks of the rostral divisions of the fifth nerve, which in consequence run in deep grooves bounded by high ridges, are other examples of the same tendency. The nasal sacs are shallow cups hewn out of the rostral cartilage instead of being appended to it by short peduncles, as in *Polyodon*.

The large vacuity which interrupts the continuity of the roof of the cranio-spinal cartilage, and which is situated mainly behind the vagus foramen, is absent in *Polyodon*, except, perhaps, in so far as the anterior termination of the supra-spinal canal may be its representative in the latter genus. In the Sturgeon the wing-like processes of the cranio-spinal cartilage run backwards and downwards from the postero-lateral angles of the cranium, which is the contrary direction to that taken by the lateral ridges in *Polyodon*. In both genera the unusual lengthening of the post-auditory and pre-orbital regions of the chondrocranium as compared with the size of the brain, causes the Olfactory, the Pneumogastric, and the most anterior spinal nerves to run respectively forwards and backwards within the cranial cavity for a considerable distance before issuing through their respective foramina. A Facial canal, similar to but much longer and wider than that of *Polyodon*, exists also in *Acipenser*.

The Visceral Arches. (Plate 57, figs. 8 and 9.)

The two first visceral arches exhibit an interesting compromise between the peculiarities of the Elasmobranch and Teleostean types, for while the mandibular arch conforms mainly to the former type, the hyoid arch as distinctly follows the latter.

As is the case with the hyostylic Selachians, the quadrate and the pterygoid process together form a continuous bar of cartilage—the so-called "palato-quadrate bar" (fig. 8, *pg.*, Q.). This bar has no connexion with cranium, either directly or by ligament, but is suspended by its posterior extremity from the distal end of the symplectic, to which it is firmly attached by two slender ligaments. These ligaments pass from the rounded posterior margin of the quadrate cartilage backwards parallel to the anterior margin of the symplectic, and are attached, one to the proximal end of that cartilage, and the other to the contiguous extremity of the hyomandibular ;

they evidently correspond to the symplectic ligament of the Shark. The palato-pterygoid cartilage is a laterally compressed bar, deep behind and concave externally, with a gently rounded and elevated postero-superior border and a transversely extended articular convexity for the proximal end of the mandible. Anteriorly, in the orbital region, the bar becomes constricted, with its upper margin gently concave and its plane a little twisted, so that its outer surface looks upwards and outwards, and its inner downwards and inwards; and, before curving inwards to join the corresponding bar of the opposite side, it becomes somewhat expanded, and at the same time tilted upwards. From the quadrate portion of the bar, a broad leaf-like outgrowth of cartilage (*or.p.*) passes forwards on the outer side of, and parallel to, the "palato-quadrate" arcade, and in close relation externally with the inner surface of the hinder part of the maxillary splint. To this curious outgrowth I shall have occasion to refer presently. The levator mandibularis muscle lies along the grooved outer surface of the palato-quadrate cartilage between it and the maxilla externally, and passing backwards from its anterior attachment curves sharply downwards behind, between the quadrate and its leaf-like process and passes to its connexion with the lower jaw.* Closely applied to nearly the whole length of the palato-quadrate lamina there is a thin splint-like maxilla. This bone (*mx.*) has an unusual backward extension, reaching, in fact, as far as the condylar end of the quadrate cartilage; its posterior third is closely adherent to the outer surface of the leaf-like process, and its middle third is separated from the main pterygoid bar by the levator mandibularis muscle, while its diminished and almost pointed anterior third clips the lower edge of the ectosteal scale (*m.pg.*) and the adjacent cartilage.

Thus the maxilla in its extension backwards to the outer side of the lower end of the quadrate corresponds to the entire infra-temporal arcade (maxilla, jugal, and quadrato-jugal) of other Vertebrata. Two other ossifications have a more intimate relation to the "palato-quadrate bar." Of these, one is composed of two thin round scale-like exostoses applied to the inner and outer sides of the expanded anterior end of the bar; each scale is marked with concentric lines of growth and separated from the other by intervening cartilage (fig. 8, *m.pg.*). The other bone is a long splint attached along the whole length of the inner side of the palato-quadrate cartilage from the inner side of the quadrate forwards to near the symphysial union of the two bars. Broad behind, the bone thins away in front, and its slightly sinuous anterior portion clips the upper edges of the two ectosteal scales. The splint is clearly a parosteal pterygoid, comparable to the bone so named in the Axolotl and other Urodele Amphibia.

Dr. TRAQUAIR† (plate vii., fig. 2) gives an inner view of the palato-quadrate cartilage, showing the pterygoid; but neither in this figure nor in fig. 1 does he represent the two ectosteal scales, or the leaf-like outgrowth from the quadrate.

The proximal end of the Meckelian cartilage (*Mk.c.*) is slightly bent upwards on its

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* *Vide* plate vii., fig. 2, in Dr. TRAQUAIR's memoir, *loc. cit.*

† *Loc. cit.*

distal moiety, so that a short ascending process is produced which carries a transverse groove for the reception of the co-adapted articular end of the quadrate; its angular margin is gently rounded. Towards the ligamentous symphysis the axial core thins away, and finally terminates in a stout, cylindrical, mento-Meckelian bone.

A long dentary splint (*d.*) is applied along the whole length of the outer side of the cartilage; by the bending inwards of its upper and lower edges a groove is formed in which the axial cartilage and the mento-Meckelian ossicle lie. The splenial is a thin splint applied to the inner side of the proximal portion of the cartilage and reaches nearly to the articular end. There is no os articulare, nor any distinct angular, supra-angular, or coronary bones. Dr. TRAQUAIR* supplies a view of the mandible with its investing splints (plate vii., fig. 2).

Though the mandibular arch has no cartilaginous or osseous metapterygoid, yet the latter is not without a representative. The spiracular opening leads by a short tube directly into the pharynx, and in front of it, in the place of the spiracular cartilage, a strong ligament passes from the smaller of the two parasphenoidal alæ obliquely downwards and backwards beneath the spiracle to be attached to the anterior margin of the upper third of the hyomandibular.

Along the spiracular margin of this ligament the pharyngeal mucous membrane is produced into a number of short branchial filaments with attached bases and free extremities. I could not detect any cartilaginous rays in these filaments nor any cartilage in the ligament.

The gill filaments constitute a spiracular or mandibular gill, homologous with the structures so named in the Selachii; and the ligament is as clearly the representative of the metapterygoid ligament of that group from which, however, the former differs in being attached to the hyomandibular instead of to the quadrate cartilage. But it may be pointed out that in the Shark some at least of the fibres of the metapterygoid ligament are attached to the hyomandibular.

In *Acipenser*, in consequence of the increased transverse width of the skull, a long spiracular canal connects the external opening with the pharynx, and along the anterior margin of the slit-like pharyngeal opening of this canal the mucous membrane is produced into a series of branchial filaments precisely as in *Polyodon*, though there is nothing in the Sturgeon comparable to a metapterygoid ligament—a fact which supports Mr. PARKER's conclusion, that this element is represented by the triangular cartilage which, with its fellow, forms the posterior part of the roof of the protrusible proboscis.

I am not aware that the presence of spiracular gills has previously been noticed in any adult Vertebrata except the Elasmobranchii. Neither *Amia, Polypterus*, nor *Lepidosteus* possesses any traces of such structures. Hence, so far as this point is concerned, *Acipenser* and *Polyodon* resemble each other and the Elasmobranchii, and differ from the remainder of the Ganoidei and from all the Teleostei.

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* *Loc. cit.*

To determine the homologies of the two ectosteal patches (*m.pg.*) previously described as developed in connexion with the palato-pterygoid bar, is a task which is not very easy, and in order to do so it is necessary to form a true conception of the nature of the "palato-pterygoid" bar as it exists in different Vertebrata. This bar has been generally regarded as a forward outgrowth from the mandibular arch, which is eventually ossified from several distinct centres and so gives rise to the palatine, mesopterygoid, and pterygoid bones, the first mentioned being an ossification in the most anterior moiety of the bar. If this view be correct our task is an easy one, and we may at once regard the bone *m.pg.* as being the equivalent of the palatine bone of other Fishes.

But Professor PARKER has suggested another view as to the nature of this bar in the Vertebrata generally. In his opinion it is formed by the coalescence of a retral growth from the ethmoidal region (palatine process) with a forward outgrowth from the mandibular arch (pterygoid process). In accordance with this view the palatine bone is an ossification in the retral process, and is a preoral structure, while the mesopterygoid and pterygoid are developed in relation with the cartilaginous pterygoid process, and are therefore postoral elements. If, therefore, the upper jaw in *Polyodon* is formed by the union of the pterygoid processes only, then the ectosteal patches represent a mesopterygoid element. The evidence in favour of this theory as to the nature of the palato-pterygoid bar is very strong, and may be briefly set forth.

In the common Toad, in the adult skull, the primitive distinctness of the palatine and pterygoid processes is well seen. The cartilaginous pterygoid process forms less than half of the curvature of the subocular bar, the anterior part of which is formed by the retral palatine process. From the inner side of this process a transverse piece of cartilage, the ethmo-palatine, is given off, and is connected by fibrous tissue with the prefrontal region (ethmo-trabecular outgrowth). In front of the ethmo-palatine the palatine process is produced into a prepalatine spur, and behind it into a postpalatine process which is in ligamentous connexion with the anterior end of the pterygoid process.* In the Frog the segmentation of the palatine and pterygoid outgrowths has not been observed, neither has the primitive formation of this bar by the coalescence of retral and forward outgrowths been observed in any embryo Anura. This important desideratum is however supplied by the developmental history of the embryo Axolotl. In young embryos of $1\frac{1}{4}$, $2\frac{1}{4}$, and $3\frac{1}{2}$ inches long, the stages A, B, and C of PARKER, the formation of these processes has been observed ('Morphology of the Skull,' PARKER and BETTANY, pp. 109–112). In stage A there is not the slightest trace of a pterygoid process in connexion with the suspensorial cartilage, but in the antorbital region a small transversely disposed rod of cartilage has appeared which is attached by ligament to the trabecular cartilage just above the broad tooth-bearing part of the palatine splint. This transverse rod is the homologue of the palatine process of the Toad. In stage B a pterygoid process has appeared growing forwards

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* PARKER, Phil. Trans., 1876, Plate 54, fig. 3.

from the anterior margin of the suspensorium ; it is quite distinct from the palatine process, and in front of it there is a small free postpalatine segment which has been segmented off from the ethmo-palatine moiety in front of it. By stage C the post-palatine process has coalesced behind with the pterygoid bar, and in front is united by fibrous tissue to the remainder of the palatine outgrowth, almost precisely as in the Toad.

This want of synchronism in the first appearance of the two outgrowths in the Axolotl is evidence sufficiently strong to effectually preclude the idea that the palatine cartilage is a segment cut off from the pterygoid bar and united to the antorbital region of the trabecula.

Having established the anatomical distinctness of the two structures in question both in the adult skull and in the embryo Axolotl, and, in addition, in the latter a sequence in the developmental origin of the two parts, we may notice briefly the condition of these parts in the Teleostei and in Elasmobranchs.

In the Siluroidei there is a continuous bony outgrowth extending from the quadrate forwards to the prefrontal region. In no specimen of this family which I have had the opportunity of examining have I been able to detect any segmentation in this osseous bar, which either ends freely in front, or is attached by fibrous tissue to the vomer or to the prefrontal. A shaft-like cartilage-tipped bone is attached anteriorly to the prefrontal, and behind overlaps the outer or inner side of the previously mentioned osseous bar.

The latter clearly corresponds to the pterygoid process of the Amphibia, while the shaft-like bone is in every way comparable to the retral palatine process of the Axolotl and the Toad. In some Siluroids there is a facet on the middle of the inner face of the palatine bone by which it is applied to the prefrontal bone; this facet represents the ethmo-palatine process of the Toad, and the shaft-like bone thus exhibits prepalatine and postpalatine regions.

In the common Eel the primitive pterygoid cartilage is not wholly converted into bone, and a short slender piece of cartilage is left creeping along the inner side of the unsegmented osseous pterygoid. The palatine process is apparently absent in the adult. In the Conger Eel, however, the palatine process is a large unossified mass of cartilage applied to the outer side of the anterior end of the bony pterygoid process, and closely united by fibrous tissue to the persistently cartilaginous ethmo-trabecular region. The attached face of the palatine is slightly grooved, and between it and the ethmo-trabecular cartilage the rostral branch of the second division of the fifth nerve passes, precisely as is the case in the Skate.

From the Siluroid to the ordinary Osseous Fish the step is a short one, and if transitional links are needed they may be found in the *Cyprinoidei* and other generalised physostome Teleostei. In the Carp the ossified palatine process is attached by ligament only to the pterygoid process, and forms in conjunction with the latter a palato-pterygoid arcade. The regional distinctness of the palatine is well marked ;

there is a prepalatine spur in front of the short ethmo-palatine connexion with the prefrontal and a postpalatine region behind which abuts against the mesopterygoid and pterygoid elements of the pterygoid outgrowth. It may be mentioned that the recorded facts of the development of the Teleostean skull show that the palatine is at first quite distinct both from the ethmo-trabecular and quadrate regions, as seen in Plates 1–3 of Mr. PARKER'S 'Memoir on the Structure and Development of the Salmon's Skull.'* Subsequently the palatine fuses behind either with the quadrate or with its pterygoid outgrowth.

In the Elasmobranchs the same structures are readily recognized. In some Selachii (*Notidanus*) and in some Batoidei (*Raja* and *Rhinchobatus*) there is a large recurrent cartilaginous process attached to the hinder wall of each nasal capsule, but in many Selachians, as in *Cestracion* and in *Mustelus*, there is no trace of any such free process. In *Raja* the thick proximal end of this process is perforated by a foramen for the transmission of the superior maxillary division of the fifth nerve, as is the case with the palatine process of the Conger Eel; and in *Rhinchobatus* it presents a division into regions strictly comparable to the prepalatine, ethmo-palatine, and postpalatine regions of the palatine process of the Teleostean and the Amphibian. In all Elasmobranchs a forward growth from the proximal half of the mandibular arch unites beneath the prefrontal region with its fellow of the opposite side to form the characteristic upper jaw of this group.

That this retral antorbital process of the Sharks and Rays is homologous with the palatine process of the Toad, the Axolotl, and the Siluroid, seems to me an incontestable fact, proved by its attachment to the prefrontal region, by its perforation in *Raja*, and by its regional subdivision in *Rhinchobatus*. But if this retral outgrowth be a true palatine element, then the forward outgrowth from the manibular arch—the so-called palato-pterygoid bar—must correspond to the pterygoid process of the Toad and Axolotl, and cannot therefore contain any palatine element. Consequently the term "palato-quadrate arcade" is not rightly applicable to the confluent pterygoid and quadrate cartilages, and ought to be replaced by the term "pterygo-quadrate."

We are now in a better position to determine the nature of the upper jaw and its ossifications in *Polyodon*. It is evident that in so far as relates to the palatine and pterygoid processes, this Ganoid so closely resembles those Selachii in which the mesially-united pterygoids are unaccompanied by even the rudiments of recurrent palatine outgrowths, that we must conclude that its upper jaws are formed in precisely the same way, *i.e.*, by the union of the pterygoid bars. The two ectosteal patches without doubt, therefore, correspond to the mesopterygoid elements of other Vertebrata.

It is impossible to say whether the palatine process has entirely aborted, or whether it has contributed to form any part of the thickened and somewhat produced posterior wall of the nasal capsule.

An alternative, suggested by the slightly upturned anterior end of each half of the

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* Phil. Trans., 1872.

upper jaw, that the palatine elements may have coalesced behind with pterygoid outgrowths and then united with each other beneath the basis cranii, has nothing to recommend its acceptance. In view of the strong resemblance in the structure of their jaws between *Polyodon* and Selachians, nothing short of actual demonstration can render this alternative explanation in any way tenable.

*The Hyoid Arch.**

The hyomandibular (*h.m.*) is a long, relatively slender, well ossified shaft-like bone, with a vertically expanded proximal end, a much constricted central portion, and a vertically compressed distal extremity. Its axis is inclined backwards at an angle of less than 30° with the cranio-spinal cartilage. The cartilage-tipped cranial end of the bone fits into the vertical groove on the external face of the otic capsule dorsad of the horizontal semicircular canal, as in Teleostei, and not ventrad as in Elasmobranchs. The symplectic (*sym.*) is a stout triangular piece of cartilage attached by its broad apex to the anterior angle of the distal end of the hyomandibular, and by its base to the lower part of the rounded posterior margin of the quadrate to which it is firmly attached by the two symplectic ligaments, while its long axis is directed forward at an angle of about 90° with that of the hyomandibular. A line drawn through the proximal end of the latter and the distal end of the former will make an angle of at least 135° with the fore part of the skull. It is this extraordinary inclination of the hyomandibular and the quadrate that carries backwards the articular end of the mandible to a point opposite to the foramina for the passage of the fourth or fifth pairs of spinal nerves, thus giving a backward extension to the gape which is far greater than that of any other living Vertebrate animal, approaching, nearest in this respect, to the amphistylic Selachians and autostylic Anurous Amphibians, but receding from the other extreme as exhibited in the relatively short gapes and forwardly-inclined suspensoria of the Teleostei.

The remainder of the hyoid arch consists of a stout curved interhyal segment (*i.hy.*) attached to the posterior margin of the symplectic near its distal end, a long and comparatively slender epi-ceratohyal segment, and an unossified hypohyal element which connects the arch to the anterior end of the first basibranchial. The only ossification in the distal half of this arch is an hourglass-shaped ceratohyal (*c.hy.*). The hyoid arch of *Acipenser* is essentially the same as that just described. In the segmentation of the proximal half of the arch into hyomandibular and symplectic pieces, and in the presence of an interhyal, *Polyodon* resembles the Teleostei, but resembles the Selachii in the non-segmentation of the epihyal from the ceratohyal, and in the attachment of the ventral half of the arch almost to the distal end of the symplectic; in the possession of hypohyals and absence of a basihyal, *Polyodon* resembles the Skate, but in the attachment of the former to the first basibranchial element approaches such Urodele Amphibians as the Axolotl and Menobranchus.

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* See also plate vii., fig. 1, in Dr. TRAQUAIR's memoir on the Palæoniscidæ, previously cited.

The membranous fold that grows backwards from the hyoid arch over the gill opening is strengthened and supported by two thin squamose plates. Of these, one is attached to the outer side of the cartilaginous epiphysis at the distal end of the hyomandibular, and the other to the outer side of the interhyal cartilage. The first, which represents the operculum of the Osseous Fish, is produced backwards into several diverging, transversely striated flexible bony rays. It has no articular attachment, but is simply applied to lower end of the hyomandibular and retained in its place by fibrous tissue. The second, though usually described as a branchiostegal ray, seems to me to correspond rather to the interoperculum of *Ceratodus*, and, like the operculum, breaks up into a number of thin flexible rays. These two branching parostoses attached to the ventral and dorsal halves of the hyoid arch very curiously similate the two groups of coalesced cartilaginous rays, which in Selachii are attached, one to the hyomandibular, and the other to the ceratohyal. Lying in a groove on the outer side of the hyomandibular, I found a small cartilaginous filament, which I imagine is a remnant of one of these rays. A flattened nodule occupies a similar position in the Sturgeon, and I have also detected in *Lepidosteus* a small ray attached to the synchondrosis between the hyomandibular and symplectic bones and applied to the inner side of the operculum. HUXLEY has described similar structures attached to the inner side of the operculum and interoperculum of *Ceratodus* and *Lepidosiren*. The coexistence of these cartilaginous rays, which are so characteristic of the Shark and Rays, with the simple or branched parostoses which eventually replace them in the Teleostei, is an interesting transitional feature in these Ganoids and Dipnoid.

Branchiostegal rays may be regarded as the parostoseal representatives of such of the free cartilaginous rays as are attached to the lower end of the ceratohyal of the Shark.

The opercular gill, which is well developed in the Sturgeon, is altogether absent in *Polyodon*.

The Branchial Arches. (Plate, 57, fig. 9.)

The five branchial arches are very curious structures. The lateral half of each complete arch consists of a dorsal and a ventral segment uniting with each other at a very acute angle. The dorsal segment, formed by the pharyngobranchial and epibranchial elements, is directed backwards, downwards, and outwards to its union with the ventral segment while the latter passes forwards and inwards to join the mesobranchial pieces.

The first three arches possess the typical number of elements, viz., pharyngobranchial, epibranchial, ceratobranchial, and hypobranchials; the fourth arch has lost its pharyngobranchial and hypobranchial branchials, and the fifth retains only its ceratobranchial element.

The first pharyngobranchial ($p.br^1.$) is a short thick piece of cartilage attached by its conical extremity to the margin of the parasphenoid in the angle between the smaller ala and the body of that splint. The epibranchial ($ep.b^1.$) forms nearly the whole of the dorsal segment of the arch, and is a thin laterally compressed lamina of cartilage with a nearly straight inner, and a curved outer margin, resembling a segment of a circle. Ossification has invaded the proximal part of the cartilage, and given rise to an hourglass-shaped osseous epibranchial, and from its upper and posterior angle a short pointed ray is given off, which is directed upwards and backwards. The ceratobranchial ($c.b^1.$), which forms the greater part of the ventral segment resembles the epibranchial in shape; there is a small semicircular ossification marked with concentric striæ near its ventral end. The short, stout, and slightly curved hypobranchial is attached to the anterior end of first basibranchial, in close contact with the hypohyal.

The second and third branchial arches, which differ but little from the first, are attached by their pharyngobranchial elements to the hinder edge of the cranial end of the epibranchial belonging to the arch in front, and not directly to the skull. The second epibranchial ($ep.b^2.$) also possesses a small cartilaginous ray, similar to that attached to the first. These hypobranchials are also attached to the first basibranchial element.

The hypobranchial of the third arch has its ventral end laterally compressed and bifid, like the proximal end of a rib, and between the two processes and the basibranchial to which they are applied, one of the lateral trunks of the aorta passes on its way to the gill filaments of the arch it supplies. The fourth arch consists of epibranchials and ceratobranchials only, though the latter may also contain a hypobranchial element. They possess ossifications similar to those in the corresponding elements of the preceding arches. The epibranchial is attached dorsally to the proximal end of the epibranchial of the third arch, and ventrally to the second basibranchial. The distal end of the fourth ceratobranchial forks into two processes, the anterior of which is applied to the junction of the first with the second basibranchial, while the posterior one is applied wholly to the latter; and between the two forks another branchial artery passes.

The fifth pair of ceratobranchials are wholly cartilaginous, and lie in the floor of pharynx; above, they are attached by ligament to the antecedent ceratobranchials, and below to the posterior end of the third basibranchial. The three basibranchials are entirely cartilaginous. The first has attached to it the hypohyal, and the hypobranchials of the next three arches; the second completes the fourth arch, and the third is attached posteriorly to the fifth pair of ceratobranchials. The coalescence of the hypobranchial elements of the third and fourth arches with second basibranchial in *Dactylethra* and *Pipa* suggests that possibly the second and third basibranchials of *Polyodon* may be the mesially coalesced hypobranchials of the fourth and fifth arches. But the existence of three basibranchial in *Acipenser*, and the fact that hypobranchial and pharyngobranchial segments are the most variable of the branchial elements, as is shown by their absence in such Urodele Amphibians as *Siredon* and *Menobranchus*, and by their suppression in the hinder branchial arches of most Fishes, seems to negative this view. It seems much more probable that the fourth and fifth ceratobranchials each contain an unsegmented hypobranchial element.

Comparison of the branchial arches of *Polyodon* with those of *Acipenser* does not reveal any important differences. In the latter the arches are more massive and rounded. The same elements are ossified, but ossification has more largely invaded the cartilage, so that the resultant bones are shaft-like and massive.

There are three basibranchials as in *Polyodon*, but the second is rudimentary, being reduced to a small nodule of cartilage imbedded in the ligament by which the hypobranchials of the third arch are connected together. The third and fourth hypobranchials have their ventral ends split for the transmission of a branchial artery. The aorta, lying ventrad of the basibranchial pieces, gives off a large lateral trunk for the supply of the gills in relation with the third, fourth, and fifth arches, which passes outwards, downwards, and backwards, between the forks of the third hypobranchial, and after giving off a branch for the third arch, passes backwards between the forks of the fourth hypobranchial to the convex side of the fourth arch.

The cartilaginous ray, which in *Polyodon* is attached to the second epibranchial, is represented in *Acipenser* by a cartilage much larger that the pharyngo-branchial, and is applied to the lateral wall of the skull immediately over the course of the jugular vein. The next arch has no such ray. These free rays in the Chondrosteous Ganoids correspond to the autogenous processes which are given off from the pharyngobranchials and epibranchials of the second, third, and fourth arches of the Salmon.

The splitting of the distal ends of the third and fourth hypobranchials, though I am not aware that it occurs in any Teleostei, is the rule amongst Ganoids. In *Amia* and *Lepidosteous*, the same branchial elements are similarly bifurcated, and I believe that such is also the case in *Polypterus*.

Thus it will be seen that, though conforming mainly to the Teleostean type, the branchial arches of *Polyodon* and *Acipenser* in possessing free rays, in the forking of the hypobranchials, and in the non-coalescence of the dorsal elements of the posterior arches, differ from those of most Teleostei. The almost entirely cartilaginous nature of the arches, and the inclination to each other of the dorsal and ventral segments, are interesting embryonic and Chimæroid features.

The bifurcation of the proximal ends of certain of the branchial arches in *Polyodon* (and in many Teleostei also), is an interesting survival of a feature which is transitory in the hyoid arch of the Shark and Skate, though permanent in the mandibular arch of Amphibia. Each free ray may be compared to free "otic process."

As has been already mentioned, the branchial and hyoid arches of *Polyodon* differ but very little from those of the Sturgeon, but as regards the mandibular arch the differences between the two genera are more marked, though mainly dependent on the adaptation of the osseous and cartilaginous elements of the palato-quadrate arcade to form the remarkable proboscidiform mouth of the latter. The metapterygoid ligament is absent in the Sturgeon, and the broad triangular plate of cartilage, which takes its place as the proximal element of the mandibular arch, is apposed by its upper extremity to its fellow in the median line beneath the basis cranii, while its lower end is applied to the posterior margin of the quadrate. In full-grown Sturgeons there are two ectosteal mesopterygoid plates, thicker than those in *Polyodon*, and applied to the inner and outer sides of the anterior ends of the two palato-quadrate bars, which are also in contact beneath the basis cranii. A massive pterygoid flanks the inner side of the axial cartilage, and a maxilla and a jugal the outer side. But a much more important difference between the two Ganoids is the presence of a palatine bone in *Acipenser*, apparently due to the ossification of the antero-lateral edge of the palato-quadrate cartilage, and of a quadrate. ossicle. In the Sturgeon there is no trace of the leaf-like process which grows forwards from the quadrate cartilage of *Polyodon*.*

The existence of this leaf-like outgrowth is one of the most interesting anatomical features possessed by this Ganoid. Its superficial relation to the levator mandibularis muscle which passes downwards between it and the main body of the pterygoid process to its insertion of the mandible, and its evident origin as an outgrowth from the proximal half of the first postoral arch at once suggests a comparison with the "orbitar process" of the Anurous Amphibia. In the Tadpoles of all the Anura whose cranial development has been worked out, a flap of cartilage grows out from the upper edge of the much-inclined mandibular pier, arches over the levator mandibularis, and either terminates freely above, as in the Frog, or joins the prefrontal region of the trabecula, as in the Toad.†

At a later stage, when the rotation backwards of the mandibular pier has caused it to take up a position nearly at right angles to the axis of the skull, the orbitar process, now much reduced in relative size, instead of being directed upwards, is now directed forwards, and appears as a styliform outgrowth from the hinder edge of the quadrate cartilage. It is this particular stage which is depicted in Plate 58, figs. 2 and 3, of Mr. Parker's memoir,‡ and, though transitory in *Dactylethra capensis*, yet appears to be permanently retained by *Polyodon*.§

In the permanent retention of this embryonic feature, *Polyodon* is almost unique amongst Ichthyopsida, the Lamprey being the only fish in which the "orbitar process" has retained its probable primitive function of acting as a suspensor to the much-inclined suspensorium of Vertebrata possessing a suctorial mouth. This correlation of a suctorial mouth with the existence of a well-marked "orbitar process," and with a backward rotation of the suspensorium during the course of their cranial development, which is so marked a feature in the embryo Anura, at once suggests the probability

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* *Vide* Parker, "Development of the Face in the Sturgeon," 'Monthly Microscopical Journal,' 1873.

† Parker, "On the Structure and Development of the Skull in Batrachia," Part II., Plate 55, fig. 3, Phil. Trans., 1876.

‡ *Loc .cit.*, Phil. Trans , 1876.

§ Mr. Parker informs me that he has found a distinct orbitar process in a half-grown *Otilophus margaritifer* and even in an adult *Bufo ornatus*.

that the young *Polyodon* possessed a suctorial mouth and passed through larval stages in several respects very similar to those of the former.

The study of the cranial development of *Acipenser* and *Polyodon* will probably reveal new and important anatomical features which are common to the Ganoids and the Amphibia.

Shoulder Girdle.

Dr. TRAQUAIR, in his memoir "On the Palæoniscidæ" (plate vii., fig. 1), supplies a sketch of the membrane bones which invest the cartilaginous pectoral girdle.* As to the identity of the bones lettered *l.cl.* and *c.l.* there can be no doubt; they correspond to the clavicle and inter-clavicle of other Ganoids. The bone marked *p.cl.* is clearly the equivalent of the bone which in *Acipenser* is called supra-clavicle by PARKER; but I agree with Dr. TRAQUAIR in regarding it as a post-clavicle, the true supra-clavicle being the long and sabre-shaped bone marked *s.cl.* The supra-clavicle is attached to the postero-external angle of the bone c^1, the squamosal of TRAQUAIR, which evidently corresponds to the bone which in the Sturgeon PARKER calls the supra-temporal. But this splint (c^1) corresponds in every respect to the post-temporal bone of other Ganoids; its horizontal and descending lamina are analogous to the two processes into which the Piscine post-temporal ordinarily divides, and like the latter serve to attach the pectoral girdle to the postero-lateral angles of the skull.

The cartilaginous part of the pectoral arch, though far less massive and of more delicate proportions, is essentially the same as in the Sturgeon; but while the latter resembles the Skate in having a distinct supra-scapular segment, *Polyodon* agrees with the Selachii in having that element confluent with the scapular.

It may be remarked that in both of these genera the scapular arch is essentially the same as in Elasmobranchs, with the exception that the large and distinct præcoracoid of the former has no representative in the latter. An examination of any ordinary Shark—*Acanthia* or *Squatina* for example—will show that the scapula segment is divided by a scapular fenestra into a scapula proper and a pre-scapular element, and, further, that there is a coraco-scapular fenestra perforating the coracoid cartilage immediately ventrad of the glenoid facets and opening into the floor of the scapular fenestra precisely as in *Polyodon* and *Acipenser*. Such a coraco-scapular fenestra is found in nearly all the Selachii which I have had the opportunity of examining.

The epicoracoid process in *Polyodon* does not extend so far along the inner surface of the inter-clavicle as in *Acipenser*.

The coraco-scapular cartilage is much the same in *Amia*, though much reduced in size proportionally to the size of the overlying derm bones; and the same may be said of these structures in *Lepidosteus*, though in the latter, scapula and coracoid are largely ossified. In both genera there is the same longitudinal clearage of the cartilage into distinct morphological elements by coraco-scapular and scapular fenestræ, having the same relations to each other as in *Polyodon*.

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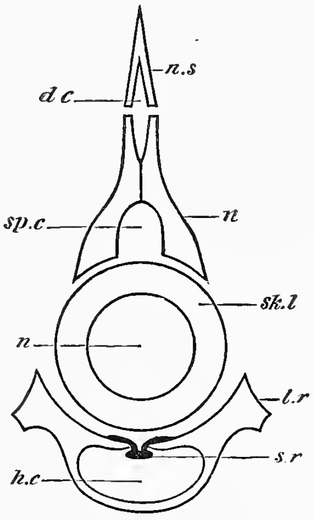
* For figures of coraco-scapular arch, *vide* GEGENBAUR, *op. cit.*, Part I., plate vi., figs. 3A, 3B, and 3C.

Vertebral Column, and the Skeleton of the Fins.

I do not propose to give a detailed description of the axial skeleton, as the destination of my specimen would not admit of the requisite dissection. The large unconstricted and unsegmented notochord is surrounded by a relatively thick skeletogenous layer which supports the neural and hæmal arches. The cartilaginous neural arches, though confluent near the cranium, are distinct in the central and posterior regions of the vertebral column. Above the spinal cord they coalesce, and dorsad of the true medullary canal a second or dorsal tube is formed between the slightly-divergent apices of the neuropophyses and the arms of the Y-shaped ossified neural spines, which terminates anteriorly on the upper surface of the cranio-spinal cartilage. A precisely similar canal is found in the Sturgeon and in *Ceratodus*, but not, so far as I am aware, in any Selachian or Teleostean. In the central and posterior regions of the column the neural spines become double, one being attached to each neuropophysis, as is not unfrequently the case in certain Teleostei. Interposed between the true neural arches there is a series of small, triangular, intercalated cartilages. In front of each intercalated piece there is a small foramen for the exit of the anterior root of a spinal nerve, the posterior root passing outwards just over the apex of each cartilage, as shown in Plate 55, fig. 3. The hæmal arches, which are not continuous with the neural arches, are attached along the ventral surface of the notochord; each arch is incomplete above, along the medio-ventral line of the notochord, but below it completely encloses the dorsal aorta, the walls of which are co-extensive with those of the cartilaginous hæmal canal. Each lateral half of each arch contributes to the formation of the continuous lateral ridge which alone repre-



n.s. Neural spine.
d.c. Dorsal canal.
sp.c. Spinal canal.
n. Notochord.
sk.l. Skeletogenous layer.
l.r. Lateral ridge.
h.c. Hæmal canal.
n. Nemopophysis.
s.r. Sub-notochordal rod.

sents the well-ossified ribs and transverse processes of the Sturgeon. Behind the anus the ridges converge slightly, and together with the well-ossified inferior caudal arches which are attached to them, form the canal in which the caudal vein is contained.

A transverse section of the vertebral column in the caudal region would disclose two canals—one immediately below the notochord, and containing the dorsal aorta; and a second ventrad of the first, containing, as previously mentioned, the caudal vein. In comparing such a section with the transverse section through the tail of an embryo *Scyllium*, given by BALFOUR ('Journal of Anatomy,' vol. xi.), it is evident that if chondrification were to affect the cells which lie between the aorta and the caudal vein, two precisely similar cartilage-bounded canals would be formed. Wedged in between the bases of each two adjacent hæmal arches, there is a small piece of cartilage exactly corresponding in position below the notochord to the intercalated cartilages above. These are interhæmal cartilages, analogous to similarly-situated structures described by GEGENBAUR* as existing in the Sturgeon.

But perhaps the most remarkable structure in connexion with the vertebral column is a stout, flattened, fibrous band which is attached to the narrow membranous interspace which exists between the broad pedate bases of the hæmal arches and projects downwards into the cavity of the aortic canal (fig. 2). Anteriorly, this fibrous band blends with the cartilage of the basis cranii, but posteriorly it may be traced nearly to the end of the tail.

As to the significance of this curious structure I can offer no suggestion, but it is not a little remarkable that it exactly corresponds in position and relations to the "sub-notochordal rod" which was first observed by GÖTTE in the embryo Frog, and subsequently and independently by BALFOUR and SEMPER in the developing *Scyllium*.

In the Shark the "sub-notochordal rod" appears to be formed from a ridge of cells which runs along the medio-dorsal line of the alimentary canal.† The cells at the extreme summit of this ridge become gradually constricted off to form a linear rod, the splitting off taking place from before backwards. The formation of this rod by segmentation from the summit of the hypoblast is an almost exact repetition of the process by which the notochord itself is derived from the same germinal layer. When the dorsal aorta is formed, the "sub-notochordal rod" becomes separated from the summit of the alimentary canal, and the aorta interposed between the two. At the period of its fullest development the rod extends anteriorly to a point in front of auditory capsules, but terminates a little behind the apex of the notochord; posteriorly, it extends nearly to the end of the tail. When at its maximum size the "sub-notochordal rod" lies immediately beneath the notochordal sheath and dorsad of the aorta; and it even projects into the cavity of the latter, which consequently appears in transverse section as reniform in outline, with the hilum directed upwards.

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* Jenaische Zeit. 1867.

† BALFOUR, 'Journal of Anatomy and Physiology,' vol. xi.

If this rod, instead of completely disappearing as it does in the Frog and the Shark, were to increase in size and persist, it would occupy a position analogous in every way to that of the fibrous band in *Polyodon.*

It may be remarked that the "sub-notochordal rod" of these Vertebrata occupies a position in relation to the aorta which is not altogether without *analogy* to the position of the "endostyle" of the *Ascidians* in relation to the blood sinus which in these animals lies on the hæmal side of the branchial sac.*

Skeleton of the Fins.

The structure of the pectoral fins has been described and figured by GEGENBAUR.† The dorsal and anal fins are each supported by two series of well-ossified, shaft-like interspinous bones, each bone supporting at least two fin rays. There is also a third series of interspinous cartilages formed by a number of small cubical pieces of cartilage which occupy the interspaces between the distal ends of the bones of the second series. A similar peripheral fringe of small cartilages, and similarly arranged, is found in the pectoral fins, and also in the ventral portion of the caudal fin attached to the free ends of the inferior caudal arches. In the Selachii a similar peripheral fringe of small cartilages is to be found in the mesial, and in some cases in the paired fins also. None of the interspinous bones or cartilages exhibited any of that tendency to coalescence which is so marked in the mesial fins of the Selachii. On the other hand, the evident formation of the ventral fins by the coalescence of a series of originally distinct cartilaginous rays is clearly indicative of a more primitive condition of these structures than can be found in any other living Vertebrate animal.‡

SUMMARY AND CONCLUSION.

In the foregoing description of the skeletal structures of *Polyodon* I have compared the latter with *Acipenser,* in so far as the special features under discussion were concerned, in order to demonstrate the close relationship that exists between the two forms in almost every detail of their anatomy. It will now be necessary to compare the two genera as to the points in which they differ from one another, and also to compare them with the remaining recent Ganoidei on the one hand, and with the Elasmobranchii and Amphibia on the other. In order to do this, it will be advantageous briefly to recapitulate the more important anatomical facts with which such a comparison is concerned, and which have been given in detail in the earlier part of this paper.

In the following paragraphs are enumerated those features in the osteology of *Polyodon* and *Acipenser* which are of greatest importance in relation to more general conclusions.

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* *Vide* also Dr. HUGO EISIG's paper on "Nebendarm d. Capitellidæ" in the 'Zoologischer Anzeiger,' No. 7, 1878.

† *Op. cit.* Plate 8.

‡ THACHER, *loc. cit.*

I. The very generalised cranial splints arranged in a medio-dorsal, a supero-lateral, and a lateral series, investing the dorsal surface of the chondrocranium, and corresponding to the dermo-ethmoid, the parietal and frontals, the post-temporal, the dermo-sphenotic, and the nasal bones of other Ganoids and Teleostei; the serial repetition of these dermal elements along the dorsal surface of a massive and prolonged prenasal rostrum; the median parasphenoid and paired vomers, which in like manner are repeated by a series of homologues applied along the ventral surface of the rostrum; while the infero-lateral series of splints includes an operculum, an interoperculum, a maxilla, and a multi-segmented premaxilla, but no preoperculum, suboperculum, branchiostegal rays, or median or paired jugular plates.

II. The massive and persistent chondrocranium entirely devoid of any such endosteal ossifications as basioccipital, exoccipital, alisphenoid, orbitosphenoid, or prefrontal elements, all of which are represented in the crania of the remaining recent Ganoidei, if we regard *Ceratodus* and *Lepidosiren* as being members of a distinct primary group, the Dipnoi; while diminutive scale-like opisthotic and pterotic exostoses alone represent in *Polyodon* the well-ossified periotic bones of other Ganoids and Teleostei.

III. The union of the anterior ends of the persistently cartilaginous quadrato-pterygoid arcades of their hyostylic skulls in a medio-ventral symphysis beneath the olfactory capsules, as in the Selachii. Associated with each arcade, which is suspended from the hyomandibular by a pair of symplectic ligaments, there are ectosteal mesopterygoid plates, and a parosteal pterygoid bone; and in addition, in *Acipenser*, a well-ossified palatine element. The proximal segment of the mandibular arch is either represented by a metapterygoid cartilage (*Acipenser*), or by a strong metapterygoid ligament passing from the skull downwards and backwards beneath the spiracle to its insertion into the upper third of the hyomandibular (*Polyodon*).

IV. The enormous backward extension of the gape, due to the great posterior inclination of the hyomandibular and quadrate; and the production of the latter into a large and persistent "orbitar process," between which and the quadrato-pterygoid bar the levator mandibularis muscle lies, precisely as is the case with the Tadpoles of the Anurous Amphibia.

V. The presence of persistent spiracles, and of mandibular branchiæ attached to the posterior margin of the metapterygoid ligament (*Polyodon*).

VI. The perforation of the outer wall of the periotic capsule by an antero-posterior canal, as in the Elasmobranchs *Trygon, Pristis,* and *Rhinchobatus;* and the fenestration of the roof of the capsule by a canal passing beneath the arch of the posterior vertical semicircular canal to communicate with the cranial cavity, which may possibly represent the orifice of the primitive involution to form the internal ear.

VII. The persistent unossified Meckelian cartilages, flanked by dentary and splenial elements, and with ossified lower labial cartilages (mento-Meckelian bones) interposed between their symphysial ends.

VIII. The cartilaginous branchial arches, consisting of dorsal and ventral segments uniting at an acute angle, and possessing small epibranchial and ceratobranchial ossifications. The absence of a basihyal element, and the consequent union of the hypohyals with the anterior end of the first basibranchial piece.

IX. The massive, unsegmented, unconstricted, and persistent notochord, extending into the basis cranii as far as the pituitary fossa, and surrounded by a relatively thick skeletogenous layer, which in *Polyodon* supports dorsally the cartilaginous neural arches, and ventrally the hæmal arches that enclose the dorsal aorta, the latter being also produced on each side into the continuous ridge which alone represents the transverse processes and ribs of other Ichthyopsida. The appearance of inferior arches in the caudal region gives rise to the formation of a second ventral canal lying beneath that in which the aorta is contained, and enclosing the azygous cardinal vein. In *Acipenser* the axial notochord and its sheath support well-ossified neural arches, transverse processes, and ribs. In both genera a curious flattened, fibrous band traverses the medio-ventral line of the notochord, projecting into the dorsal aorta, and presenting interesting analogies in its position and relations to the "sub-notochordal rod" of the developing Frog and Shark. In both *Polyodon* and *Acipenser* the elements of the anterior part of the vertebral column are almost indistinguishably fused with each other and with the cranium into a continuous cranio-spinal mass.

X. The presence of rudimentary intercalated neural arches lying behind the foramina for the exit of the anterior roots of the spinal nerves, and below those for the passage of the posterior roots; and also of interhæmal cartilages having a similar relation to the true hæmal arches.

XI. Cartilaginous scapular arches that conform to the Selachian type, but which are accompanied by post-temporal, supraclavicular, postclavicular, clavicular, and interclavicular derm bones.

XII. In the structure of the pectoral, ventral, anal, and caudal fins, *Polyodon* and *Acipenser* to some extent resemble the Selachii, and notably in that their fins possess a marginal fringe of small cubical cartilages, in the segmentation of their interspinous bones and cartilages, and also in the fact that the latter are less numerous than the fin-rays which they support.

A. *Comparison of* Polyodon *with* Acipenser.

Though so closely related to *Polyodon*, and agreeing with that genus in most of the anatomical features to which attention has been directed in the preceding paragraphs, the Sturgeon differs from the former in several points, of which the most important are—the ossification of the retral palatine outgrowths as palatine bones; the representation of the proximal elements of the mandibular arches by large metapterygoid cartilages, which are so coadapted as to form the hinder part of the roof of the protrusible proboscidiform mouth, and not merely by metapterygoid ligaments as in

Polyodon; the possession of jugal and quadrate ossicles, and well-developed osseous transverse processes, neural arches, neural spines and ribs. The massive growth of the cranial cartilage in *Acipenser* has obliterated the parietal foramina and the cavity of the prenasal rostrum, diminished the antero-posterior extent of the cranial cavity, and caused the abortion of the cranial prolongation of the notochord. The posterior median fontanelle of *Acipenser* has no existence in *Polyodon,* and in the former the fibrous tracts ossify as distinct splints, and not continuously, as in the latter. To these more or less important differences we may add that in *Acipenser* there is a well-developed opercular gill, a siphonal stomach, and a simple non-cellular air-bladder; whereas in *Polyodon* there is no opercular gill, the stomach is cœcal, and the air-bladder cellular. With these, for the most part relatively unimportant exceptions, the agreement between the two genera, even in the minutiæ of their skeletal anatomy, is very striking. It will be noticed that, in so far as the Sturgeon recedes from the *Polyodon* type, it becomes more specialised, and approaches the higher Ganoids and the Teleostei.

B. *Comparison of* Polyodon *with the Elasmobranchii.*

Not the least interesting feature in the osteology of *Polyodon* is its possession of a remarkable combination of characters, usually regarded as specially distinctive of the Plagiostome Elasmobranchs. Of these, we may mention the unossified chondrocranium; the formation of the upper jaw by the union of the pterygo-quadrate bars in a median symphysis beneath the basis cranii; the attachment of the quadrate cartilage to the hyomandibular by means of symplectic ligaments; the abortion of the proximal segment of the mandibular arch and its representation by a metapterygoid ligament which bounds the spiracle in front, supports a mandibular gill, and is attached behind and below to the hyomandibular; the presence of a Facial canal; the rudimentary cartilaginous ray attached to the hyomandibular, as in *Acipenser,* which it is possible may represent the hyoid rays of the Selachii; and the parosteal operculum and interoperculum, resembling in their branched and subdivided condition the cartilaginous rays which they have replaced.

The presence of intercalated neural arches, the prolongation of a fleshy lobe into the upper division of the caudal fin, the abortive condition of the ribs, and the structure of the shoulder girdle and pectoral fins, are also evidences of a like affinity. Of the more numerous minor points to which the same significance must be attached, I may mention the absence of a basihyal as in the Rays, the non-segmentation of the epihyal from the ceratohyal, and the prolonged prenasal rostrum, with its probably coalesced first pair of labial cartilages. On the other hand the condition of the ventral fins and of the vertebral column is far more primitive than in the Selachii.

The lateral position of the anterior and posterior nares, the structure of the pectoral, ventral, anal, and dorsal fins, as well as the segmentation and disposition of the interspinous cartilages and their relation to the more numerous fin rays, are to some extent a transition towards the structure of the same parts in the typical Elasmobranchii. In nearly all of those characters *Polyodon* differs from all other living Ganoids, with the exception of *Acipenser* and its nearest living and fossil allies. The extent to which this close correspondence in structure between the chondrosteous Ganoids and the Elasmobranchii bears on the presumed phylogenetic relationship of the former to the latter will be referred to subsequently.

C. *Comparison of* Polyodon *with the Higher Ganoids and with the Teleostei.*

Comparison of *Polyodon* with the more specialised Ganoids and with the Teleostei shows that the former resembles the latter in possessing cranial and pectoral splints, rudimentary periotic ossifications, and in the partial ossification of its visceral arches; in the presence of opercular bones, in the division of the proximal half of the hyoid arch into hyomandibular and symplectic segments, and in presence of an interhyal element. I may also mention the absorption of the inner wall of the otic capsule, and the development of a lamina perpendicularis; the absence of labial or extra-branchial cartilages with the doubtful exception of such as may be confluent with the anterior end of the produced prenasal rostrum. The non-lobate character of the pectoral fins, and the lateral position of the anterior and posterior nares, as well as the fact that the interspinous bones, though thrice segmented, are not more numerous than the segments of the axial skeleton, as indicated by the neural arches, may also be regarded as instances of specialisation in the same direction.

Previous to discussing the relations of *Polyodon* to the Amphibia we may consider the bearing of the facts elucidated in preceding detailed comparison of the genus with the Elasmobranchii, and with the other Ganoids, on its systematic position. Several anatomists have attempted to give expression to the important structural differences between the two genera of *Polyodon* and *Acipenser* and the remaining recent and fossil Ganoids in various schemes of classification, and it will be advantageous to give a brief sketch of their views.

AGASSIZ,[*] in his newly constituted order of the Ganoidei, included *Acipenser* and *Polyodon* in the family Acipenserides.

HEINRICH MÜLLER,[†] in his classical paper on the anatomy of the recent Ganoids, classified them as follows:—

α. Holostei.—Osseous vertebral column.

Family I.	Lepidosteini . . .	*Lepidosteus.*
Family II.	Polypterini . . .	*Polypterus.*

β. Chondrostei.—Cartilaginous vertebral column and persistent notochord.

Family III.	Acipenserini . . .	*Acipenser* (*Scaphirhynchus*).
Family IV.	Spatulariæ . . .	*Polyodon* (LACÉP.); *Planirostra* (RAF.).

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[*] AGASSIZ, Pois. Foss.

[†] 'Ueber den Bau und die Grenzen der Ganoiden.' Abhandl. Ak. Wiss. Berlin, 1864.

GIEBEL,[*] in his classification of fossil Fishes, retained H. MÜLLER's subdivisions of Holostei and Chondrostei, the genera *Polyodon* and *Acipenser* being relegated to the last-mentioned group.

VOGT,[†] in a system of classification of recent and extinct Ganoids, divided the order into the three suborders of (I.) Loricata, which contained the families Cephalaspida, Acipenserida, and Spatularida; (II.) Rhombifera, including Lepidosteidæ, Polypteridæ, and the extinct Ganoids with rhomboidal scales; (III.) Cyclifera, comprising the Amiadæ and the cycliferous fossil genera.

HUXLEY,[‡] in his essay on 'The Systematic Arrangement of the Fishes of the Devonian Epoch,' and subsequently in his 'Anatomy of Vertebrate Animals,' arranged the recent and fossil members of this order in the following suborders:—

I. Amiadæ.
II. Lepidosteidæ.
III. Crossopterygidæ.
IV. Chondrosteidæ.
V. Cephalaspidæ.
VI. Placodermi.
VII. Acanthodidæ.

LÜTKEN,[§] declining to separate the Ganoids from the Teleostei, regarded the former simply as a subdivision of the Physostomi, thus:—

Subclass I. Teleostei.
  Order I. Air-duct absent (*Physoclysti*).
  ,, II. Air-duct present (*Physostomi*).
    Suborder I. Containing MÜLLER's families of the physostome Teleostei, and, in addition, the *Amiidæ*, the *Leptolepidæ*, the *Megaluri*, and the *Caturi* of the Jurassic period.
    ,, II. *Ganoidei.*—Nearly all the recent and extinct Ganoids.
    ,, III. *Sturiones* (*Acipenser*, *Polyodon*).

It will be noticed that, in so far as *Polyodon* and *Acipenser* are concerned, LÜTKEN agrees with H. MÜLLER in regarding them as constituting a group equal in morphological value to that in which all the other Ganoids are contained.

CARUS[‖] retains the Chondrostei as a distinct suborder for the reception of the two genera, while the suborder Phractosomata contains the Placodermi and the

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[*] 'Fauna der Vorwelt,' vol. I., pt. 3. Leipzig, 1848.

[†] 'Zoologische Briefe,' vol. II. Frankfurt, 1852.

[‡] Mem. Geol. Survey, sec. X.

[§] *Vide* German translation in 'Palæontographica,' Band xxii., 1873. Mag. Nat. Hist. 1871.

[‖] 'Handbuch der Zoologie' (CARUS und GERSTÆCKER).

Cephalaspidæ, the remaining recent and fossil Ganoids being distributed among seven other suborders.

OWEN[*] included *Cephalaspis, Pterichthys, Pteraspis, Coccosteus,* and the Sturionidæ (gen. *Acipenser, Chondrosteus, Polyodon*) in the suborder Placoganoidei; while the remaining Ganoids, both recent and extinct, are included in his second suborder of Lepidoganoidei.

Dr. GÜNTHER,[†] in his catalogue of living Fishes, retains H. MÜLLER's subdivisions of Holostei and Chondrostei, and uses them in the sense originally suggested by their author, the latter group including the families of the Acipenseridæ (*Acipenser* and *Scaphirhynchus*), and *Polyodon.* Dr. GÜNTHER further removes *Polyodon* from the Acipenseridæ and elevates it to the rank of a distinct family, under the name of Polyodontidæ.

It will be seen from the foregoing brief account of the various schemes for the classification of the extinct and living Ganoids which have been proposed, that in the majority of cases the systematic importance of the Chondrosteous Ganoids is regarded as being equivalent only to that of each of the other families of Ganoids. But I submit that the facts elucidated in the earlier part of this paper, and briefly summarised above, afford abundant evidence in favour of adhering to H. MÜLLER's two primary divisions of Ganoidei (Chondrostei and Holostei), or to LÜTKEN's two groups of 'Ganoidei proper' and 'Sturiones.'

Though there can be but little doubt as to the primary group of Fishes to which the Chondrostei should be relegated, yet the remarkable combination of characters generally regarded as more or less distinctive of the Teleostei, the Elasmobranchs, and the Amphibia possessed by *Polyodon* and, though to a less extent, by *Acipenser* also, even if they do not justify the elevation of these genera to a primary subdivision of the class, at all events render it necessary that these facts, which so clearly demonstrate their distinctness from all other Ganoids, should receive adequate expression in any natural system of classification.

Omitting for a moment any reference to the Amphibian tendencies of *Polyodon*, it seems probable that the true position of *Polyodon* is that of an annectent group, between the Elasmobranchs on the one hand, and the divergent stems of the Teleostei and Ganoidei on the other. The existence of such transitional families as the Chondrosteidæ and the Palæoniscidæ, by which the Acipenseroid and Polyodontoid types are related to the more typical Ganoids, renders doubtful the necessity of establishing a primary group for the reception of those types. While, therefore, the Chondrostei may be retained with the remaining Ganoids in the same order, yet it seems necessary that the distinction between the two groups should be definitely expressed. We may, I think, retain the two subdivisions first instituted by H. MÜLLER, though it seems advisable to remodel the definitions on which they were based. As before stated, the

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[*] Palæontology. 1861.

[†] 'British Museum Catalogue of Fishes,' vol. viii.

most distinctive feature about *Acipenser* and *Polyodon* is the remarkably Selachian character of their mandibular arches, and it is to those structures that we must look for definitions of the subsidiary groups. In both genera the pterygoid outgrowth from the proximal segment of each mandibular arch is united with its fellow in a median symphysis, as in the Selachii; in all the remaining recent Ganoids these outgrowths are connected through the intervention of a palatine bone with the prefrontal region of the skull.

We may therefore classify Ganoids into two main groups as follows :—

Ganoidei,

α. *Selachoidei.*—Pterygoid processes united in a median symphysis. Persistent and unsegmented notochord. Persistent spiracles and mandibular branchiæ.

Genera—*Polyodon, Acipenser, Scaphirhynchus,* and *Chondrosteus.*

β. *Teleosteoidei.*—Pterygoid processes not united with each other, but connected directly, or indirectly through the intervention of a palatine bone, with the prefrontal region of the cranium. Vertebral column generally ossified into distinct vertebræ; notochord aborted; no mandibular branchiæ.

Genera—*Amia, Polypterus, Calamoichthys,* and *Lepidosteus.*

In deciding upon the claims of the fossil Ganoids to be included in one or the other of these two groups, we are compelled to rely altogether on the structure of the upper jaw. Tested in this way, we must refer the Crossopterygidæ, Lepidosteidæ, Palæoniscidæ, and Platysomidæ to the group of Teleosteoid Ganoids. The Acanthodidæ probably belong to the Selachoid group.

The distinctness of the Selachoid and Teleosteoid groups is materially lessened when an attempt is made to include the fossil genera of Ganoids in either of them. *Acipenser* and *Chondrosteus* in the former group, and the Palæoniscidæ and the Platysomidæ in the latter, partially bridge over the gap which exists between the two when only the recent forms are considered.

Dr. Traquair,* in his valuable paper "On the Ganoids of the Carboniferous Formations," refers the families Palæoniscidæ and Platysomidæ to the suborder Acipenseroidei, which also includes *Acipenser, Chondrosteus,* and *Polyodon.*

The principal characters by which this suborder is defined and distinguished from the other suborders of Crossopterygii, Amioidei, and Lepidosteoidei, as given by Dr. Traquair, are as follows ;—

"Suborder II. *Acipenseroidei.*—Tail completely heterocercal; notochord persistent; paired fins non-lobate, infra-claviculars present, rays of dorsal and anal fins exceeding in number their supporting interspinous bones; præoperculum when present tending to extend forwards over the cheek; branchiostegal rays in most, but large jugulars in

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* *Op. cit.*

none. Skin in many furnished with osseous scutes, or with rhomboidal scales, in some naked, save on the prolongation of the body along the upper lobe of the caudal fin."

Though Dr. Traquair may have adduced sufficient evidence to justify the separation of Palæoniscidæ and Platysomidæ from the Lepidosteoid forms with which they have so long been associated, nevertheless I venture to think that he has not been so successful in his efforts to establish the claims of those two families to be included in the same group with the Acipenseroid forms. Most of the characters on which he relies for justifying the union of the Palæoniscidæ with the Selachoid Ganoids, are certain generalised structures which are common to widely diverse groups of Fishes, and the force which these facts possess as evidences of affinity is very considerably lessened by considerations based upon more specialised structures. For example, paired interclavicles are common not only to the special groups under discussion, but also to the recent and extinct Crossopterygidæ ; segmentation of the interspinous bones or cartilages is characteristic of nearly all Ganoids, of Ceratodus, and of Elasmobranchs ; while the fin-rays are more numerous than the bones or cartilages which support them in such diverse forms as certain Crossopterygii, Ceratodus, and Lepidosiren, and the same may be said of the corneous fibres which represent those structures in the Holocephali and Elasmobranchii. Similar remarks might be made on the persistence of the notochord, the absence of ossified ribs, and the condition of the caudal fin. On the other hand, the more specialised structures of the Palæoniscidæ point strongly to the conclusion that they are more closely related to the Teleosteoid than to the Selachoid Ganoids. The dermal armature of Palæoniscidæ is essentially the same as that of the Crossopterygii and the Amiidæ, and the latter groups further agree with the former in possessing a complete series of opercular bones. The hyomandibular of *Palæoniscus* is strikingly like that of *Polypterus*, and the former also resembles the latter in the character of the preoperculum which extends forwards over the cheek towards the orbit, as well as in the possession of a transverse chain of supra-temporal ossicles behind the posterior margin of the cranial buckler.

And in addition, the Palæoniscidæ possess well-ossified, quadrate, metapterygoid, and articular elements ; and there is also evidence that the chondrocranium was not altogether devoid of ossific centres. Most of the characters which appear to support the view that the Palæoniscidæ are closely related to *Polyodon* and *Acipenser* seem to be only evidence of generalisation or the persistence of ancestral features common alike to these forms and to many other Ganoids, and not of any special affinity between any of the groups which may happen to possess them. But far outweighing the evidence in favour of such affinity is that furnished by an examination of the structure of the upper jaw in *Palæoniscus* and its allies. Dr. Traquair has shown that in that genus the upper jaws somewhat resemble the same parts in *Polyodon*, and notably in possessing long parosteal pterygoid bones, between which and the backwardly-produced maxilla the levator mandibularis muscle must have passed precisely as was described in the earlier part of this paper as existing in *Polyodon*. Nevertheless, in Palæoniscus it would appear that the pterygoids were attached to the prefrontal region, as is the case in all the Telesteoid Ganoids, but differed in this respect from the disposition of those parts in *Polyodon* and its allies. While, therefore, it may be admitted that the Palæoniscidæ and the Platysomidæ are to some extent annectent forms between the two main subdivisions of the Ganoids, yet at present the balance of evidence is against their being included with the Selachoid forms, and in favour of their classification with the Teleosteoidei, though whether they should provisionally be classed with the Lepidosteoid forms, or form the nucleus of a new group is not, as yet, quite clear.

A careful comparison of the dermal armature of the Placodermi and the Cephalaspidæ with that of *Polyodon* and *Acipenser*, failed to suggest the existence of any near relationship between these widely different forms, or to throw any light on the nature of the cranial buckler in the two Palæzoic families.

D. *Comparison of* Polyodon *with the Amphibia.*

A comparison of the skull of *Polyodon* with the skulls of the Anura and Urodela leads to interesting though somewhat perplexing results, which are not without significance in relation to the phylogeny of the three most important groups of Ichthyopsida. The majority of living Ganoidei, both in their osteology and myology, exhibit many indications of affinity to the Amphibia, and *Polyodon* is no exception to this rule. There are several interesting points of agreement with the Urodela. The fenestration of the roof of the periotic capsule in *Polyodon* is comparable in position and in its relations to the arch of the posterior vertical semicircular canal to a similar vacuity, which in *Siren lacertina* represents the primitive involution to form the internal ear; and it is noteworthy that in this point both forms agree with each other and differ from the Plagiostomi where the parietal foramina are situated to the inner or mesial side of the vestibular sac and its canals. *Polyodon*, in common with all other Ganoids, agrees with the majority of Urodela in the absence of a basihyal and the attachment of the hypohyals to the first mesobranchial segment. With the Anura, *Polyodon* agrees in the ossification of its lower labial cartilages as mento-Meckelian bones. But perhaps the most remarkable feature in which *Polyodon* resembles the Anurous Amphibia is the possession of a forwardly directed "orbitar process" associated with a suspensorium so much inclined backwards that the angle of the gape is extended even beyond the posterior limits of the skull. The Marsipobranchii are the only other adult Ichthyopsida in which this singular process has as yet been discovered. So far as I am aware no such structure, or even a rudiment of it, has yet been detected in any of the Elasmobranchii, the Teleostei, or in any of the Urodela, either in the adult or in the embryo. The precise significance of the structure in question is not very clear, but one suggestion may be made. The condition of the "orbitar process" as an apparently functionless rudiment in *Polyodon* does not throw any light on its primary origin or function; but its existence and relations in the adult Lamprey, as well as its transitory condition in the developing Frog, suggest that originally it may have acted as a kind of anterior suspensor to the much inclined mandibular pier of such primitive Vertebrata as happened to possess a suctorial mouth, prior to the adaptation of the palato-pterygoid bar to that purpose. Hence the existence of an "orbitar process" in the permanently suctorial mouth of the Marsipobranchii, and as a rudiment in the temporarily suctorial mouth of the young Tadpole. These facts, coupled with the rotation of the quadrate cartilage, which from the direction and relations of the "orbitar process" we may infer to have taken place, are sufficient to render extremely probable the existence of a close parallelism between the developmental history of the cranium in the embryo *Polyodon* and such otherwise dissimilar forms of Anurous Amphibia as *Bufo ornatus, Dactylethra capensis, Rana temporaria*, and *Otilophus margaritifer*, though I am not aware that anything has yet been made known about the cranial embryology of the former.

The existence in this very generalised Ganoid of so remarkable a combination of Selachian and Amphibian characters at once suggests an inquiry into the probable ancestral relations to the progenitors of those two groups, and the first question for solution is, whether the Ganoids are more closely related phylogenetically to the Amphibia or to the Elasmobranchii? The embryology of the Ganoids is at present altogether unknown, and from this source, at all events, no clue can be obtained as to the phylogeny of the group; consequently we are dependent upon such conclusions as may be drawn from comparative anatomy. An analysis of the structural features common to any two of the three groups seems to me to lend very strong support to the view that the Ganoids and the Amphibia are of monophyletic origin.

The two groups agree with each other in—

I. The possession of morphological or functional lungs.

II. The rudimentary condition of the cerebellum.

III. The persistence of the embryonic connexion between the efferent renal ducts (Wolffian ducts) and the Mullerian ducts, due to the incomplete splitting of the primitive segmental duct.

IV. The development of parosteal and endosteal ossifications in relation with their originally cartilaginous crania.

V. The possession of a conus arteriosus with valves.

VI. The existence of functional external gills in the embryos.

In all of these points both the Urodela and the Anura agree with the Ganoids, while the anatomical facts to which reference is made in paragraphs III., V., and VI., are common alike to the Ganoids, the Amphibia, and the Elasmobranchs.

The Urodela and the Ganoids have also certain minor characters in common, which are not shared by the Anura, and these are—the possession of a dentigerous splenial, the fenestration of the roof of the otic capsule (*Polyodon* and *Siren lacertina*), the absence of a basihyal and the attachment of the hypohyals to the first basibranchial segment. On the other hand, nearly all Ganoids in possessing mento-Meckelian ossicles, and the Ganoid *Polyodon* in possessing an "orbitar process," agree with most adult or embryo Anura. In *Polypterus* among Ganoids, in all Anura, and in not a few Urodela, there is a tendency towards the development of a more or less extensive interorbital ossification comparable to the "os en ceinture" of the Frog. But perhaps the most important of all the characters by which the Ganoid-Amphibian stem is distinguished from the Elasmobranch stem is the presence of a cœcal diverticulum from the fore part of the alimentary canal, which in its varying degrees of development forms the functional lung of the Amphibian, or the swim-bladder of the Ganoid—a structure which, though partially respiratory, is mainly mechanical in function.

Finally may be mentioned the existence of such transitional forms as the Dipnoi, which in so many features are connecting links between the Amphibia and the more generalised Crossopterygian Ganoids. It may now, perhaps, be permissible to assume that the primitive ancestors of the Ganoids and Amphibians were differentiated from the Elasmobranch stock prior to the evolution of their two main groups of descendants.

The fundamental similarities between the main divisions of the Amphibia lead us to believe that they have been derived from some one ancestral form. That the divergence of the Anura and Urodela took place at a very early period is evident not only from the differences observable when their adult structures are compared, but also from the extreme dissimilarity of their larval forms. Even if we admit that differences which now exist between the larvæ are the result of adaptive modification in one or other of them subsequent to the evolution of the two types, there will still remain those existing between the adults which are scarcely less important.

As far as our deficient knowledge of the development of the cranium in the Urodela will admit of any attempt to make general statements, it would appear that the very characteristic metamorphosis of the mandibular arch of the young Tadpole which results in the attachment of the forwardly inclined mandibular pier to the prefrontal region by a vertically disposed palato-pterygoid bar, and in the development of an "orbitar" outgrowth from the quadrate, while the non-functional Meckelian cartilages are superseded in their use as jaws by the labial cartilages, never occurs in any of the caudate Amphibia. In the latter also there are no labial cartilages, and even when horny jaw sheaths are present, as in the adult *Siren*, they are not supported by labial cartilages, as in the Tadpoles of the Anura, but by the dentary and premaxillary bones. But the wide difference between the two types is most decisively shown when such Perennibranchiate forms as *Proteus* and *Menobranchus* are compared with any of the Anura.

In *Menobranchus*[*] the suspensorium retains its primitive position at right angles to the cranial axis, and the palatine and rudimentary pterygoid processes are widely sundered; the trabeculæ retain their embryonic condition as simple nearly parallel

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[*] HUXLEY, Proc. Zool. Soc., 1874.

bars entirely devoid of any ossific centres, and, as is the case with no other Vertebrate animal, the parachordals are permanently distinct from one another, the notochord lying between them in a stratum of connective tissue. In *Proteus*, and also in *Menopoma*, the proximal segment of the hyoid arch persists as a large and distinct hyomandibular cartilage, and is especially large and attached by ligament to the periotic capsule in the former, so that the distal part of the arch is directly connected with the skull by its own proper proximal segment. In addition, there may be mentioned the absence of an opercular fold for the protection of the gills in all Perennibranchiata, with the exception of a rudiment in *Siredon;* the persistence of external gills combined with the non-development of internal branchiæ, the absence of eyelids, and the imperfect condition of the inter-auricular septum. Unless many of these features are due to the degradation of the more immediate predecessors of the recent Urodela, we must believe that the primitive common ancestor of the Urodela and the Anurous Amphibia was in many respects of a lower organization than any living Marsipobranch or Selachian. On the other hand, there are reasons for believing that the Perennibranchiate Urodela owe much of their simplicity of structure to the fact that they are the descendants of Caducibranchiate forms which have undergone a certain amount of retrograde metamorphosis. The comparative ease with which the former under the influence of suitable natural or artificial conditions become Caducibranchiate, as shown by the interesting experiments of Fräulein MARIE VON CHAUVIN,* suggests that the change is nothing more than the assumption of a more immediate ancestral habit of life. As to the relation of the Ganoidei to this ancestral form, or to either of its two groups of descendants, two suggestions may be made. The close analogy between the Polyodontidæ and the Anura in the nature of their cranial metamorphoses which I infer to exist from the existence and relations of the "orbitar process" in both groups, combined with the fundamental differences which exist both between the larval and the adult Anura and Urodela, is somewhat unexpected and inexplicable, on the view that the two latter are more closely related to one another than is either of them to any of the Ganoid *Polyodon*. On the other hand, is it possible that *Polyodon* may have been an offshoot from the first evolved Anura subsequently to the divergence from the Urodela, and that while the Ganoid has retained its piscine habit, the two other groups have been modified along parallel lines and adapted to an air-breathing life? But an affirmative answer to this question involves the inadmissible conclusion that the phylogenetic relationship of the Anura to the Polyodontidæ, and presumably therefore to all other Ganoids and to the Teleostei, is closer than that of the former to the Urodela. Such a conclusion is also negatived by many other considerations. The close correspondence which can be traced between the caudate and the non-caudate Amphibia in the arrangement of their generative and vascular systems, and in the structure of the vertebral column, pectoral and pelvic arches, and their limb-skeletons, is so marked that it can hardly be explained in any other way than by

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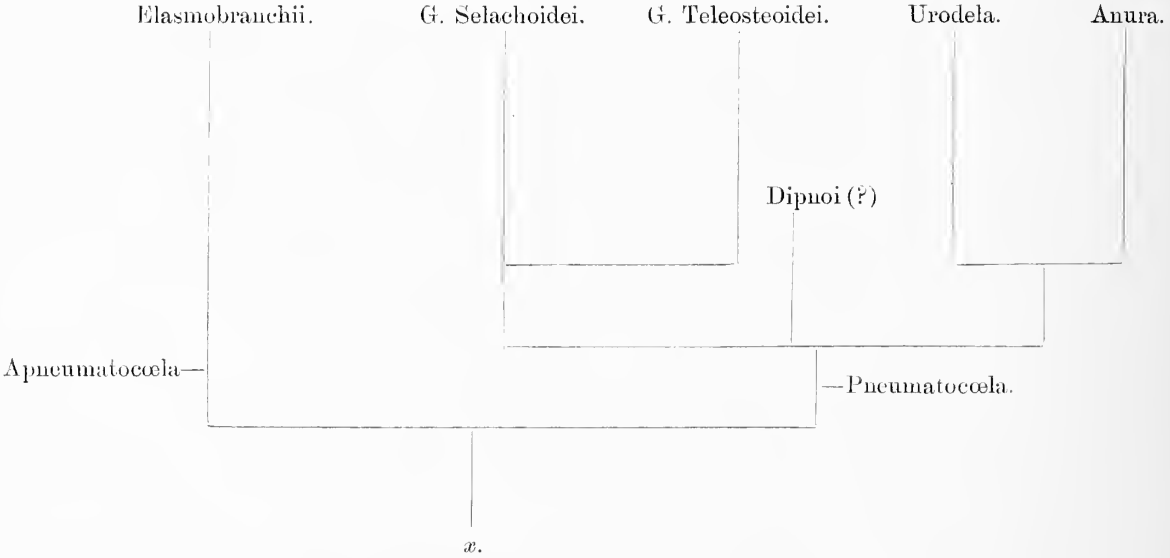
* Zeitschrift für Wissenschaftliche Zoologie. Bd. xxvii., 1876.

admitting their community of descent. We must therefore conclude that the important differences between the two types are probably due to their early separation from each other, and to the fact that the lowest Urodela have undergone much retrograde metamorphosis in their cranial structure, while adaptive changes have profoundly modified the adult and larval forms of both groups since they first came into existence. An alternative suggestion which may be made is to regard the Ganoid-Amphibian phylum as splitting into its two main constituent groups, the primeval Amphibia afterwards differentiating into Urodela and Anura, while the primitive root-stock of the Ganoids gave rise to the Teleosteoidei and to the Teleostei proper, the Selachoidei types persisting with relatively little modification as representatives of those early progenitors of their order.

We may venture more clearly to express the foregoing conclusions as to the relations of *Polyodon* and the other Ganoids to the Amphibia and Elasmobranchii by the aid of the following diagram :—



As to the nature of the primitive ancestor (*x.*) from whence the divergent Elasmobranch, Ganoid, and Amphibian stems have been derived, we may infer that it possessed an amphistylic skull, perhaps not differing essentially from that of the recent *Notidanus;* a persistent notochord supporting dorsally a series of neural and intercalated arches, which were related to the anterior and posterior roots of the spinal nerves, as in the embryo Selachian and in the adult Marsipobranch, and also in relation ventrally with a similarly double series of hæmal arches enclosing the dorsal aorta ; that respiration was carried on by external gills, which were developed in connexion with the mandibular as well as from the other visceral arches, and were probably destitute of an opercular covering ; persistent segmental openings may have been present, and also an archipterygium composed of three proximal segments (propterygium, mesopterygium, and metapterygium), each supporting a series of radial cartilages, but entirely devoid both of morphological and functional lungs. Such an ancestral type may be supposed to have very early differentiated into the two groups of Apneumatocœla and Pneumatocœla, the former becoming the root-stock of the modern Elasmobranchii, which have retained not a few of the characteristics of their prototype, while the latter, by acquiring rudimentary and more or less functional lungs, became the primitive double-breathers from which have been derived the Ganoids and Amphibia. Of these primitive Pneumatocœla it is possible that the recent *Lepidosiren, Protopterus*, and *Ceratodus* are the nearest, though much modified, living allies. The Amphibia developed more complex lungs and a chiropterygium, and became adapted to a terrestrial life, while what remained of the Ganoid-Amphibian stem after the separation of the Amphibia, differentiated into the Teleosteoid Ganoids, the Teleostei proper, and into the progenitors of the Selachoid Ganoids, all of which acquired an ichthyopterygium, and in the more specialised forms the originally complex and more or less respiratory primitive lungs became much simplified in structure and mainly mechanical in function. If the view which I have here ventured to suggest be correct, then we may regard these forms as being the relatively little modified descendants of the common stock from which the other Ganoids and the Teleostei have been differentiated, and as bearing a relation to the latter comparable to that in which the Tragulina stand to all the other recent ruminating Artiodactyla. That the Selachoid Ganoids have departed less from the ancestral type (*x.*) than any of the other groups, with the exception of the Selachii, is evinced by the structure of their vertebral column, fins, and chondrocranium, by their retention of mandibular gills, and by several other less important features. Two facts in the cranial structure of *Polyodon* are not easy to explain, viz., the formation of the upper jaw, and the existence of the "orbitar process." The union of the pterygoid processes in a median symphysis may have been the primitive condition of the jaw in the ancestral form (*x.*) and in the primitive Pneumatocœla, but that while persistent in Elasmobranchs and in *Polyodon*, it has been superseded by a different arrangement, viz., the union of the pterygoid processes with retral palatine outgrowths, in all other Ganoids and in all Teleostei and Amphibia; or we may conceive that, whatever may have been the primitive condition of these structures, the development of similarly constituted jaws in the Shark and in *Polyodon* is simply due to a parallelism in adaptive modification, though this latter alternative is far less probable. Neither is it easy to account for the retention of the "orbitar process" by *Polyodon*. It may have been an adaptive modification, correlated with a suctorial mouth in the larval forms of those Ganoids that were first differentiated from the Ganoid-Amphibian stem, and independently developed; or, and this is more probable, it may have been possessed by certain primitive suctorial-mouthed Pneumatocœla, but has become obsolete in all their descendants except *Polyodon* and the embryos of the Anurous Amphibia.*

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* The probability that the primitive common ancestors of the Ganoids and Amphibians possessed suctorial mouths seems to me to be very great, and if this view be correct, the curious differences which

Thus it would seem that the Polyodontidæ constitute a remarkably central group of Fishes. They resemble the Elasmobranchs in retaining many of the features which were characteristic of their presumptive common ancestor; and they further resemble the Amphibia and the higher Ganoids in having undergone a certain amount of specialisation in the same direction, and that they are not altogether without indications of having undergone a certain amount of retrogression is evident from the localised distribution of their rhomboidal scales.

HAECKEL'S[*] pedigree of the "non-amnionate craniota" is somewhat different from that which I have ventured to suggest. He appears to regard the Amphibia, the Dipnoi, and the Halisauria as constituting one line of descent from a primitive pre-Selachian stem, while the Ganoids and the Teleostei are derivatives of the group of Plagiostome Elasmobranchs, the Chondrosteous Ganoids forming a transition type between the latter and the higher Ganoids. If this view be correct, we are required to believe that morphological or functional lungs have been developed quite independently on precisely the same plan, and with every grade of transition between their extreme modifications in two diverging groups of Ichthyopsida, modified for entirely different habits of life. The *à priori* improbability of such a course being adopted seems to me to be very great. The complex double swim-bladder of the Crossopterygian Ganoids is connected by a gradual transition in almost every anatomical detail with the simple, undivided air-bladder of the Physoclisti on the one hand, and through the double, cellular and functional lungs of the Dipnoi, with the more perfect and exclusively air-breathing structures of the Amphibia on the other. The best explanation of such facts seems to me to be the assumption that the two extremes of lung structure have been derived by progressive modification in opposite directions from some intermediate type of structure which, as I have suggested, may have been possessed by the primitive Pneumatocœla. Such evidence in favour of the monophyletic origin of the Pneumatocœla drawn from the consideration of a single organ, is, as I have tried to show, supported and strengthened by the existence of numerous other points of agreement, in both osteology and myology, between their two main groups of descendants. Moreover, if we accept HAECKEL'S views on the ancestral relations of the Dipnoi, it becomes exceedingly difficult to explain the existence of numerous points of agreement with the Crossopterygidæ which is so striking a feature in the anatomy of those Fishes. On the other hand, if the Dipnoi are looked upon as the much modified descendants of the primitive Pneumatocœla, there is much less difficulty in explaining their affinities to the early Ganoids in the structure of the vertebral column and fins, and their Amphibian tendencies, which are so clearly shown in the structure of their lungs, heart, nostrils, &c.

exist in the cranial metamorphoses of the larval Urodela and Anura are to some extent explicable. The embryos of *Polyodon* and the Anura retain for a time this primitive suctorial mouth, but in the larval Urodela it has been replaced by a masticatory mouth, the suctorial stage being altogether suppressed.

[*] "Naturliche Schöpfungsgeschichte," 1873.

The remarkable correspondence in so many features between the Elasmobranchs and the Selachoid Ganoids is readily explicable on the view of their community of descent, and the retention of many ancestral structures in both groups; and any support which this agreement may lend to the view of the direct descent of the Ganoids from the Elasmobranchs is very much diminished in value by the existence of other facts, previously referred to, which strongly points to the monophyletic origin of the Amphibian and Ganoid stocks.

DESCRIPTION OF THE PLATES.

The figures are nearly all of life-size, and the lettering is uniform throughout. The cartilage is tinted blue, the cartilage bones are coloured yellow, while the membrane bones remain white.

a^1. Dermo-ethmoid.
a^2, a^3, and a^4. Rostral splints belonging to the medio-dorsal series.
b^1. Parietals.
b^2. Frontals.
b^3, b^4, and b^5. Rostral splints belonging to the supero-lateral series.
b.br^1. First basibranchial element.
b.tp. Basitemporal ala.
c^1. Post-temporal.
c^2. Dermo-sphenotic.
c^3. Nasals or dermo-prefrontals.
c.br^1., *c.br*^{4}. Cerato-branchials.
c.h. Ceratohyal.
D. Dentary.
ep.br. Epibranchial.
h.m. Hyomandibular.
h.hy. Hypohyal.
h.br. Hypobranchial.
i.hy. Interhyal.
i.op. Interoperculum.
mg.o. Dorsal or ventral marginal ossicles of rostrum.
m.pg. Mesopterygoid.
mt.pg. Metapterygoid ligament.
mx. Maxilla.
Mk.c. Meckel's cartilage.
op. Opisthotic.

or.p. Orbitar process.
op. Operculum.
pg.p. Pterygoid process.
p.br. Pharyngo-branchial.
pt.r. Pterotic ridge.
p.f. Parietal foramen.
pt. Pterotic.
pa.s. Parasphenoid.
p^2, p^3. Rostral homologues of the parasphenoid.
Q. Quadrate cartilage.
sp.r. Sphenotic ridge.
sy.l. Symplectic ligaments.
Vo. Vomer.
V^2 and V^3. Rostral homologues of the vomers.

Nerves.

II. Optic.
m.o. Motores oculi.
tr. Trochlearis.
V, V″. Foramina for exit of branches of trigeminus.
VII′, VII′. Anterior and posterior openings of facial canal.
IX. Glossopharyngeal.
Vg. Vagus.

PLATE 55.

Fig. 1. Dorsal aspect of the skull. Reduced one-half.
Fig. 2. Ventral view. Reduced one-half.
  hy.g. Articular groove for head of hyomandibular.
  x. Prolongation of first visceral cleft.
  h.c. Hæmal canal.
Fig. 3. Side view of the cranium. Natural size.
  a.n. Anterior nares.
  p.n. Posterior narial opening.
  o.f. Orbital floor.
  i.h. Inter-hæmal cartilages.
  i.n. Inter-neural cartilages.
  a.r. Anterior root of spinal nerve.
  p.r. Posterior root.
  z. Foramen in side wall of rostrum.

PLATE 56.

Fig. 4. View of ventral surface of cranium. Life size.

Fig. 5. View of upper surface of chondrocranium when the insisting parostoses have been removed. Life size.

a.m.f. Anterior median fontanelle.
f.g. *f*-shaped groove.
hy.g. Groove for head of hyomandibular.
p.f. Parietal foramen.
γ. Superior opening of first visceral cleft.
ϕ. Upper and posterior extremity of the oblique ridge.

Fig. 6. Vertical and longitudinal section. Reduced one-half.

B. Canal lying above spinal canal.
ar′. and *pr′.* Anterior and posterior root of first spinal nerve.
p.sc., *a.sc.*, *h.sc.* Foramina by which the anterior and posterior vertical and the horizontal semicircular canals leave the recess in which the membranous vestibular sac lies.
Ca. Carotid foramen.
Py. Pituitary fossa.
Fa. Foramen by which the facial nerve leaves the cranium.
h.c. Hæmal canal.
n.c. Notochord.

PLATE 57.

Fig. 7. Side view of the otic capsule, which has been dissected to show the semicircular canals and their relations to the parietal foramen, and pterotic and opisthotic bones. Natural size.

α. Oblique ridge, terminating above and behind in the decurrent flap of cartilage ϕ.

Fig. 8. Mandibular and hyoid arches, and opercular bones. Life size.

mt.pg.l. Metapterygoid ligament.

Fig. 9. Branchial arches.

r.r′. Rays attached to first and second epibranchials.

XXII. *Contributions to the Anatomy of the Central Nervous System in Vertebrate Animals.*

By ALFRED SANDERS, *M.R.C.S., F.L.S., late Lecturer on Comparative Anatomy at the London Hospital Medical College.*

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[PLATES 58–65.]

Part I.—ICHTHYOPSIDA. Section 1.—PISCES. Subsection 1.—TELEOSTEI.

Introduction.

INVESTIGATIONS into the histology of the nervous system in Vertebrata, have hitherto been chiefly confined to that of the Mammalia, or if applied to the lower members of that sub-kingdom, have not been undertaken so much with reference to the morphological relationship of animals, as to their bearings on physiology and the art of medicine. It may be stated in general terms that those who have worked at the morphology of the nervous system have not paid much attention to its histology; and, per contra, those who have investigated the histology have neglected its morphological bearing. STIEDA, however, has investigated the brain in both aspects of the question, and LOCKHART CLARKE has made some reference to the nervous system of the lower Vertebrata in his classical investigations, but this bears but a subordinate proportion to the whole. FRITSCH has recently published a treatise on the histology of the central nervous system in Fishes, in which the homology of the various parts is especially considered.

It occurred to me that the proper method of research in such complicated organs as the central nervous system, would be to proceed from the more simple to the more intricate, and I have selected the Teleostei as a good starting point in the class Pisces, as they form a sort of central group from which the other orders diverge either in a progressive or in a retrograde direction.

The species which I have more particularly investigated has been the Grey Mullet (*Mugil cephalus*), and unless the contrary is mentioned, the statements in the following pages apply to that species.

For a great part of the material used, I am indebted to the kindness of Dr. EISIG, of the zoological station at Naples, who provided me with living specimens, without which it would be impossible to obtain an accurate idea of the histological elements, since the nervous tissue deteriorates with great rapidity after death.

The method of procedure was to disable the animal by cutting through the spinal cord behind the pectoral fins, and as rapidly as possible after opening the cranium to dissect out the brain, and to place it at once in absolute alcohol, to which some drops of tincture of iodine had been added so as to give it the colour of pale sherry. This method, which was first suggested by STIEDA (58) (see bibliographical list at p. 769 of this paper), and was adopted by him in his investigations, appears to have the effect of fixing the nervous elements at the moment of molecular death.

The hardening fluid used was chromic acid; and the great point to be attended to was not to have it too strong, and to place each brain into a large quantity of liquid. The proportions used were one part of chromic acid to 800 parts of water, and the allowance for each moderate sized brain was one grain of chromic acid with its proper quantity of water; by this means it was possible to make the medulla oblongata and the tuberosities of the brain of the same degree of hardness, so that a section could be cut including both parts. A stronger solution would cause the tuberosities of the brain to become brittle before the medulla oblongata had attained sufficient firmness to be cut. By this process a brain would take from ten days to a fortnight, according to the size, to gain the proper consistence. If it happened that it was not convenient to make sections when the brains were ready, they were kept in a solution of ten grains of potash bichromate to one ounce of water, which was occasionally changed. A short stay in this solution was rather an improvement, as the bundles of nerve-fibres thus acquired a yellowish tinge, and were not so easily coloured, so that they appeared more distinct. But a longer delay in this liquid rendered the brains very brittle, unless they were soft when first put in, and made the neuroglia become too granular, thus obscuring the view of the nervous arrangements and rendering it necessary to make thinner sections.

The colouring matter employed was rosaniline, which gives the sections a beautiful blue tint, and which is decidedly better than carmine, being easier to use and showing the minute structure of the cells much more distinctly. Hœmatoxylin also has a good effect.

As confusion is often caused by applying the terms of direction used in human anatomy to the lower animals which habitually move in a different position, perhaps it would not be superfluous to remark that the terms used in this paper have reference to that position; so that, instead of "posterior," the words "dorsal," "upper," or their derivatives will be used; for "anterior," the words "ventral" or "lower;" "anterior" or "forward" will mean towards the head; "posterior" or "behind" towards the tail, instead of the meaning attached to them in anthropotomy.

Macroscopic Anatomy of the Brain.

The brain of the Teleostei when viewed from above is composed of three pairs of tuberosities placed in a series from before backwards, and one unpaired tuberosity. The homologies of these tuberosities are not yet settled. MIKLUCHO-MACLAY (62) has within the last few years proposed an arrangement differing considerably from that ordinarily received—an arrangement which was endorsed by the great authority of GEGENBAUR (67) in his 'Grundzüge der vergleichenden Anatomie,' to which I find that he still adheres in his recently published 'Grundriss.' Notwithstanding this, it appears to me that the interpretation adopted by STIEDA (58) is more likely to be correct, and that author, in a subsequent paper (64), has given a detailed argument against their opinion.

The first pair of the tuberosities in the brain of fishes may be looked upon as the homologues of the bulbi olfactorii (fig. 1, *ol.*). They are pear shaped, and are not (at least in *Mugil*) connected together by a transverse commissure, but are each united separately to the second pair of tuberosities, to the anterior end of which they are closely applied. In some species—as, for instance, in the members of the Cod family, and others—the lobi olfactorii are not arranged as above mentioned, but are placed immediately over the nasal sacs, and are connected to the cerebral lobes by a long commissure; in which case the latter has the appearance of occupying the most anterior part of the cranial cavity.

The second pair of tuberosities, which I will term the cerebral lobes (figs. 1 and 2, *ce.*), are more or less rounded; they vary in size in different species. In some fishes, such as the Basse (*Labrax Lupus*) and *Scorpæna Porcus*, these bodies present furrows and ridges on their surface which might be looked upon as rudimentary convolutions. They are united by a transverse commissure which is situated on their inferior edge, and nearer the posterior than the anterior end.

Many of the older writers, who having dissected only fishes of the Carp family, in which the first pair of tuberosities are situated far in advance of the remainder of the brain and outside the cranial cavity, looked upon this second pair as the lobi olfactorii. Of these, HALLER (4) considered that they subserved the function of smell together with the lobi inferiores or hypoaria; others—as CAMPER (7), EBEL (9), WEBER (17), KUHL (21), SOMMÉ (24), GOTTSCHE (34), MAYER (49), and KLAATSCH (44)—thought that they alone constituted the lobi olfactorii; while TREVIRANUS (13), who at first termed them "Riechforsatz," subsequently altered his interpretation, and in accordance with a theory which he developed at a later period termed them "Vordere Hemisphäre." PHILIPEAUX and VULPIAN (45) considered that they represented the caruncula mammillaris at the middle root of the olfactory nerve; but the latter (56) subsequently came to the conclusion that they were the hemispheres of the brain. On the other hand, SERRES (25), BÜCHNER (33), LEURET (35), and BAUDELOT (61) homologised them with the hemispheres of the brain without reserve. GIRGENSOHN (42) was not decided, but thought that they might represent the first rudiments of those bodies.

TIEDEMANN (15) was the first who compared them to the corpora striata with the hemispheres, and their commissure to the anterior commissure of the brain—an opinion which was justified when V. BAER (32) discovered that in the course of their development these tuberosities were at first hollow and contained a small tubercle at the bottom of their ventricle, which tubercle occupied the position of the corpora striata.

Of the remaining writers, HOLLARD (55), evidently unacquainted with the researches of V. BAER, confined the interpretation of these lobes to the region of the insula. To conclude the consideration of the cerebral lobes, MIKLUCHO-MACLAY (62) and FRITSCH (66) follow V. BAER; the former termed the tuberosities in question the "vorderhirn," and the latter "hemisphœria" or "stirnhirn."

The third pair of tuberosities are the largest bodies in the brain of fishes. They are hollow, and consist of the following parts: first, a thin vault of nervous substance, the "tecta loborum opticorum" of STIEDA, which may be termed for shortness "tectum." These are united together in the mid-line by a transverse commissure, and are closed in posteriorly by a tubercle closely attached to the unpaired tuberosity behind. This is the "tuberculum cordiforme" of HALLER, and is more or less developed in different families of the Teleostei.

Along the inner contiguous edges of the tecta a pair of longitudinal ridges of a somewhat triangular shape are attached, through the bases of which the before-mentioned commissure runs: these form the fornix of GOTTSCHE (34), and the commissure was termed by the same author the "corpus callosum." On the floor of the ventricle formed by these vault-like structures two hemispherical swellings are placed, which being in some fishes of a somewhat curved form, were named by HALLER "tori semicirculares."

The complicated structure of these tuberosities induced many writers on this subject to look upon them as the homologues of the hemispheres of the brain; although one would have supposed that the position of the pineal gland and the origin from them of the optic nerve would have formed an insuperable objection to this interpretation.

CAMPER (7) was the first who made any serious attempt to decipher the homologies of the brain in fishes.

These third pair of tuberosities (which for the sake of convenience will be called here the optic lobes) were for him the cerebral hemispheres; the commissure of the tecta was the corpus callosum, and the tuberculum cordiforme homologised the corpora quadrigemina: an opinion which was also held by HALLER; and the ventricle of the optic lobes was the third ventricle.

KUHL (21), SOMMÉ (24), CUVIER (28), and LEURET (35) followed this interpretation.

EBEL (9) thought that these lobes, combined with the unpaired ganglion behind them, were equivalent to the cerebrum.

WEBER, in his first paper (17), considered that they were the hemispheres, but subsequently (22 and 27) he termed them "sehnervenhügel," by which he probably meant no more than that they were optic lobes, without absolutely homologising them with the thalami optici.

With GOTTSCHE (34) the tuberculum cordiforme corresponded to the corpora quadrigemina, and the commissure of the tectum was the corpus callosum, as with CAMPER.

The projecting ridges on the inner contiguous edges of the tecta, he termed the fornix.

CARUS (12) was the first to discover these bodies and also applied this name to them, but more on account of their shape, and without any reference to the fornix of anthropotomy.

GOTTSCHE considered the communication between the two sides of the ventricle of the optic lobes, beneath the two ridges above-mentioned, to be the foramen of MONRO, and the tuberosity on the floor of the ventricle to be the thalamus opticus.

TREVIRANUS (13), following CAMPER, homologised these lobes with the cerebral hemispheres, the tuberculum cordiforme with the corpora quadrigemina, and the tori semicirculares with the corpora striata combined with the thalami optici. Subsequently (29) slightly changing his interpretation, he compared them to the posterior part only of the hemispheres combined with the corpora quadrigemina. Many anatomists, with better reason, considered these lobes as the equivalent of the corpora bigemina in birds; the first of these was CARUS (12), who was followed by TIEDEMANN (15), who came to this conclusion from embryological considerations.

SERRES adopted this opinion on account of the position of the pineal gland and of the origin of the nervus patheticus, which, as he remarked, invariably arises between the corpora quadrigemina and the cerebellum.

This author refuted the objection that these bodies in the higher Vertebrata are solid, whereas in fishes they are hollow; by the assertion, that in the embryo of those higher Vertebrata they have a cavity in them as in the Teleostei.

GIRGENSOHN (42), BAUDELOT (61), and VULPIAN (56) also follow this interpretation; the latter having changed from his former opinion (45), in which paper he said that they were "le cerveau proprement dit."

According to V. BAER these lobes are the "zwischenhirn" (thalamencephalon), and the tuberculum cordiforme the "mittelhirn" (mesencephalon) which in some unexplained way had become thrust under the former in the process of development.

JOHANNES MÜLLER also followed this opinion; he considered that these optic lobes were equivalent to the corpora quadrigemina conjoined to the "lobus ventriculi tertii" of the fœtus. At the point where the tecta arch over the tuberculum cordiforme, just described, there is a fissure which communicates with the ventricle of the optic lobe; this KLAATSCH (44) considered to be the fissure of Bichat: an opinion which is obviously untenable.

The unpaired ganglion situated behind the optic lobes has almost unanimously been looked upon as the cerebellum (fig. 1, *cbl.*, and figs. 6, 7, and 8). This tuberosity overhangs the medulla oblongata and closes in the fourth ventricle; it varies in shape in different species, being tongue-shaped in Whiting (*Merlangus vulgaris*), almost square in *Mugil cephalus*, presents two distinct tuberosities in *Labrax Lupus*, and in *Scorpæna Porcus* is reduced to comparatively small dimensions; its anterior border is prolonged into a body (fig. 1 *v.c.*), which projects into the ventricle of the optic lobe, and was described by HALLER under the name "tuberculum cordiforme." This is essentially a prolongation of the unpaired tuberosity, which passing into the ventricle over the aquæductus sylvii, nearly as far forward as the anterior end of the same, is there turned back, and reaching the posterior and upper end of the tectum joins the so-called fornix.

In the Carp, which was the species investigated by HALLER, the lateral portions or alæ (figs. 4 and 5, *a.v.c.*) of this body are highly developed, and form several folds, the sum total of which were termed by that anatomist "cornu ammonis," probably from their form, and not as compared to the part of the same name in the human brain. This cornu ammonis is found considerably developed also in *Crenilabrus*, where it fills the whole cavity of the ventricle of the optic lobe.

It is probably the enormous development of this part which forms the extraordinary structure in the brain of *Mormyrus Cyprinoides* (Nilhecht), described by ECKER (47) and by MARCUSEN (51), and later by OEFFINGER (57). MARCUSEN thought that it was most probably the cerebellum, and MIKLUCHO-MACLAY supports that idea. As I have never had the good fortune to meet with specimens of this fish, and as the authors just mentioned do not touch upon the microscopic anatomy (except MARCUSEN, who does not do so very minutely), I am unable to give a decided opinion; but I should think that it was not the cerebellum precisely, with which this remarkable structure homologised, but that part of it which goes under the name of the valvula cerebelli. Of course this opinion is a mere guess, but the inspection of a few thin sections of a fresh subject would soon decide the point.

WEBER's was one of the few exceptions to the generally received opinion that this lobe is the cerebellum; in his first paper (17) he termed it "corpus quadrigeminum seu Impar Majus," but subsequently he changed his views, and denominated it "vorderer unpaar Hügel des Kleingehirns;" while that part of the medulla oblongata which covers over the narrow part of the fourth ventricle behind it, and which in some species is developed into a distinct tuberosity, and which HALLER named "pons mammillaris," WEBER termed "Hinterer unpaar Hügel des Kleingehirns." MAYER (49) followed his opinion; his name for this part was "epiencephalon seu cerebelli lobus anterior," but he differed from WEBER in placing his "cerebelli lobus posterior" in the pair of tuberosities on the dorsal side of the medulla oblongata, from which the nervi vagi take their origin. These tuberosities are situated on the dorsal side of the medulla oblongata behind the cerebellum. They enclose between them a deep fissure, which corresponds to the posterior part of the fourth ventricle. Anteriorly in some fishes they are united together in the midline by a single tuberosity, which is situated over the narrow part of the fourth ventricle, immediately behind the crura cerebelli. Usually this tuberosity is replaced by a thin layer of granular substance occupying the same position.

MIKLUCHO-MACLAY (62) has within the last few years attempted to place the homology of the brain in fishes on a new basis firmly centred, as he supposed, on embryology. His opinion has been endorsed by no less an authority than that of GEGENBAUR (67), which even in the 'Grundriss' just published he has not seen fit to alter. The author also informs his readers that V. BAER gave to his theory a verbal assent.

This being the case, his opinion, perhaps, merits a more detailed consideration. With him, therefore, the cerebral lobes are the "vorderhirn," the optic lobes are the "zwischenhirn" (thalamencephalon), the cerebellum the "mittelhirn" (mesencephalon), and the pons mammillaris the "hinterhirn," and the posterior part of the medulla oblongata the "nachhirn." It is thus seen that the cerebellum in many fishes is reduced to a mere bridge of granular material, covering the narrow part of the fourth ventricle.

Against this scheme STIEDA (64) has placed on record several very important considerations: 1st, that the optic nerve, which in Amphibia and Birds arises from the mesencephalon (corpora quadrigemina), would, according to this mode of interpretation, arise from the thalamencephalon; 2nd, then the trochleares would arise wrongly between the thalamencephalon and the mesencephalon; 3rd, the thalamencephalon of fishes would not correspond to that of the amphibia, where it is a small flattened segment open above; but in fishes, if MIKLUCHO-MACLAY be right, it becomes developed into a large closed segment, which is often paired; 4th, the pineal gland would be wrongly placed.

With regard to the third objection, MIKLUCHO-MACLAY appears to have overlooked the real thalamencephalon, which is the territory surrounding that narrow fissure, the greater part of which is in front of the optic lobes, so that in point of fact he searched for the "zwischenhirn" too far back.

The microscopic structure of the "mittelhirn" of MIKLUCHO-MACLAY, as was also pointed out by STIEDA, and as will be seen in the following pages, so entirely corresponds to that of the cerebellum in higher Vertebrates, that any comparison of it with any other part of the brain is wholly precluded.

The brain has hitherto been considered from above. On examining its lower surface the following structures are to be observed from before backwards, viz., in front, the entrance of the optic nerve, behind which is situated the pituitary body (fig. 1, *pi.*, figs. 3 and 4) or hypophysis cerebri, placed on a slight eminence, the trigonum fissum of GOTTSCHE; on each side a large tuberosity (fig. 1, *hy.*, figs. 3, 4, 5, and 6), the lobus inferior, or, as I think preferable, the "hypoarium," used by OWEN in his 'Anatomy of the Vertebrata:' a term having the same meaning but being less clumsy.

The posterior parts of the groove (fig. 5, *s.v.*) indicating the separation between the two hypoaria is occupied by a vascular sac, the "saccus vasculosus" or "hæmatosac." This sac in *Mugil cephalus* is not visible from the surface, being contained in a sort of chamber formed between the contiguous walls of the two hypoaria.

HALLER appears to have looked upon the hypoaria as having relation to the sense of smell, for he termed them "olfactoria inferiora" or "tubercula reniformia."

Most of the authors in the list, homologised these bodies with the corpora albicantia. CARUS (12) was the first to compare them to the tuber cinereum, with which part they agree in position. SERRES and LEURET also adopted this view, but KLAATSCH confined this interpretation to the slight eminence (the trigonum fissum) to which the hypophysis is attached. CUVIER termed them "lobes optiques;" MAYER compared them to the thalamus opticus and corpus striatum combined; while HOLLARD (55) homologised them to the corpora striata, because, as he pointed out, the fibres which correspond, as he supposed, to the corona radiata of the cerebrum pass into them to end there; but this fact was denied by VULPIAN (56), who stated that these fibres pass through them without terminating, and go on into the medulla oblongata. This, I think, is most probably an error.

GIRGENSOHN (42) emitted the curious opinion that these bodies were together a highly developed hypophysis: an opinion which is evidently based on the supposition that the latter is composed of nervous tissue. In many fishes the hypophysis is easily broken off, and does not remain attached to the brain when the latter is removed from the skull; in that case, a small fissure is seen on the trigonum from which it is torn, which is the lower opening of the infundibulum.

There is no apparent line of separation between the medulla oblongata and the spinal cord, the latter gradually passing into the former; but the posterior end of the vagal tuberosities may be taken as a landmark for want of a better.

The anterior part of the spinal cord (fig. 14) has somewhat of a four-sided figure with the angles cut off; the dorsal and the ventral fissures are represented only by shallow depressions, the dorsal one becoming deeper on approaching the posterior end of the medulla oblongata. At this part the central canal has an oval section, the long axis of which is placed at right angles to the long axis of the cord. This canal (*c.ca.*, fig. 14) becomes smaller at a point nearer the fourth ventricle and also circular in outline; it then enlarges in a funnel-shaped manner, and becoming notched on its upper margin joins the gradually deepening dorsal fissure and enters the posterior section of the fourth ventricle. Occasionally a rod is seen in sections through this central canal of the cord, which most probably is the coagulated liquid contained therein, as STIEDA suggested; it is only in a few sections that this rod is seen, for it usually falls out, not being retained in its place by any attachments.

The fourth ventricle (fig. 10, *s.r.*) consists of two separate portions; the posterior part is a deep trench, situated between the two lobes of the vagus, the walls of which are nearly perpendicular and the floor rounded. Superiorly, this trench is covered by a layer of pia mater which passes over from one tuberosity to the other. Anteriorly, it passes by means of a quadrangular passage into the main part of the ventricle (fig. 9, *s.r.*), which is a wide space beneath the cerebellum and between the crura thereof (fig. 10). The passage between the two divisions of this ventricle is formed by a bridge of nervous tissue, which in some species* becomes enlarged into a tuberosity. Before

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* Carp family.

entering the wider chamber the narrow part of the fourth ventricle becomes triangular in shape, with the base of the triangle turned upwards. The segment of this ventricle, which forms the chamber beneath the cerebellum, has the same relations that the fourth ventricle has in the human subject. It is shaped like a cocked hat when seen from above; the top of the hat being directed backwards, and the sides running out into pointed extremities; below, there is a longitudinal furrow on the floor; above, its roof at first flat afterwards becomes furrowed in the centre. There is a large triangular opening in its roof which places this ventricle in communication with the outside of the brain beneath the cerebellum; anteriorly, a lozenge-shaped passage (fig. 5, *aq.Sy.*), the aquæductus Sylvii, forms a communication between this ventricle and the ventricle of the optic lobes. Before opening into the latter the aqueduct spreads out into a wide depressed fissure. In the roof a longitudinal furrow extends from the anterior part of the fourth ventricle, and in the floor a similar furrow extends from the middle of the narrow part of the ventricle as far forward as the entrance of the aqueduct into the ventricle of the optic lobe.

With regard to this last space, GOTTSCHE (34) considered that it corresponded to the third, combined with the lateral ventricles, and FRITSCH (66) even is doubtful whether it does or does not homologise with the lateral ventricles; but I think that after all it is merely an expansion of the aquæductus Sylvii. That it does not correspond to the third ventricle appears nearly certain, since the fissure which properly corresponds to that ventricle is in front, and communicates with the space in question by a small foramen, which opens behind the posterior commissure. The third ventricle (fig. 2, *v.th.*) is situated between the two bundles of fibres which pass from the cerebral lobes into the hypoaria, and also between the two optic tracts. In front, this fissure communicates through a narrower passage with a small oblong chamber, placed close behind the anterior limits of the brain, between the bases of the two cerebral lobes, which seems to correspond to the lamina terminalis of the fœtus. The above-mentioned chamber and the narrower passage are lined by a stratum of nerve-cells; inferiorly, the third ventricle contracts into a funnel-shaped passage, the infundibulum, which leads into a space surrounded by the hypophysis, and which receives on each side the openings of the passage from the ventricles of the hypoaria.

Apparent Origin of the Cranial Nerves.

In the Teleostei, all the cranial nerves, with the exception of the hypoglossal and spinal accessory, may be distinguished, if not as separate roots, at least as divisions of some of the others. Thus, the nerve which is usually considered to be the facial is a branch of the trifacial; and the supposed glossopharyngeal is sometimes a branch of the acusticus. The four anterior nerves do not require much discussion; they arise in the same manner in all the Teleostei that I have examined.

The olfactory (fig. 1, *n.* 1), perhaps, in those fishes in which the lobe is applied close to the anterior end of the cerebral lobes may be considered as a true nerve.

The optic nerve (fig. 1, *n.* 2) is folded on itself transversely several times, so as to give its section a pectinated appearance,—a fact which was first mentioned by MALPIGHII (2).

The motores oculorum (fig. 5, *n.* 3) invariably arise from the point of junction of the hypoaria with the ventral edge of the medulla, where a band of transverse fibres exist, which were described by GOTTSCHE under the name of "commissura ansulata;" these nerves pass outwards, and appear in the groove between those bodies and the lower edge of the optic lobes at about the centre of their length.

The trochleares (fig. 6, *n.* 4) arise from the valvula-cerebelli, whence they pass out behind the optic lobe, and curving round the posterior edge of that body pass forward towards their destination

DESMOULINS (26) has given the origin of this nerve from the ventral surface of the medulla oblongata, where, he says, the filaments of the root of one nerve were contiguous to those of the nerve on the opposite side. What could have induced this author to make such a mistake it is impossible to imagine. No nerve, except the motores oculorum, and that not strictly, answers to his description; and that nerve is precluded by the context.

The remainder of the cranial nerves vary in origin in different species of Teleostei.

The trifacial (fig. 2, *n.* 5; fig. 7, *n.* 5; fig. 8, *l.c.* 5; fig. 9, *l.c.* 5 and *u.c.* 5) in *Mugil cephalus* has the following arrangement: it arises by three closely contiguous roots from the side of the medulla oblongata, beneath the anterior edge of the crura cerebelli; the casserian ganglion is developed on the middle root only; the anterior passes forward over the ganglion, and joins the middle root in the foramen of exit, which in this species is through bone; the posterior root passes forward to a separate foramen, through which it emerges as an independent nerve, having first received a branch from the under surface of the ganglion; it then pursues its course obliquely through the palato-quadrate arch, and its fibres are lost on the outer surface of the quadrate bone. It is this root which is considered by anatomists to be the facial nerve. In the *Pleuronectes*, according to STANNIUS, there are four or five roots to the trifacial, and more than one of them contribute to the formation of the facial.

The abducens (fig. 8, *n.* 6; fig. 9, *n.* 6) has a very constant origin; each root arises by two small cords from the ventral surface of the medulla, beneath the anterior root of the vagus.

The acusticus arises from the medulla oblongata, on a lower level than the trifacial; the anterior end of its root is placed exactly ventrad of the posterior root of the trifacial, while its posterior edge is in front of the anterior root of the vagus. This nerve immediately divides into two branches: one, the anterior, goes to the anterior and horizontal semicircular canals, while the posterior goes backwards to be distributed to the sac of the otolithes; in its course the latter gives off a small branch, which supplies the ampulla of the posterior semicircular canal.

There is a small nerve which is often merely a branch from the posterior division of the acusticus, but which in *Mugil* has a distinct origin from the medulla oblongata, immediately below the anterior root of the vagus. This nerve crosses the last-described branch of the acusticus at right angles, and passes through a distinct foramen in the outer wall of the ear sac, where it forms a moderately sized ganglion. From this ganglion two branches are given off: one supplies the anterior branchial arch; the other, passing forward in close juxtaposition to the outside of the skull, joins the posterior root of the trifacial above mentioned. This nerve is usually described as the glossopharyngeal (STANNIUS, 38, 43, BÜCHNER, 33), but WEBER (22) terms it the nervus auditorius accessorius in the Carp.

The vagus (figs. 2, 9, 10, and 11, *n.* 10) arises above the level of the acusticus and the last-mentioned nerve by two roots, the anterior of which emerging from the medulla oblongata a short distance behind the posterior edge of the crus cerebelli, passes back, and having received a communicating branch from the posterior root, emerges from the skull through a distinct foramen in the exoccipital bone. The posterior root rises opposite the posterior end of the vagal tuberosities, and after giving off the above-mentioned branch to the anterior root, passes out of the skull through a separate foramen beneath that root. Thus the two origins of the vagus do not entirely unite until they arrive at the outside of the skull in *Mugil cephalus.*

The nervus recurrens connecting the trifacial with the vagus and the so-called hypoglossal, inside the skull beneath the brain, which is described by WEBER and BÜCHNER in the Carp, is not present in this species.

In the Whiting Pollack (*Merlangus Pollachius*) all the roots of the trifacial pass through the casserian ganglion, which is placed in the course of the nerve partly within and partly without the cranial cavity.

The acusticus, as in *Mugil cephalus,* divides into two main branches, the anterior of which goes to the ampullæ of the anterior and horizontal semicircular canals, and the posterior divides into two branches; of these, the posterior joins the inferior surface of the vagus immediately beyond or at the point of junction of the two roots of that nerve; the anterior, after supplying the ampulla of the posterior semicircular canal, passes out of the cranial cavity through a foramen in the external wall of the ear cavity, where, as in *Mugil,* it developes a ganglion which is placed closed to the bone at the exit of the nerve. From this ganglion two large branches are given off: one goes back to supply the first branchial arch, the other goes forward, closely applied to the external surface of the skull, and joins the base of the casserian ganglion at a point between the branching off of the ophthalmic and the two maxillary nerves. In its course this branch supplies twigs to the anterior ends of the kidneys, and to the muscles on the inner side of the branchial arches.

This division of the acusticus is evidently the same nerve as that described in the *Mugil,* and corresponds to the glossopharyngeal of authors. It has no relation to the nervus recurrens of the Carp, which is, in point of fact, the sympathetic nerve, and remains inside the skull while the nerve in question is on the outside.

STANNIUS (38) gives an account of this nerve in the *Gadus callarias,* in which species it also developes a ganglion on the outside of the skull; but that author does not mention any connexion between it and either the acusticus (for as in the *Mugil* it has a separate origin from the brain) or the trifacial.

I would suggest that the branch of the acusticus just described homologises with the facial and not with the glossopharyngeal. From this point of view, the ganglion in the course of this nerve would be the ganglion geniculare or intumescentia, which on account of the non-development of the tympanum in Teleostei, comes to be placed outside the skull. The branch to the first branchial arch would be the twig of communication between the facial and the glossopharyngeal, and the anterior branch joining the casserian ganglion would be the auriculo temporal, or, perhaps, the main trunk. The true glossopharyngeal would be found in the branch of the vagus to be presently described.

In the evolution of the higher Vertebrate animals, when the glossopharyngeal and the facial become independent nerves, I should imagine, on the supposition that the above considerations were well founded, that the posterior branch of the ganglion in question would degenerate into the communicating twig between the facial and the glossopharyngeal; while the anterior branch would increase in size, *pari passu* with the increasing development of the muscles of the face, and at the same time become detached from immediate connexion with the trifacial.

The vagus arises here also by two roots as in *Mugil cephalus,* but they join before arriving at the foramen of exit, instead of outside the skull as in that species. The ganglion of the vagus is formed partly in the foramen, but the greater part of it is situated outside. From the upper edge of the ganglion, two nerves are given off: one to the lateral line, the other to the skin over the dorsal part of the branchial chamber at the point where it makes a fold to line the operculum. Two branches are given off from the distal and outer side of the vagal ganglion, which supply the second and third branchial arches; then the greater part of the ganglion tapers off at its distal extremity into the main trunk of the vagus, which after giving a branch to the fourth branchial arch, follows the œsophagus.

It seems quite as reasonable to consider as the glossopharyngeal all the above-mentioned nerves to the branchial arches, which appear to a great extent to be distinct from the main trunk of the vagus; as to confine the interpretation to the branch which supplies the first branchial arch alone, which is usually the custom.

The nerve which is situated behind the vagus is generally looked upon as a spinal nerve, although in many Teleostei, in the present species among the number, it passes out of the skull through a special foramen in the exoccipital bone; here it is clearly a double nerve. It arises by two dorsal and two ventral roots; these all join into one trunk, which has a ganglionic enlargement as it emerges through the foramen. Immediately outside it gives off a dorsal branch like a spinal nerve; after which it divides into two trunks, which run a parallel course dorsad of the kidney, and then unite together and with the main trunk of the succeeding nerve, which is the first nerve that emerges through the vertebral column. It may be looked upon as the second spinal nerve, or if the first is considered to be two nerves this would be the third. The trunk formed by the union of these nerves constitutes a sort of brachial plexus, and supplies the muscles of the pectoral fin, after having given off a branch which runs down the anterior edge of the shoulder-girdle towards the muscles surrounding the glossohyal bone.

This nerve is certainly a spinal nerve, as far as its origin goes, for it has both dorsal and ventral roots, and in this species it appears to be a combination of two nerves, as is shown by its four roots, and by its trunk dividing into two and joining again. In some other species--the *Scorpœna Porcus*, for instance—this nerve gives a branch which is distributed quite as far as the anterior part of the glossohyal.

In examining a dissection of the nerves from the upper surface one is struck with the resemblance between the trifacial, the vagus, and the spinal nerves, in respect to their dorsal branches. Thus we find that in proceeding from behind forward these branches are given off at a gradually decreasing distance from their ganglia; in the spinal nerves their dorsal branch is given off distad of the ganglion; in the first spinal nerve and the vagus it comes off from the ganglion itself, and in the trifacial it divides from the main trunk proximad of the ganglion.

In the Whiting (*Merlangus vulgaris*) the arrangement of the three nerves in relation to each other differs more from that of the *M. Pollachius* than would be supposed possible in species of the same genus. In this species, the so-called facial of Stannius arises in common with the remainder of the trifacial; being the penultimate root of that nerve, it passes over the casserian ganglion as a distinct cord without in any way communicating with it. It then emerges through the foramen with the remaining branches of the trifacial; after that, it receives the branch from the ganglion of the so-called glossopharyngeal, which is well developed in this species; it then divides into two branches: one pursues its course along the posterior and outer edge of the orbit; the other, after perforating the palato-quadrate arch, passes down on the outside of that structure to disappear from view outside the quadrate bone.

The acusticus does not send a communicating branch to the vagus, but its posterior cord is closely applied to the anterior root of the latter, until it arrives at the ampulla of the posterior semicircular canal, when it leaves its companion to be distributed on that structure. The two trunks are very easily separated from each other, and there is no nervous connexion between them.

In this species both roots of the vagus pass through the ear cavity over the otolithic sac: a disposition which is caused by the extreme backward prolongation of the latter. The so-called glossopharyngeal, or branch to the first branchial arch, is given off by the posterior root of the vagus immediately before its junction with the anterior root of that nerve; it passes forward along the inner surface of the outer wall of the ear cavity, and emerges through the foramen in that wall, outside of which it forms the ganglion, as in *M. Pollachius*. From this ganglion the nerve for the first branchial arch is given off from its posterior end, while the branch which joins the facial root of the trifacial comes off from its anterior side.

Thus we have here a corresponding nerve given off from two distinct sources in allied species, so that it seems not beyond the bounds of possibility, with such varying elements (evolution being granted), that the branchial divisions of the vagus should be detached from that trunk, and absorbing the branch of the acusticus to the first branchial arch should become developed in the higher Vertebrata as the glossopharyngeal; and, on the other hand, that the facial division of the trifacial should become detached from that origin, and attach itself entirely to the anterior cord of the same branch of the acusticus; the two halves of the latter being separated. Then the communicating twig between the glossopharyngeal and the facial remains as evidence of their former union.

Microscopic Anatomy of the Brain.

Lobi Olfactorii.

The lobi olfactorii (fig. 1, *ol.*) consist essentially of three layers. Of these the external is thicker in front, and is formed by the fibres of the olfactory nerve (fig. 1, *n.* 1), which on entering diverge in all directions and form a sort of envelope for the anterior part of the lobe. More internal comes a layer of finely granular neuroglia, which surrounds on the upper, anterior, and lower sides a mass of small cells, which occupy the central and posterior part of the lobe, being separated from the hinder edge by a narrow stratum of fibres, which descend to the postero inferior angle, and pass out to form the commissure between this lobe and the cerebrum (fig. 15, *ab.*).

The cells of the central group are small in size, resembling to a certain extent those of the cerebrum. Many are oval or circular in outline, but generally they are more or less pear-shaped. Each cell has a nucleus of comparatively large size, which is invariably situated at the broader end of the cell; the protoplasm or cell-contents occupy the narrower side, which terminates in a more or less blunt point, from which a single free fibril emerges. The nucleus contains a single spot-like nucleolus, situated near the centre, which occasionally shows symptoms of breaking up into its constituent granules. The total average length of these cells is 0·007 millim. or 0·008 millim., their diameter 0·004 millim. or 0·005 millim.; the nuclei are generally round, or nearly so; and their diameter averages 0·004 millim.

Many of these cells occupy spaces in the neuroglia which probably correspond to the spaces surrounding the cells of the cerebrum described by OBERSTEINER (78) and BEVAN LEWISS (84). Occasionally, nearly the whole of the cell projects into this chamber, but more generally only the broad end, so that the nucleus alone would be bathed in lymph in the latter case. The granules which to a great extent compose the neuroglia of the olfactory lobes become aggregated together, and form a smooth surface on the walls of these spaces; they do not actually form an epithelial layer, but seem to be a rudimentary form of that structure.

A layer of neuroglia surrounds this group of cells, as before mentioned, on all sides except posteriorly, in which with high powers only very fine granules are to be observed. The above described cells occur very sparingly here.

The external portion of the lobule (fig. 1, *o.l.*) is formed principally by the fibres of the olfactory nerve. These fibres enter at the anterior end, and occupy about half the length of the lobe; they do not go straight, but the bundle dividing at the apex forms an interlacing layer which encloses the fore part of the lobe as in a sheath, and envelopes small rounded masses of coarse granular neuroglia, which may be looked upon as representing the glomeruli in the bulbus olfactorius of Mammalia, described by MEYNERT (81).

Larger cells are seen to occupy the inner edge of this layer of nerve-fibres, where it begins to pass over into the stratum of finely granular neuroglia above described; at this part the neuroglia is coarser, and the cells in question occupy spaces therein in the same way that the small cells do in the central group. These cells (fig. 15, *e.f.*) are mostly tripolar, with sometimes one, and sometimes two broad protoplasmic processes, the others being fine and probably axis-cylinder prolongations. They measure 0·013 millim. long by 0·010 millim. broad, the nucleus measuring 0·007 millim. by 0·006 millim.; some have a distinct spot-like nucleolus, which, however, in many specimens cannot be so easily distinguished. Besides these, other unipolar cells (fig. 15, *c.*) occur in which the protoplasm greatly preponderates, and where the nucleus is not much larger than that of the small cells of the central group. The specimen figured has a spot-like nucleolus placed nearly in the centre of the nucleus. In addition to the larger cells, which as before mentioned occupy the border of the layer of fibres, some of these fibres themselves show cell-like swellings (fig. 15, *d.*) in their course, which somewhat resemble the cells described by MEYNERT (81) in the glomeruli olfactorii of the human subject. These cell-like swellings in the course of the fibres are like some kinds of bipolar cells: they have large oval nuclei and conspicuous nucleoli. The length of the one figured is 0·020 millim. and the width 0·006 millim., the nucleus measuring 0·006 millim. by 0·004 millim.

In the Whiting Pollack (*Merlangus Pollachius*) the olfactory lobes are situated beyond the cranial cavity, close over the nasal sacs, and are connected to the cerebral lobes by two long commissures. Here the structure very much resembles that of those lobes in the Grey Mullet (*Mugil cephalus*), above described. In these, also, the group of small cells is found to be surrounded by the layer of finely granular neuroglia; and outside of all comes the fibres of the olfactory nerve.

That these lobes homologise the bulbi olfactorii of Mammalia will be seen by comparing a section through the olfactory lobe of the Teleostean, with a similar section through the bulbus olfactorius of the monkey, as described by MEYNERT (81). According to that author a section from outside inwards shows first, the olfactory nerve layer; next, a stratum glomerulosum containing glomeruli olfactorii, then a stratum gelatinosum in which the ganglion cells are more closely aggregated internally, finally a layer of medullary fibres. A section through the olfactory lobe of a Teleostean observed from before backwards, is comparable to the above-described section made from without inwards; first come the layer of fibres from the olfactory nerve comparable to the external layer, then the layer of nodulated masses of neuroglia and coarse granules comparable to the stratum glomerulosum, then the finely granular neuroglia with sparsely scattered cells, comparable to the external part of the stratum glomerulosum, then the central group of small cells comparable to the internal part of the same stratum, and finally the thin layer of fibres which form the commissure to the cerebral lobes, which are comparable to the internal layer of medullary fibres above mentioned.

Cerebrum or Cerebral Lobes.

The lobi cerebri are invested under the pia mater by a single layer of epithelial cells of the columnar variety; each of these cells gives off from its inner extremity a long fine process, which passing into the substance of these lobes is lost in the neuroglia. A finely granular neuroglia occupies the greater part of the cerebral lobes, in which cells of various sizes are scattered in varying proportions; these cells range in size from 0 014 millim. long by 0·010 millim. broad, to 0·007 millim. by 0·005 millim.; the smallest cells (fig. 16, *a.*) are generally pyriform in shape, each with a comparatively large nucleus, to one side of which a conical, more or less elongated mass of protoplasm is attached, from the pointed end of which a nerve-fibre emerges.

Occasionally, cells (fig. 16, *b.*) occur which have two diverging processes, the nucleus being attached to the middle and projecting two-thirds of its circumference from the protoplasm. These smaller cells seldom show a distinct nucleolus, but more often only scattered granules, as if the nucleolus had been broken up. This kind of cell (fig. 17) occurs more frequently near the outer edge of the cerebral lobes, and in some places forms a layer of cells beneath the epithelium three or four deep, with scarcely any neuroglia between them. They do not form a connected layer throughout the border of the cerebral lobes, but only occur in places; at other parts the neuroglia extends quite to the inner surface of the epithelial cells.

The neuroglia increases in quantity towards the centre of the cerebral lobes, while the cells decrease in number, but at the same time become larger in size; these are either bipolar or tripolar cells (fig. 16, *c.*), in which distinct nuclei are visible, which are always placed either on one side of the tripolar or at one end of the bipolar cells. It thus happens that some part of the nucleus always projects, bare of protoplasm, into the space of neuroglia which surrounds the cells on all sides; generally they are only held in position by their processes, for which the neuroglia often forms hollow sheaths, in which the processes occasionally extend some distance. It seldom happens that a distinct nucleolus is observable in the nucleus, but usually only granules, which sometimes are found grouped together in the centre, as if the nucleolus had just broken up.

That the spaces mentioned above are lymph spaces has been conclusively shown by Obersteiner (78), who not only observed lymph corpuscles in the corresponding vacuities of the mammalian cerebrum, but actually injected one of them,—a fact which demonstrates that STIEDA's idea that they are artificial productions is erroneous; besides, BEVAN LEWISS (84) also shows that they are real structures. The inner surface of these spaces is rendered smooth by an agglomeration and close apposition of the granules of the neuroglia, as in the olfactory lobes.

The cells above described occupy a position intermediate between the small cells on the periphery and the central part of the lobes, which is occupied by white medullary fibres; these fibres converge towards the postero-inferior angle, where they form a medullary cord. They are comparable to the "corona radiata" of the cerebrum; they pass downward and backward to be lost in the anterior end of the hypoarium.

The cells become much less numerous in passing towards the centre of these lobes and are seldom found in the "corona radiata;" those that do occur, however, have an elongated shape (fig. 16, *d.*), both protoplasm and nucleus appearing as if they had been pressed out by the surrounding fibres. The neuroglia, which occupies a large part of these lobes, is composed of a granular substance, in which extremely fine fibres ramify in all directions, forming a network uniting together the smaller cells above described. The transverse commissure which connects the lobe of one side to that of the other is formed of two bundles of fibres placed one behind the other; the anterior bundle passes on each side forward into the anterior part of each lobe, and is lost in the walls of the fissure between the two lobes, while the posterior bundle passes more directly into the central parts.

It will be seen that the structure of these lobes differs considerably from that of the lobi olfactorii. Here, there are no incipient glomeruli olfactorii, neither are there cell-like swellings of the nerve-fibres, nor a central group of cells; but these bodies are more dispersed, are mostly of a different character, and are collected more towards the circumference. Thus, histological investigation does not support the opinion of some of the older anatomists, that these tuberosities homologise the bulbi olfactorii. The crura of the lobi olfactorii (fig. 1, *cc.*) pass from the posterior end of those lobes in a curved manner to the posterior end of the cerebral lobes, crossing over the fibres of the "corona radiata;" they cross these fibres, but have no direct communication with them, but continue their course into the posterior lateral lobule of the cerebrum, where they are lost.

Tectum Lobi Optici.

The structure of the tectum lobi optici differs in different parts of its extent; taken at about the centre of its arch, it shows (fig. 18) seven layers, commencing on the outside.

The first layer consists of finely granular neuroglia placed in immediate contact with the pia mater covering the tectum.

The second layer consists of coarse fibres apparently transverse but really oblique, that is, going in a direction between transverse and longitudinal; this layer contains sparsely distributed fusiform cells with their long axes placed radially.

The third layer consists of granular matter, with closely packed fibres, arranged radially so as to give it a smoothly striated appearance.

The fourth layer has the radial fibres not very closely placed, and the remainder of the layer consists of obliquely directed fibres, as in the second layer.

The fifth layer consists of transverse fibres derived from the crura lobi optici. This layer is clearer in colour than the remainder of the tectum, owing to the absence of granular matter, and also to the fibres not becoming so readily coloured as the other parts of the tectum; some radial fibres run across this stratum also.

The sixth layer consists of small cells arranged on branched stems which are prolonged into the radial fibres occurring in the other layers.

The seventh is a layer of connective tissue of varying thickness in different parts of the tectum; it forms on its external edge a support for the cells of the sixth layer, and internally terminates by a single layer of epithelial cells which form a smooth surface towards the ventricle of the optic lobe; this "ependyma" forms a support for the radiating fibres of the crura lobi optici in their passage from the torus semicircularis to the tectum; this connective tissue is composed of an inextricable network of fibrillæ derived from its epithelial layer of cells.

Stieda (58) makes eight strata in the tectum by the expedient of dividing the ependyma into two, making the epithelial layer of cells distinct from the connective tissue beneath them; but as the fibrillæ of this layer are derived from the cells, it seems scarcely correct to separate them. In other respects his division corresponds very closely to mine.

The cells of the sixth layer are of two different forms: those situated on the inner edge which make the terminal enlargement of the radial fibres, and those of the deeper part of the stratum which are attached to the sides of the fibres of the first, like grapes to their stalk.

The cells (fig. 18, *b.*) which make up the inner row of this layer may be described as fusiform, oval, or rhomboidal in shape; they generally show a distinct oval nucleus in which also a distinct spot-like nucleolus is visible: they vary in length from 0·010 millim. to 0·015 millim., and in width from 0·002 millim. to 0·005 millim.; the nucleus measures in length from 0·004 millim. to 0·005 millim., and from 0·003 millim. to 0·004 millim. in width; the connective tissue of the seventh layer is attached to the inner end of some of the cells, while their outer end passes off into a fibre which runs radially towards the external surface of the tectum, and probably extends as far as the second layer.

The fibres of these cells have a tendency to run in bundles formed of several united together; the bulk of this stratum is made up of cells (fig. 18, *c.*) of a smaller size than those described, and generally more rounded; these are attached to the fibres of the former cells sometimes by a short stalk, and in other cases they are sessile; they usually measure 0·004 millim. by 0·003 millim., with but slight variation in size.

There is present in most cases a distinct nucleus, which is generally rounded, but sometimes it presents a tendency to the oval form ; these nuclei often show a distinct nucleolus. About ten or twelve of these cells occupy the thickness of this sixth layer ; towards the anterior part of the tectum this layer passes continuously into the fornix. (I shall adopt this term in preference to "torus longitudinalis" used by Fritsch, without attaching any homological meaning to it, but simply as indicating the form of the part to which it is applied ; it has been in use since Carus first discovered the part, and therefore has the sanction of antiquity in its favour.) The other species (fig. 18, *d.*) of nervous elements in the tectum is found in the second layer ; these are long, fusiform, cell-like swellings of the radial fibres, which are finer at the inner end than at the outer ; it is somewhat difficult to measure their length owing to the gradual transition between the fibre and the cells, it may however be said to vary between 0·018 millim. and 0·040 millim. ; the width is more constant, being seldom more than 0·005 millim., but occasionally reaching as much as 0·007 millim. ; the nucleus measured about 0·005 millim. by 0·003 millim. These cells are not enclosed in a space, but the neuroglia is in apposition to their external surface. The fibre which passes from the inner end of these cells is the finer, and can be traced into the smoothly striated third layer ; the fibres from the sixth layer can be traced into the same stratum on the other side ; from which circumstance the presumption arises that the small cells of the internal layer stand in connexion with the fusiform bodies just described. Stieda (58) places these cells in his third or striated layer, and considers that they belong to the neuroglia "grundsubstanz." This does not appear to be quite correct ; they are situated, in fact, in the layer which he terms "die äussere Langsfaserschicht." Their nervous character can scarcely be doubtful ; they resemble in fundamental structure the cell-like swellings on the fibres of the olfactory lobe, and also the Purkinje cells of the second layer of the cerebellum, as the latter would appear if drawn out and stretched so as to be made long and thin. The outer process of these cells can be traced under favourable circumstances into the outer finely-granular layer of the tectum.

The modifications undergone by these layers in various parts of the tectum are as follows :—

At the outer side of the entrance of the crura lobi optici the fifth layer disappears as a distinct stratum, being only formed by the fibres of that part running in an inward direction towards the central commissure of the tectum. Anteriorly towards the inner end the first layer disappears. On the outer side all the layers, except the first, third, and sixth, are obliterated by the fibres of the optic tract. Further back, at about the middle of the tectum, longitudinal fibres appear which are intercalated between the first and second layers ; others occur between the fourth and fifth layers towards the inner edge ; these latter gradually die out and extend only about one-third of the width of the tectum.

The cells of the sixth layer are continuous internally with those of the fornix at the anterior termination of the tectum. The fornix consists of two longitudinal ridges of a somewhat triangular shape projecting into the ventricle of the optic lobe, and are placed along the inner and contiguous margins of the two halves of the tectum. The border of each ridge, which is turned towards the ventricle, is lined by a delicate layer of columnar epithelial cells, which lie immediately upon the nervous elements, without the intervention of any connective tissue.

The cells (fig. 19) which constitute the fornix are mostly of a spherical form, consisting almost entirely of nuclei with only a narrow rim of protoplasm round them; they generally contain a spot-like nucleolus; they usually measure 0·003 millim. or 0·004 millim. in diameter. Occasionally larger cells occur, which present a triangular shape from the greater quantity of protoplasm belonging to them. One of these is seen in the figure. These cells are arranged in rows, or in single files radiating from the upper and inner angle of the fornix; each row is separated from its neighbour by bundles of fibrillæ, which also radiate from the same point; these bundles are thicker at the proximal end, and gradually become smaller by giving off radiating fibrils in their course. The cells are attached to these fibrils sometimes by short branchlets, and sometimes they are sessile. The fornix forms a link connecting the sixth layer of the tectum to the granular layer of the valvula cerebelli, by which they are further continuous with the third layer in the cerebellum.

Tori Semicirculares.

The tori semicirculares (fig. 20, *a.*) may be considered as the anterior termination of the medulla oblongata; they are tuberosities of a more or less semicircular shape, which project into the floor of the ventricle of the optic lobe; they are principally composed of grey matter, through which the bundles of the crura lobi optici pass on their way to the internal surface of the tectum lobi optici. The surface turned towards the ventricle is lined by a stratum of connective tissue (fig. 20, *ep.*), which is continuous with the corresponding lining of the tectum; its margin is bordered by a layer of columnar epithelial cells, the internally directed extremities of which pass into and form the fibrillæ of the connective tissue, as in the seventh layer of the tectum. This appears to correspond to the "ependyma ventriculorum." The principal characteristics of the tori semicirculares are small cells (fig. 20, *b.*), elongated, narrow, and tapering at their pointed extremities into long fibres, which can be traced through the substance of the torus to the level of the bundles of nerve-fibres which belong to the crura lobi optici; they have a circular or oval nucleus which usually occupies the whole width of the thickest part of the cell; the nucleolus is generally represented by a few granules. They vary in length from 0·009 millim. to 0·016 millim., and in width from 0·004 millim. to 0·006 millim.; they are arranged in a row along the margin of the torus, and in the interior there are two or three, more or less complete, nearly horizontal rows of the same species of cells. The deeper part of this tuberosity is occupied by bundles of nerve-fibres belonging to the crura lobi optici, which pass through them on their way to the internal surface of the tectum. Between them, and beneath the more superficial bundles, are scattered cells which resemble the larger cells in the cerebrum in size and shape.

In the Basse (*Labrax Lupus*) the smaller cells on the surface are not placed in a row, but in groups, leaving spaces between them bare of cells, or in which only a few isolated ones occur; they are also found in groups in the interior, but not in connected rows. As if to make up for this deficiency a few large cells are found in the interior, which slightly exceed in size the larger ones in the cerebrum; they are pyriform in shape, and have a nucleus and a distinct nucleolus, resembling, on a smaller scale, the cells in the spinal cord. In a section taken through the whole length of the torus about six of these cells were found in one plane. The parenchyma of the tori semicirculares is composed of coarse granular neuroglia, in which fibres, mostly longitudinal, are indistinctly visible, chiefly towards the more superficial part. As in *Mugil*, the deeper part of the torus is occupied by the bundles of fibres of the crura lobi optici. Beneath the more superficial bundles of these, there occurs a thin layer of grey substance in which other cells are found, which are slightly larger than those just mentioned; they have a nucleus, nucleolus, and two or three processes; each cell occupies a space in the neuroglia; they are much less numerous than in the *Mugil*, and occur in groups of two or three. Two groups and a single cell occur in a longitudinal section through the whole torus in one plane.

The Hypoaria.

The parenchyma of the hypoaria is composed of finely granular neuroglia, in which the ramifications of extremely small fibres form a network of inextricable tenuity. In this neuroglia bundles of nerve-fibres radiate from the posterior and upper side in distinct and well-formed cords. The nerve cells (fig. 21, *a*.) occur throughout the neuroglia scattered singly, but increasing in number towards the free margin of the hypoarium. These cells are pyriform in shape; the thicker end is occupied by a nucleus, in the centre of which, in most cases, a spot-like nucleolus is seen; occasionally, however, this is represented by a group of granules. Each cell partly occupies a distinct chamber in the neuroglia, into which only the nucleus projects, the pointed end being closely surrounded by neuroglia; occasionally a process, given off from the thicker end, converts them into bipolar cells. Their average length is 0·008 millim., and their width 0·005 millim.; the nucleus usually has a diameter of 0·004 millim.

The external surface of the hypoaria is not covered by an epithelial layer. Each lobe presents a ventricle which commences close to the inferior surface, where it extends from near the anterior to near the posterior end; behind, this ventricle is narrow and slightly curved inwards; anteriorly, it expands and becomes broader; each ventricle gives off a passage from its anterior end, which passes with a curve upwards and inwards until the passage of one side meets that of the other in the infundibulum.

These ventricles are lined by a distinct layer of epithelium, which is a continuation of the epithelium lining the central canal of the spinal cord. They are surrounded by a layer of small circular or pear-shaped cells (fig. 21, *b.*), which extend for some distance into the substance of the hypoaria; these cells abound more especially along the ventricles themselves, close to the lower surface of these tuberosities, but they also occur in the course of the passages which communicate with the infundibulum. They measure about 0·006 millim. in length and 0·005 millim. in width, and have comparatively large nuclei of about 0·004 millim. in diameter. Some have a spot-like nucleolus, which in other specimens is represented only by a group of granules. Cells (fig. 21 *c.*) somewhat resembling those of the terminal row of the sixth layer of the tectum occur beneath the epithelial lining of the infundibulum and the ante-chamber, or ventricle as it may be called, of the hypophysis cerebri; they are bipolar, tripolar, or quadripolar; but one process, that directed away from the infundibulum, can be traced much further than the others; they vary from 0·008 millim. to 0·022 millim. in length, and from 0·006 millim. to 0·009 millim. in width; they have distinct, well-marked nuclei, the diameter of the larger being 0·006 millim., that of the smaller specimens being 0·004 millim.; in most cases the nucleolus is represented by a few granules, either scattered through the nucleus or collected into a group.

In addition to these small cells, others of a much larger size (fig. 21, *d.*) occur, which are situated more particularly under the epithelium of the ventricle of the hypophysis cerebri, and the adjacent anterior edge of the hypoarium; some are pear-shaped, others are expanded at one end and flattened like an enlarged columnar epithelial cell, but they are not of the nature of epithelium, for they lie beneath that layer. The largest measure 0·030 millim. in length, by 0·014 millim. in width; the nucleus is large, and the nucleolus also is disk-like and measures 0·002 millim. in diameter.

Each hypoarium contains a peculiar spherical body (fig. 22), denominated by FRITSCH "nucleus rotundus." This is placed nearer the posterior edge and the mid-line than the external surface and the anterior end; it is composed of interlacing fibres and granules, in which are embedded cell-like bodies which differ very greatly in size, some measuring as much as 0·07 millim. by 0·06 millim., some as little as 0·04 millim. by 0·03 millim.; many are nearly round, others longer than broad; they are composed of loosely aggregated granules of about 0·001 millim. or 0·002 millim. in diameter, which are crowded round the circumference, leaving a space in the centre, which in many cases is occupied by a smooth oval or pyriform body, having somewhat the aspect of the protoplasm of a cell, but in which no nucleus is observable; sometimes this body is missing in very thin sections, there remaining only a clear space, which it had probably occupied. Capillaries are occasionally to be seen passing through the loose granules of the circumferential portion.

What these bodies are I am at a loss to determine; they are of about the same size as the larger cells of the spinal cord. The granules also which compose their cortex resemble those of the protoplasm of those cells, in size and aspect, except that they are more loosely aggregated: they might be looked upon as cells in which the granules of the protoplasm are more loosely arranged than in an ordinary cell ; or the body in the centre might be regarded as a cell, surrounded by a mass of granules for reinforcing or extending its nervous energy.

In *Mugil cephalus* these bodies occupy the whole of the "nucleus rotundus," but in *Crenilabrus* there is a space left in the centre occupied only by granules and a network of fibres.

In *Crenilabrus* the circumference of this body is bounded by a layer of small cells (fig. 22, *c.*), varying in length from 0·010 millim. to 0·016 millim., and in width from 0·004 millim. to 0·005 millim. ; they are elongated in shape, and have a circular nucleus, which contains several granules instead of a single nucleolus. This nucleus often projects from one end of the cell, giving to the latter a high-shouldered appearance ; the side of the cell opposite its nucleus ends in one or two processes, and is always turned towards the centre of the "nucleus rotundus." These cells absorb colouring matter to a greater extent than most others. In *Mugil* they exist, but do not form a connected layer round the body in question, as is the case in *Crenilabrus.*

Cerebellum.

The cerebellum presents a structure comparable to that found in Mammalia. In a section (fig. 23) through this tuberosity, four layers are observable from the external surface to the centre.

The first layer (fig. 23, *a.*) consists of straight fibres, arranged perpendicularly to the long axis of this division of the brain ; these fibres are embedded in a coarse granular neuroglia.

The second layer (fig. 23, *b.*) is composed of large bipolar cells of peculiar structure, corresponding to the cells of Purkinje in the cerebellum of Mammalia. These form a stratum three or four cells in thickness throughout the cerebellum, except at the superior and inferior angles, where they are accumulated to a greater extent.

The third layer (fig. 23, *c.*) is constituted by a mass of minute cells, and ramifications of fine fibrillæ.

The fourth layer is formed by a bundle of fibres which go into the anterior crus cerebelli.

The bipolar cells have the usual structure of those of Purkinje (fig. 23, *d.*) in the mammalian cerebellum. They are elongated bodies, containing a large nucleus, in which a well-developed nucleolus is observable ; one end tapers into a fine process, while the other passes off more gradually to a thick one. This latter, after passing some distance in the same layer, either towards the anterior or posterior end, parallel to the long axis of the cerebellum, enters the external layer, and, dividing dichotomously, becomes finer and still more fine, as it continues at right angles to the surface, and passes towards the outer edge, where it eventually disappears from view. At the point of bifurcation of these fibres a swelling occurs which fills up the angle formed by the divergence of the two branches; at the external edge the fibres are sometimes seen to form loops turning back towards the centre.

The Purkinje cells in Teleostei differ from those in the cerebellum of Mammalia in being less complicated, inasmuch as their thick processes do not divide into two branches, giving off a forest of straight fibres, but continue as a single trunk for some distance, and enter the external layer before dividing, and that only dichotomously. Moreover, they are placed with their long axis parallel to the long axis of the cerebellum, instead of transversely to that axis, as in Mammalia. They resemble, in fact, the cells of the corresponding layer in the cerebellum of the newly-born infant in the simplicity of the arrangement of their prolongations.

The third layer (fig. 23, *c.*) is composed of cells of extreme minuteness, which possess a nucleus, a nucleolus, and a small quantity of protoplasm. They are usually pear-shaped, and from the pointed end an extremely fine fibre proceeds; which fibre, with those of the other cells in this layer, join together in a network which forms ramifications in the neuroglia of this stratum. Occasionally some of these cells are seen to be attached by their points to the side of a fibrilla.

The theory I adopt as to the arrangement of the elements of the cerebellum is this: The broad processes of the cells of Purkinje divide continuously until they arrive near to the external edge, by which time they have become extremely fine; they then turn back, and form a network in this layer, and, passing between the cells of Purkinje, finally join the corpuscles of the third layer, uniting them together in an inextricable network.

The above theory is based on the following circumstances. In teased-out preparations, the broad fibres of the Purkinje cells are found to be covered by coarse granules, which are attached to them at a certain distance from the cell, at points nearer to which they are smooth. These granules resemble those found in the neuroglia of the external layer. The cells of the third layer are seen, when teased out, to be united together by fine fibres, which are quite smooth, having no attached granules. Thus it is found that the characteristic elements of the external layer are fibres to which coarse granules are attached, giving them a granulated appearance; while the third layer is distinguished by cells the fibrils of which are smooth.

In some preparations of teased-out Purkinje cells the broad fibre is seen to become more and more decreased in size, and the finest fibres have granules attached. On the other hand, preparations of the cells of the third layer have been made in which fibrils longer than usual have been preserved unbroken; these, smooth near their origin, have become covered with granules, resembling those of the first layer, at a greater or less distance from the cell. From this it may be inferred that the latter fibres pass from the third layer into the first. But on the above supposition, what becomes of the fine fibres of the Purkinje cells? GERLACH (71) made out that they joined the cells and network of the third layer, which if the above description be anywhere near the truth cannot be the case, and his opinion is not confirmed by DEITERS (73).

The latter author and KOSCHENIKOFF (75) demonstrated that they join double contoured nerve-fibres,—a fact which would accord well with the above-mentioned views, and would imply that they join, not the small cells of the third, but the fibres of the fourth layer, and thus a junction would be formed in a roundabout manner between the minute elements of the third layer and the commissures of the other parts of the brain. According to HADLICH (77) the arrangement is similar to that above given, but he does not explain what becomes of the finer fibres of the Purkinje cells. OBERSTEINER (78) describes small cells or corpuscles in the outer or first layer in the cerebellum of the fœtus, to which the fibres from the broad processes of the Purkinje cells are attached,—a fact which SANKEY (83) has confirmed in the adult. I have not found any cells of this description in the external layer of the cerebellum in Teleostei; but their presence would not interfere with the interpretation adopted here, for the fibres in question could yet pass through those corpuscles, and behave as above mentioned.

The whole cerebellum in the Teleostei appears to correspond to a single lobulus in that division of the brain in the human subject.

In *Mugil cephalus* there are indications of the commencement of a second lobulus, which in *Labrax Lupus* are more decided.

There is a vertical fissure in the central line of the cerebellum in *Mugil*, forming a ventricle, which extends from the posterior end as far forward as the posterior edge of the crura cerebelli; it has an appearance as if the two halves had not properly united.

The Valvula Cerebelli.

The valvula cerebelli has precisely the same structure that the cerebellum itself has; the layer of straight fibres, the layer of Purkinje cells, and the layer of minute cells are all present: the only difference to be seen lies in the arrangement of these strata.

The layer of straight fibres is a direct continuation of the external layer of the cerebellum, which, passing forward into the ventricle of the optic lobe nearly as far as its anterior extremity, turns back on itself and forms a fold; so that the edges which would be external in the cerebellum face each other, leaving a narrow fissure between them which communicates with the external surface between the posterior end of the tectum, and the anterior surface of the cerebellum, by a foramen which is closed in by pia mater and gives access to vessels. The Purkinje cells bear the same relation to the layers of straight fibres that they do to the external layer of the cerebellum. Then the mass of small cells resembling those found in the third layer come to be placed on the surface facing the ventricle of the optic lobe, and extend as wings on each side. In the *Mugil* these wings are simple rounded bodies, but in the *Labrax* a deep transverse fold of straight fibres divides them into two lobes, and in the *Crenilabrus* two such folds divide them into three lobes. This also appears to be the case in the Carp. They are the bodies termed by HALLER "cornu ammonis," and I have reason to suppose that they form the peculiar organ found in the brain of the *Mormyridæ*, as I have before mentioned.

Tuberosities of the Vagus and the Medulla Oblongata.

The tuberosities of the vagus (fig. 10, *t.v.*) resemble very much the tori semicirculares in structure. The sides forming the walls of the fissure constituting the posterior part of the fourth ventricle are lined by a layer of connective tissue bounded internally by a single row of epithelial cells, precisely in the same way that the surface of the torus facing the ventricle of the optic lobe is covered. A vertical row of cells is placed externally to this ependyma of the same kind as those found beneath that structure in the torus. The only difference that exists between these two tuberosities is, that in the one belonging to the vagus the groups of cells project more beyond the level of the nervous tissue into the ependyma than in the torus, and also that the larger cells which exist in the latter are not found in the former. That part of the medulla oblongata which bridges over the narrow part of the fourth ventricle consists entirely of granular matter, and no cells of any kind are to be seen in it.

Transverse Commissures of the Central Nervous System.

In the anterior part of the spinal cord only two commissures are visible—one about half way between the central canal and the ventral edge of the cord; this is the ventral commissure (fig. 1, *v.t.c.*) which connects the ventral horns of grey matter of the two sides. The other is visible only in sections which pass through the dorsal roots of the nerves, and connects the dorsal horns of grey matter of the two sides. This latter commissure is more pronounced nearer the fourth ventricle, and immediately behind the fissure of the medulla oblongata it comes to be closely applied to the dorsal edge of the central canal.

The two commissures mentioned by MAUTHNER (50) are not visible in *Mugil;* the place which he indicates for them being occupied in this species by connective tissue: the "substantia gelatinosa centralis" of STILLING. This connective tissue is composed of fibres derived from the epithelial layer surrounding the central canal of the cord.

Immediately behind the posterior fissure of the fourth ventricle a series of transverse commissures come to view, which gradually take the place of the ventral commissures of the cord, although they do not occupy exactly the same relative position. They connect the tuberosities of the vagi together and increase in size, pari passu, with the latter; they pass downward and across to the other side ventrad of the central longitudinal column.

At about the middle of the vagus territory they unite the root of one nerve to that of the other. At this point one of the branches of the above-mentioned commissures turns back at the dorso-exterior angle of the tuberosity of the vagus, passes downward along the external edge of the medulla to its ventral edge, whence it passes over to the corresponding point on the opposite side. The first-mentioned transverse commissures extend at intervals as far forward as the posterior end of the ganglion of the motores oculorum; the most anterior bundles become smaller, and pass above instead of beneath the central longitudinal column. Those situated in the region of the crura cerebelli could not be traced directly into them.

Commissura Ansulata. (Figs. 1, 2, and 5, *c.a.*)

There is a double system of transverse commissures at the region of the origin of the motores oculorum and the posterior end of the tectum which are derived from the anterior part of the latter body. They are the commissura ansulata of Gottsch (34). Commencing the description of the outer circle of these commissures from the system of transverse fibres passing across the base of the fornix, we can trace the fibres through the fifth layer of the tectum, then by numerous bundles into the torus, where they form the system of the crura lobi optici, then downwards and backwards along the external edge of the medulla to its ventral surface, on the roof of the furrow between the medulla and the hypoarium, where after giving a few fibres to the motor oculi of the opposite side, they cross over to return to the region whence they started. The other part of this double commissure follows the same course as the last, until it attains the lower part of the external edge of the medulla, and there it turns inwards towards the mid-line; it then crosses obliquely to the other side, and attains the inner part of the torus, near the point of junction of the alæ of the valvula cerebelli with that body, and extends nearly as far forward as the anterior end of the optic lobe. The latter commissure extends in the medulla to about one quarter the length of the space occupied by the former; they are derived from bundles of longitudinal fibres which first appear on the lateral borders of the medulla oblongata opposite the narrow part of the forth ventricle (fig. 9, *c.a.*).

Some writers compare this commissure to the pons varolii; this can scarcely be a true homology, since none of the fibres come from the cerebellum. It seems to me more probable that they homologise the decussation of the anterior pyramids; they resemble to a slight extent the figures of that decussation given by Lockhart Clarke (70). Baudelot (61) was also of this opinion; he termed them "commissure antérieure des pyramides;" he appears only to have been acquainted with the first described one. The objection that occurs to me is that they are too far forward in front of, instead of behind the fourth ventricle; they also come from the optic lobes.

At the anterior end of the floor of the optic lobes are two transverse commissures, one superficial and one deep. The superficial commissure (fig. 3, *p.c.*) passes over the posterior part of the third ventricle, and the passage that puts it into communication with the ventricle of the optic lobe. This passage is, in fact, the anterior end of the aqueduct of Sylvius. The commissure in question is derived from two separate regions. The anterior part connects the two sides of the anterior end of the floor of the ventricle of the optic lobe (*Thalamus opticus, Stieda*). The posterior part connects the anterior termination of the central longitudinal columns of the cord, or at least of some few fibres which are the continuation forward of those columns. The deeper commissure (figs. 3 and 24, *c.pr.*) is derived from the region about the anterior part of the floor of the optic lobe; it then passes downwards and backwards, and passing through the "nucleus rotundus" of the hypoarium, turns downwards and forwards, and crosses over in front of the infundibulum to the corresponding point on the opposite side.

CUVIER (28), GOTTSCHE (34), and other anatomists considered that the former homologised the anterior commissure of the cerebrum, but it appears to me that its position corresponds better with the posterior commissure; its situation at the posterior end of the third ventricle, and behind the infundibulum, points in that direction; and if, as STIEDA maintains, the region round this ventricle comprising the anterior part of the floor of the optic ventricle homologises the thalami optici, this interpretation would be strengthened, for the commissure in question connects the two sides of that region.

Longitudinal Columns and their Anterior Prolongations.

The anterior columns of the cord are composed of two bundles of large fibres (fig. 1, &c.), one of which is situated close to the ventral edge, the other is situated internally beneath the substantia gelatinosa centralis; they are separated from each other by the ventral commissures; the former may be called the ventral longitudinal columns, and the latter the central longitudinal columns. In the latter are situated the giant fibres, two in number, one in each column. These were first described by MAUTHNER (50); they are oval in section, measuring 0·016 millim. by 0·012 millim.; each is contained in a separate tube of connective tissue, which is much larger and more regular in outline than the tubes surrounding the remaining fibres of these bundles; the other nerve-fibres in these columns vary in size from 0·010 millim. by 0·005 millim. to 0·003 millim. in diameter: thus there is a gradation in size from the small nerves to those nearly as large as the Mauthner fibres. The smaller ones are more numerous in the lateral part of the ventral columns. The Mauthner fibres do not seem to correspond, as MAUTHNER thought, to the large fibres in the spinal cord of the myxinoid fishes, which are described by MÜLLER (36) as flat band-like fibres, which these are not.

The Mauthner fibres (fig. 8) decussate at about opposite the posterior end of the origin of the trifacial, after which they disappear entirely, and I could trace them no farther; probably they break up into fine fibrillæ of which they are most likely composed. In support of this opinion I may cite a case in which I saw one of these fibres in the spinal cord of *Scorpæna* distinctly break up into fibrillæ for a short distance, and then resume its original appearance.

The ventral longitudinal column rises up gradually from contact with the lower surface of the medulla (fig. 1), and the central longitundal column joins it beneath the posterior part of the cerebellum, beyond the decussation of the Mauthner's fibres; which decussation is not participated in by the accompanying bundles.

The combined columns pass straight forward and are lost in the internal part of the floor of the optic lobe at about the posterior end of the third ventricle.

The columns situated on each side of the central portion of the substantia gelantinosa centralis, which may be termed the lateral columns (fig. 12, *l.c.*), pass forward and are lost in the floor of the optic ventricle near to but outside the last-mentioned columns. At the point where the ventral longitudinal bundles rise up towards the dorsal surface, other fibres are substituted for them along the ventral and lateral borders of the medulla. The more lateral of these, as before mentioned, pass into the commissura ansulata; those in the mid-line, however, pass into the posterior end of the hypoarium through the commissura ansulata, and curve over the anterior end of the furrow between that body and the medulla.

Anterior Crura Cerebelli. (Fig. 24.)

There are three bundles of fibres which may be collectively termed the anterior crura cerebelli, since they pass out of the cerebellum and then turn forward.

The most anterior extending downwards and forwards beneath the anterior layer of the Purkinje cells of the cerebellum, immediately turns forwards and is lost in the lateral wings of the valvula cerebelli.

The most posterior of the three bundles passes obliquely downwards and forwards towards the hypoarium and is lost near the outer side of the "nucleus rotundus" of that lobe.

The middle cord of the anterior crura cerebelli is the largest of the three; it separates into two divisions at the base of that section of the brain. The superior division enters the lateral wing of the valvula cerebelli beneath the anterior bundle of the crus.

The other division, which is the main body of this bundle, enters the deeper parts of the torus, and then making a gentle curve with the concavity turned upwards, it enters the fourth layer of the tectum; this may be looked upon as the processus e cerebello ad testes.

Many bundles of fibres are derived from the deep commissure which was described with the transverse commissures of the brain; of these one passes off when it arrives at the superior edge of the "nucleus rotundus." This bundle on leaving the abovementioned commissure borders the posterior margin of that body, and is joined at the lower edge by several small bundles of fibres, which leave the deep commissure in the "nucleus rotundus," also by bundles of fibres which are derived from the region slightly in advance of the origin of that commissure; these latter cords pass down in front of the "nucleus rotundus," close to its anterior edge, forming a border thereto. The whole of these fibres stream out from the lower edge of the "nucleus rotundus," and disappear in the posterior part of the hypoarium.

Deep origins of the Cerebral Nerves.

The origin of the olfactory nerve (fig. 1, *n.* 1) has been described in the account of the histology of the lobi olfactorii.

The optic nerves (figs. 3 and 4, *n.* 2) are derived from three sources. A few of the inferior fibres come up from the anterior end of the hypoarium. The superior fibres are derived from the three outer layers of the tectum lobi optii; in some sections the fibres for this nerve form an additional distinct stratum of longitudinal fibres intercalated between the first and second layer; this shows more particularly towards the middle of each tectum.

The third root is formed by fibres, which arise in the interior of the torus semicircularis, take a direction outwards towards its external edge, and form a bundle which passes through the crura cerebri, and being intercalated between them and the outer margin of the torus, descend towards the anterior inferior end of the tectum, where it joins the optic tract.

The motores oculorum (figs. 1, 2, and 5, *n.* 3) are derived from a group of medium-sized cells, situated beneath the floor of the aquæductus Sylvii immediately behind its entrance into the ventricle of the optic lobe; from these cells the fibres pass down close to the mid-line towards the ventral surface of the medulla, where they pass through the commissura ansulata; they then emerge from the angle between the medulla and the hypoarium, having received some fibres from the opposite side of the commissure. This nerve has two separate origins: the main origin from the ganglion on its own side, above described, and a smaller one from the region of the crura lobi optici of the opposite side.

The trochleares (fig. 6, *n.* 4) arise from the anterior end of the cerebellum on a level with the base of the valvula cerebelli, where each nerve decussates with its fellow of the opposite side, and becomes intermingled with the transverse commissure of the cerebellum above the aqueduct; the nerve then passes down the margin of the medulla, outside the crura cerebelli, and emerges behind the posterior end of the optic lobe.

The trifacial (figs. 2, 7, 8, and 9, *n.* 5) has three distinct points of origin, of which two belong to the anterior part of its root, and one to the posterior part. The anterior root is made up of two distinct species of fibres; the ventral part of the root is composed of fine slender, while the dorsal half has large, coarse, double contoured fibres. The regions of origin whence these are derived are placed at some distance apart; the latter come in a nearly straight line from a large ganglion situated beneath the floor of the widest part of the fourth ventricle. The cells forming this ganglion are of large size, like those found in the cord. This ganglion bears about the same relation to the floor of the anterior part of the fourth ventricle that the larger cells of the ganglion of origin of the vagus do to the floor of the posterior part of the same ventricle, being, if anything, a little nearer the floor and farther from the mid-line. The fine fibres which form the ventral side of this root are derived from a bundle, which comes forward from the external part of the lateral columns of the spinal cord, and crosses the posterior root of the vagus close to its emergence from the medulla oblongata; it passes ventrad of the anterior root of the latter, then between the origin of the acusticus and the posterior part of the root of this nerve, and then joins the ventral part of the root under discussion. This bundle is very distinct and well-formed between the vagus and the point where it emerges from the medulla, but behind the former it gradually disappears, and is probably derived from the ventral horn of grey matter in the cord. In passing over the vagus it presents a delusive appearance of joining that nerve, all the fibres turning towards the external edge at that point; but a careful inspection of a series of horizontal sections proves that such is not the case, but that the fibres pass on backwards as described. The posterior root of the trifacial arises as a bundle of fibres, which come forward from the central portion of the vagal tuberosity, and are collected into a cord which passes forward along the side of the narrow part of the fourth ventricle, until it arrives at a point close behind the expanded portion of the same, and opposite the decussation of the fibres of Mauthner, where it turns outward and slightly downward to form part of the posterior roots of the trifacial.

The abducens (figs. 8 and 9, *n.* 6) arises by two small roots from two little ganglia placed in the ventral grey matter of the medulla oblongata, beneath the narrow part of the fourth ventricle. A small nervous cord emerges from each of these ganglia, and the two unite together to form the trunk of the nerve at its exit from the ventral side of the medulla.

The acusticus (fig. 9, *n.* 8) is derived from the lateral part of the medulla oblongata in the region of the anterior end of the ganglion from which the posterior root of the vagus arises. Fibres from this source pass upwards and inwards, and forming a distinct bundle become applied to the upper and outer side of the central longitudinal column in the medulla oblongata. This bundle follows that column, keeping entirely distinct therefrom, as far forward as the posterior end of the anterior part of the fourth ventricle; here it turns outwards and downwards with a curve, and emerges from the medulla as a nerve trunk placed at a lower level than the last-described root of the trifacial.

The vagus (figs. 2, 9, 10, and 11, *n.* 10) arises by two roots. The anterior is derived from two distinct sources—viz., from the cerebellum, and from the grey matter covering the narrow part of the fourth ventricle. The former appears as two bundles of nerve-fibres which pass back from that part of the cerebellum immediately covering the anterior chamber of the fourth ventricle; the latter comes from the region of grey matter which covers over the narrow passage of that ventricle, and partly from the transverse commissure at that part. The so-called glossopharyngeal also comes out from beneath this root, and is principally derived from the same part and from the lower portion of the same transverse commissure.

The posterior root of the vagus is also derived from two distinct sources. One is a conspicuous ganglion which extends beneath the whole length of the floor of the posterior part of the fourth ventricle, and reaches even for a short distance behind it. This ganglion is narrower transversely than from above downwards; it is composed of cells of a pyriform shape, the points of which run out into fibres which are partly directed outwards and partly outwards and downwards. The fibres from the cells situated on the dorsal part of the ganglion join the nerve more directly than those which are derived from cells situated nearer the ventral side. The vagus receives eight or ten bundles of fibres from this ganglion.

The other source of origin for this root of the vagus is the vertical row of small cells which occurs in the walls of the posterior end of the fourth ventricle or the vagal tuberosity. From these cells bundles of fine fibres pass across to the outer edge of the medulla; just as in the torus the fibres from the corresponding cells pass through to the deep surface of that part. These fibres all join this posterior root of the vagus, which also derives some reinforcement from the transverse commissure of the medulla at this point. If the idea of OWSJANNIKOW (46) be correct, that the sensory roots of the nerves are derived from small cells, while the motor roots come from the larger cells—an idea which receives countenance here from the origins of all the nerves except that of the trochleares—then these are the origins of the sensory fibres of the vagus, while the bundles from the larger-celled ganglion would be motor roots.

LOCKHART CLARKE (60) thinks that this ganglion of the vagus properly belongs to the hypoglossal, because, he says "that the ganglion in question bears the same relation to the floor of the fourth ventricle that the hypoglossal ganglion bears to the central canal;" he also remarks that a branch of the vagus supplies the tongue in fishes. With regard to the first statement I would point out that the ganglion in question is not the only one which bears that relation to the floor of the fourth ventricle or to the central canal; and that from the extent of the territory supplied by the vagus it is scarcely probable that its greatest and most important source of origin would be devoted to such a comparatively unimportant part (in the fish) as the tongue. With regard to the second point, I have never myself found a branch of the vagus going to the tongue, neither have I met such a statement in the writings of any author that I have consulted. The terminal twigs of the nerves that supply the branchial arches are not in point here, as they would belong to the glossopharyngeal nerve and not to the hypoglossal.

The spinal nerves (figs. 12 and 13), including the first spinal, or as some authors call it, the hypoglossal, have all dorsal and ventral roots. The ventral root arises by three bundles, one of which comes from the ventral transverse commissure, the other two come from the ventral horn of grey matter. I could not find a distinct connexion between these roots and the cells existing in the grey matter. In *Mugil* there are generally nine of these cells to be seen in one section, when any were to be seen at all. The *Scorpæna Porcus*, although a sluggish fish, has more cells in the ventral horn of grey matter than the *Mugil*, which is more active.

The dorsal roots of the spinal nerves arise by three or more bundles, which emerge from the outer and dorsal side of the dorsal horn of grey matter. Some of these fibres came from the dorsal commissures, others from the grey matter itself.

No cells are to be seen in this horn in the *Mugil*, but in *Scorpæna Porcus* the unusual circumstance occurs that its spinal cord, at some distance behind the medulla oblongata, presents at its dorsal part, and close to the mid-line, a pair of ganglia, one on each side. These ganglia are formed of large cells, which resemble those only found on the ventral side of the cord in other fishes; they have a distinct nucleus and nucleolus.

Conclusion.

With regard to the homologies of the brain of Teleostei, I have come to very nearly the same conclusion as STIEDA, although my opinion being based solely on researches into the Teleostean nervous system may require some modifications hereafter when I have an opportunity of investigating the brain of Elasmobranchii and Ganoids. The posterior unpaired tuberosity of Teleostei I consider to correspond to the cerebellum, in the ordinary acceptation of the term. The optic lobes would be the corpora quadrigemina, or rather bigemina, and would homologise with the anterior pair. FRITSCH—whose work 'On the Minute Structure of the Brain in Fishes,' only recently published, I have received since the foregoing pages were written, and to which unfortunately I have not yet had time to devote the amount of study which its importance demands—has opposed to this interpretation the consideration that their structure is too complicated; but I think that, if the other conditions of the homologies are satisfied, structure alone would not form a sufficient objection. The other conditions are satisfied: their position at the anterior end of the aqueduct of Sylvius, behind the third ventricle and in front of the cerebellum, points in that direction. The fact that they are comparatively highly organised simply shows that in fishes they perform functions which in higher animals are relegated to other organs. This being the case, the ventricle of these lobes would be simply an extremely enlarged aqueduct of Sylvius, and would correspond to the ventricle of the corpora quadrigemina in the fœtus. The third ventricle is the fissure between the crura cerebri, and would extend between the cerebral lobes as far as the commissure which connects them, which would then homologise the anterior commissure, while the transverse bundle of fibres on the floor of the ventricle of the optic lobe has the relative position of the posterior commissure. FRITSCH has marked the last-named cavity the lateral ventricle, but this appears to be quite untenable; not only its position behind the third ventricle, and the pineal gland, and at the mouth of the aqueduct, is against it, but V. BAER had seen the ventricle in question in the cerebral lobes of the embryonic Teleostean, and WILDER (65) has pointed them out in the same tuberosities in the adult fish. The region round the fissure of the third ventricle would be the thalamus opticus; and the torus semicircularis I would, with some doubt, refer to the corpus geniculatum externum. FRITSCH has determined the deeper part of this torus as the corpus quadrigeminum, and the superficial part as the thalamus opticus. The interpretation of the latter may be partly correct, but the relation of the former to the aqueduct of Sylvius forms a valid objection to its homology as given by this author. Of the remaining parts of the brain to be considered, the passage extending from the base of the third ventricle to the pituitary body is easily recognised as the infundibulum, and the glandular body into which it enters is clearly the pituitary body. The relation of the former to the latter is another instance of a fœtal arrangement of higher animals, surviving as an adult arrangement in the lower. The hypoaria occupy strictly the position of the tuber cinereum, and the part called valvula cerebelli that of the valve of Wieussens, both being cases of structures better developed in animals of an inferior scale than in those of a superior. Finally, the cerebral lobes homologise the corpora striata, combined with the hemispheres of the brain; and although I have not discovered in the species examined by me the ventricles in those bodies described by WILDER (65), yet I do not doubt that in other species they may exist.

The presence of the fissure between the crura cerebri in the brain of fishes has some bearing on the theories lately propagated by SEMPER and DOHRN as to the origin of Vertebrata from the Annelida. The former author has not yet published any researches on the relation of the Vertebrate nervous system to that of the Invertebrata, but DOHRN (82) sees in the fissure belonging to the posterior end of the fourth Ventricle the remains of the space included in the commissures between the supra-œsophageal and infra-œsophageal ganglia of insects through which the œsophagus of the Invertebrate animal passes. There are several objections to this, one serious one being the amount of space to be filled up with nervous matter, of which no indications exist in the Teleostean brain. Mere likeness alone between the parts in the embryo fish and the insect would not suffice, as is shown by the curious resemblance between the cerebral cortex of a mouse and the tectum lobi optici of a Teleostean fish seen in a horizontal section, yet the optic lobes do not homologise the cerebral hemispheres. Another objection FRITSCH has pointed out is, that if the œsophagus passed through this point, the jaws and mouth would have been supplied by a trunk from the supra-œsophageal ganglion, the fifth nerve being in advance. But if the theory be true, some other foramen must be found (if the œsophagus when it disappeared left any trace of its former existence) through which it could have passed; such a foramen presents itself in the third ventricle, which, in conjunction with the infundibulum, forms an opening quite through the nervous tissue, being closed only below by the pituitary body and above by the pineal gland. The existence of the pituitary body has never been explained. A glandular structure developed from the mucous membrane of the throat attached to the brain, it seems quite out of place; but considered as the remains of a former œsophageal or pharyngeal gland its raison d'être seems accounted for. The epiphysis cerebri is more obscure, but appears to be the remains of some vascular body useful in an ancestral state of existence, for it certainly has no functions to perform in living species. I would submit, then, that if the œsophagus did once pass through the brain as in Invertebrata, it passed through the infundibulum and the third ventricle.

Bibliography of the Nervous System in Teleostei.

I have not been able to procure copies of books marked thus, *.

1. Willis, Thomæ, Cerebri Anatome. 1664.
2. Malpighii, Marcelli, De Cerebro epistolæ ad Carolum Fracassatum. 1664.
3. Collins, S., System of Comparative Anatomy. 1685.
4. Haller, Alberti v., Opera Minora, tom. iii. 1768.
5. Haller, A. von, Elementa Physiologiæ corporis humani, tom. ii., p. 591.
6. Vicq d'Azyr, Mém. de Mathématique et de Physique, tom. vii. Paris, 1776.
7. Camper, P., Sämmtliche kleinere Schriften. Trans. into German by T. F. M. Herbell. 1785.
8. Monro, Alex., Structure and Physiology of Fishes. 1785.
9. Ebel, Godofredi, Observationes Neurologicæ. 1788.
10. *Scarpa, Antonio, Disquisitiones Anatomicæ. 1789.
11. *Chaussier, Exposition du Cerveau. 1807.
12. Carus, Carl Gustav, Versuch einer Darstellung des Nervensystems, &c. 1814.
13. Treviranus, G. R. and L. C., Vermischte Schriften, Anatomische, Bd. iii. 1816.
14. Tiedemann, F., Deutsches Archiv f. die Physiologie. Bd. ii. 1816.
15. Tiedemann, F., Anatomie und Bildungsgeschichte des Gehirns im Fœtus des Menschen. 1816.
16. Arsaki, Apostole, Commentatio de Piscium cerebro et Medulla Spinali. 1836.
17. Weber, Ernesto Henrico, Anatomia comparata Nervi Sympathetici. 1817.
18. *Wittjack, Joach., De Piscium cerebro. Berolini, 1817.
19. Zagorsky, De Syst. Nerv. Piscium. 1833.
20. *Wenzel, De Penitiori Structurâ cerebri.
21. Kuhl, Heinrich, Beiträge zur Zoologie und vergleichenden Anatomie. 1820.
22. Weber, Ernesti Henrici, De Aure et Auditu hominis et animalium. 1820.

23. BAILLY, E. M., Recherches d'Anatomie comparée du Système nerveux, &c. 1823.

24. SOMMÉ, C. S. Recherches sur l'Anatomie comp: du Cerveau. 1824.

25. SERRES, E. R. A., Anatomie comparée du Cerveau. 1824.

26. DESMOULINS, A., Anatomie des Systèmes nerveux des Animaux vertébrés. 1825.

27. WEBER, ERNST HEINRICH, Meckel's Archiv. 1827.

28. CUVIER et VALENCIENNES, Histoire Naturelle des Poissons. 1828.

29. TREVIRANUS, G. R., Über die hintern Hemisphären des Gehirns der Vögel, Amphibien und Fische, Tiedemann und Treviranus Zeitschrift, Bd. iv. 1831.

30. GILTAY, CAROLI MARINI, Descriptio Neurologica Esocis Lucii. Annales Acad. Lugduno Bataviæ, tom. xvii. 1833.

31. *STEIN, de Thalamo Optico et Origine N. Optici in homine et animalium. 1834.

32. BAER, KARL ERNST, v. Untersuchungen ü. d. Entwickelungsgeschichte der Fische. 1835.—Also, Entwickelungsgeschichte der Thiere. Zweiter Theil.

33. BÜCHNER, G., Mém. sur la Système nerveux du Barbeau (*Cyprinus Barbus*), Mém. de la Société d'Histoire Naturelle de Strasbourg, tom. ii. 1835.

34. GOTTSCHE, Müller's Archiv. 1835.

35. LEURET, FR., Anat. Comp. Système nerveux. 1839.

36. MÜLLER, JOHANNES, Vergleich. Anat. der Myxinoiden. 1840.

37. VOGT, C., Embryologie des Salmones in Hist. Naturelle des Poissons d'eau douce, par L. AGASSIZ.

38. STANNIUS, H., Peripherisches Nervensystem des Dorsch (*Gadus callarias*), Archiv für Anatomie. 1842.

39. GUILLOT, NATALIS, Exposition Anat. du centre nerveux, &c., Mém. Cour. de l'Acad. Royale de Bruxelles, tom. xvi. 1843.

40. *FLORMAN, ARVID, Recherches Struct. du Cerveau. Paris, 1843.

41. *FOVILLE, Traité complet Anat. Syst. nerveux. 1844.

42. GIRGENSOHN, O. G. L., Anat. u. Phys. des Fisch-Nervensystems. Mém. des Savants étrangers, St. Pétersbourg, tom. v. 1845.

43. STANNIUS, HERM., Das Peripherische Nervensystem der Fische. 1849.

44. KLAATSCH, H. M. A., de Cerebris Piscium. 1850. Halis.

45. PHILIPEAUX ET VULPIAN, C. R., tom. xxxiv.

46. OWSJANNIKOW, PHILIPPUS, Disquisitiones Microscopicæ de Medullæ spinalis texturâ in Piscibus factitatæ. 1854.

47. ECKER, ALEX., Anat. Beschreib. des Gehirns vom Karpfenartigen Nilhecht. 1854.

48. KÖLLIKER, A., Zeitsch. f. Wissen. Zoologie, Bd. ix. 1857.

49. MAYER, F. J. C., Ueber den Bau des Gehirns der Fische. Verhandl. der K. Leop. Car. Akad., Bd. xxx. 1859.

50. MAUTHNER, LUD., Untersuch. ü. d. Bau d. Rückenmarks der Fische. Sitzber. d. K. Akad. der Wissensch., Bd. xxxiv. Wien, 1859.

51. Marcusen, Johann, Die Familie Mormyren, Mém. de l'Acad. Imp. des Sciences de St. Pétersbourg, vii^e série, tom. vii. 1864.
52. Duméril, Aug., Hist. Naturelle des Poissons. 1864.
53. Swann, Joseph, Illustrations of the Comparative Anatomy of the Nervous System. 1864.
54. Baudelot, Functions of the Encephalon of Fishes: Annales des Sciences Naturelles. 1864.
55. Hollard, Comptes Rendus, tom. lx., p. 768. 1865.
56. Vulpian, A., Leçons sur la Physiologie Générale et Comparée du Système nerveux. Paris, 1866.
57. Oeffinger, H., Neue Untersuchungen ü. d. Bau des Gehirns vom Nilhecht. Archiv. f. Anatomie und Physiologie. 1867.
58. Stieda, Ludwig, Studien ü. das Central-Nervensystem der Knochenfische. Zeit. f. Wiss. Zool., Bd. xviii. 1868.—And published separately.
59. Kupffer, Beob. ü. d. Entwick. der Knochenfische. Arch. f. Microscopische Anatomie, Bd. iv. 1868.
60. Clarke, Lockhart, Researches into the Intimate Structure of the Brain. Phil. Trans., Vol. 158. 1868.
61. Baudelot, Etude sur l'Anatomie Comparée de l'Encephale des Poissons. Mém. de la Société des Sc. Nat. de Strasbourg, Bd. vi. 1870.
62. Miklucho-Maclay, N. von, Beiträge zur vergleichenden Neurologie der Wirbel-Thiere. Leipzig, 1870.
63. Schapringer, Alois, ü. d. Bildung des Medullarrohrs bei den Knochenfischen. Sitzber. d. K. Akad. der Wiss. Bd. lxiv. Wien, 1871.
64. Stieda, L., ü. die Deutung der einzelnen Theile des Fischgehirns. Zeit. f. Wiss. Zool., Bd. xxiii. 1873.
65. Wilder, Professor Burt G., Proc. Acad. Nat. Science, Philadelphia. 1876.
66. Fritsch, Gustav, Untersuchungen über den feineren Bau des Fischgehirns. Berlin, 1878.
67. Gegenbaur, C., Grundriss der vergleichenden Anatomie. 1878.—And, Grundzüge der vergleichenden Anatomie. Leipzig, 1870.

List of Miscellaneous Works on the Nervous System referred to in this Paper.

68. Stilling, B., Ueber den Bau des Kleingehirns. 1846.
69. Jacubowitsch and Owsjannikow, Microscopische Untersuchungen. Bull. Phys. Math. de l'Acad. de St. Pétersbourg, tom. xiv. 1854.
70. Clarke, Lockhart, Phil. Trans., Vols. 148, 149, and 152.
71. Gerlach, J., Microscopische Studien. 1858.
72. Clarke, Lockhart, Bau des Bulbus Olfactorius. Zeit. f. Wiss. Zool., Bd. xi. 1861.
73. Deiters, Otto, Untersuchungen ü. Gehirn u. Rückenmark. 1865.

74. JOLY, F., Zeit. f. Wiss. Zool. 1867.

75. KOSCHENIKOFF, Axencylinder-Vorsatz der Nervenzellen im Kleingehirn des Kalbes, Archiv f. Mikroscopische Anatomie, Bd. v. 1869.

76. HENLE, und MERKEL, Ü. die Bindesubstanz des Nervensystems. Zeit. f. Rationelle Medicin, Bd. xxxiv. 1870.

77. HADLICH, HEINRICH, Untersuchungen ü. das Kleingehirn. Archiv f. Mikroscopische Anat., Bd. vi. 1870.

78. OBERSTEINER, HEINRICH, Bau des Kleingehirns. Sitzb. der Wiener Akad., Bd. lx., Abth. ii., 1870; und Bd. lxi., Abth. i., 1870.

79. KOLLMANN, Sitzber. der Math. Phys. Klasse der Kais, Akad. der Wissench. München. 1872.

80. WEBE, MICH., Sitzber. der Akad. der Wiss. München. 1872.

81. MEYNERT, THEODOR, Stricker's Manual of Human and Comparative Histology, Sydenham Society, vol. ii. 1872.

82. DOHRN, Anton, der Ursprung der Wirbelthiere, &c. 1875.

83. SANKEY, H. R. D., Cerebellum, Quarterly Jour. of Microscopical Science, vol. xvi. 1876.

84. LEWISS, BEVAN, Lymph Spaces in Brain. Proc. Roy. Soc. 1877.

EXPLANATION OF PLATES.

The following letters have the same signification throughout:—

aq. Sy.	Aqueduct of Sylvius.
a.v.c.	Ala of the valve of the cerebellum.
c.a.	Commissura ansulata.
c.c.	Crura cerebri.
c.ca.	Central canal of the spinal cord.
cbl.	Cerebellum.
c.cbl.	Crura cerebelli.
ce.	Cerebral lobes.
c.l.c.	Central longitudinal column.
c.l.o.	Crura lobi optici.
c.pr.	Commissura profunda.
d.c.	Dorsal commissure of spinal cord.
d.h.	Dorsal horn of grey matter.
d.r.	Dorsal roots of spinal nerves.
ep.	Ependyma.
fo.	Fornix.

g.c.s. Ganglion cells of the substantia gelatinosa centralis.
gn.ab. Ganglion of the nervus abducens.
gn.mo. Ganglion of nervus motor oculi.
gn.tf. Ganglion of the trifacial.
gn.th. Ganglion of the thalamencephalon.
gn.v. Ganglion of the vagus.
hy. Hypoarium.
in. Infundibulum.
l.c. Lateral columns of the cord.
l.c. 5. Lower column of the trifacial.
l.op. Lobi optici.
m. Medulla.
m.f. Mauthner's fibres.
n. 1. Olfactory nerve.
n. 2. Optic nerve.
n. 3. Motores oculorum.
n. 4. Trochleares.
n. 5. Trifacial.
n. 6. Abducens.
n. 7. Facial.
n. 8. Acusticus.
n. 9. Glossopharyngeal.
n. 10. Vagus.
n. 11. First spinal nerve.
n.r. Nucleus rotundus.
ol. Olfactory lobe.
op.tr. Optic tract.
p.c. Posterior commissure.
pi. Pituitary body.
pr.c.t. Processus e cerebello ad testes.
p.m. Pia mater.
r.c. Restiform column.
s.g.c. Substantia gelatinosa centralis.
s.r. Fourth ventricle or sinus rhomboidalis.
s.v. saccus vasculosus.
t. Tectum lobi optici.
t.c.cbl. Transverse commissure of the cerebellum.
t.s. Torus semicircularis.
t.v. Tuberosity of the vagus.
u.c. 5. Upper column of the trifacial.
v.c. Valvula cerebelli.

v.h.	Ventricle of the hypoarium.
v.h.g.	Ventral horn of grey matter with ganglion cells.
v.l.c.	Ventral longitudinal column.
v.op.l.	Ventricle of the optic lobe.
v.pi.	Ventricle of the pituitary body.
v.r.	Ventral root of the spinal nerve.
v.t.c.	Ventral transverse commissures of the medulla.
v.th.	Third ventricle.
v.v.c.	Ventricle of the valvula cerebelli.

Except when otherwise stated, all the illustrations are taken from specimens of Mugil cephalus.

Fig. 1, Pl. 58. Longitudinal and vertical section through the brain of Grey Mullet (*Mugil cephalus*) made close to the mid-line. × 20.

Fig. 2, Pl. 58. Horizontal section through the brain of the Grey Mullet (*Mugil cephalus*) made on the level of the third ventricle. × 20.

Fig. 3, Pl. 59. Transverse section through the anterior end of the optic lobes, showing third ventricle. Infundibulum and pituitary body.

Fig. 4, Pl. 60. Transverse section through the optic lobe, valvula cerebelli and torus semicircularis, and anterior end of the aqueduct of Sylvius.

Fig. 5, Pl. 61. Transverse section through the origin of the nervus motor oculi and the commissura ansulata.

Fig. 6, Pl. 62. Transverse section through the trochlearis nerve. This section was composed from the inspection of several consecutive sections. It goes through the anterior end of the cerebellum and the posterior extremity of the optic lobe.

Fig. 7, Pl. 63. Transverse section through the anterior root of the trifacial.

Fig. 8, Pl. 64. Transverse section through the posterior end of the trifacial nerve, through the acusticus and through the abducens. This is taken from two or three contiguous sections of the same subject.

Fig. 9, Pl. 65. Transverse section through the narrow part of the fourth ventricle behind the crura cerebelli, through the anterior part of the ganglion of the vagus, and through the ganglion of the abducens. The fibres which eventually form the commissura ansulata first make their appearance in this section.

Fig. 10, Pl. 59. Transverse section through the posterior part of the fourth ventricle and the tuberosity and ganglion of the vagus.

Fig. 11, Pl. 60. Transverse section through the spinal cord immediately behind the fourth ventricle and through the posterior end of the ganglion of the vagus.

Fig. 12, Pl. 62. Transverse section through the spinal cord at the point where the dorsal root of the first spinal nerve emerges.

Fig. 13, Pl. 62. Transverse section through the spinal cord showing both dorsal and ventral roots of the first spinal nerve.

Fig. 14, Pl. 61. Transverse section through the spinal cord.

All sections from figs. 3 to 14, inclusive, are magnified 43 diameters.

Fig. 15, Pl. 63. Cells from the lobi olfactorii.

a and *b*. Cells from the central group.
c. A cell from the outer edge between the coarse neuroglia and the outer layer.
d. Cell-like swelling from the outer layer of fibres.
e.f. Tripolar cells from the inner edge of the outer layer of fibres. × 945.

Fig. 16, Pl. 63. *a* and *b*. Smaller cells near the external surface of the cerebral lobes. *c*. Cells from the central portion. *d*. A cell from the corona radiata. × 945.

Fig. 17, Pl. 63. Section through the superficial part of the cerebral lobes; showing the external layer of epithelium and the smaller cells.

Fig. 18, Pl. 64. Transverse section through the tectum lobi optici. × 170.

a. An entering bundle of the crura lobi optici.
1. External layer of granular matter.
2. Obliquely directed coarse fibres with fusiform cells.
3. Striated layer.
4. Layer of inner obliquely-directed fibres.
5. Transverse fibres of the crura lobi optici.
6. Layer of minute cells.
7. Ependyma.

b. The most internal cells of the sixth layer, with the connective tissue attached.
c. Deeper cells of the same layer.
d. Fusiform cells of the second layer, with neuroglia attached.

Fig. 19, Pl. 64. Cells from the Fornix. × 945.

a. Towards the proximal end.
b. Towards the distal extremity.

Fig. 20, Pl. 64. *a*. Longitudinal section of the torus semicircularis of the Basse (*Labrax Lupus*). × 170.

b. The superficial cells magnified. × 945.
c. The larger cells, resembling those in the spinal cord.
d. Sections of the crura lobi optici.

Fig. 21, Pl. 65. Cells from the hypoaria. All × 945.

a. A cell from the general parenchyma.

b. A group of cells from the neighbourhood of the ventricle of the hypoarium.

c. Cells from beneath the epithelial lining of the infundibulum.

d. Larger cells from the ventricle of the pituitary body.

Fig. 22, Pl. 65. *a.* Section of the nucleus rotundus of the hypoarium of *Crenilabrus.*

a′a′. Placed opposite capillaries. × 61.

b. One of the bodies in the nucleus rotundus of same specimen. × 565.

b′b′. Placed opposite capillaries.

Fig. 23, Pls. 61 and 64. Longitudinal section of the cerebellum of the Grey Mullet. × 170, showing three of the layers which are found in that body.

Fig. 23. *a.* External layer of straight fibres.

b. Second layer, consisting of cells of Purkinje.

c. Third layer, consisting of minute cells.

d. A Purkinje cell. × 565.

e. Two minute cells from the third layer. × 945.

Fig. 24, Pl. 65. Longitudinal and vertical section through the brain of *Mugil,* taken more externally than fig. 1. Showing the commissures between the cerebellum, the optic lobe, and the hypoarium.

XXIII. *On the Determination of the Constants of the Cup Anemometer by Experiments with a Whirling Machine.*

By T. R. Robinson, *D.D., F.R.S., &c.*

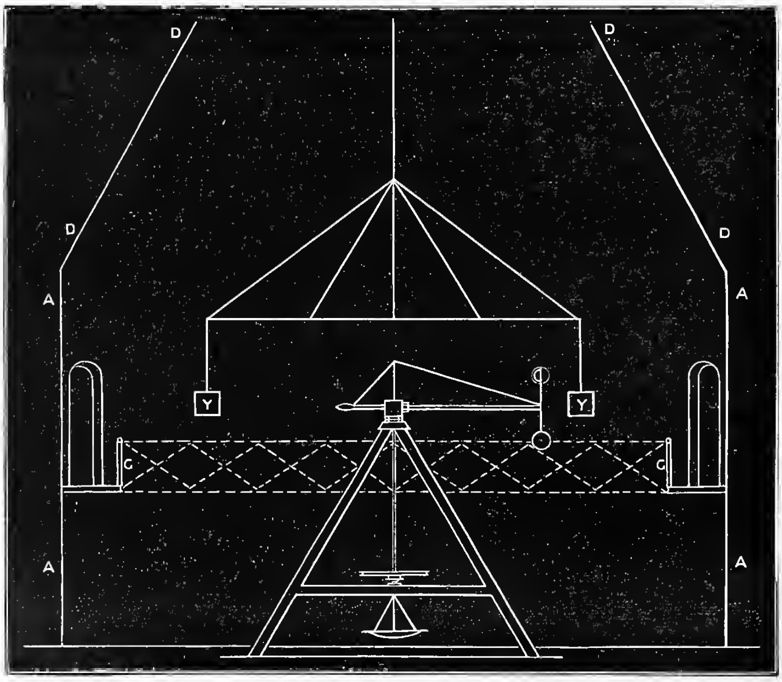
Received March 14—Read April 4, 1878.

[Plates 66–70.]

In a communication to the Royal Irish Academy (Proceedings of the Royal Irish Academy, December, 1875), in which I examined the experiments made by M. Dohrandt to determine the constants of the cup anemometer by means of Robins' whirling machine, I expressed a hope that I might have an opportunity of making similar experiments free from some influence which seemed to me objectionable. This opportunity was soon afforded me by a liberal grant from the Donation Fund of the Royal Society. I lost no time in having the necessary apparatus constructed by Mr. Howard Grubb, of Dublin, and I now propose to give the results which I have obtained with it.

(1.) I was at first embarrassed by the difficulty of finding a suitable locality for the machine. In the St. Petersburg experiments the anemometers passed in their rotation at unequal distances from the walls of the building, and were too near its roof, which seemed likely to introduce extra disturbances into the air vortex which is formed by the whirl of the machine, and which, as will be seen, is a notable element of uncertainty in these investigations. But this was removed by Mr. H. Grubb placing at my disposal the central dome of his works at Rathmines, then recently constructed for mounting the great equatorial which he is making for the Vienna Observatory. Its basis is a regular dodecagon, 42 feet least diameter, and 42 feet from its floor to the summit of the dome. Round it, at 10 feet from the ground, runs a gallery 2 feet wide, with a light iron railing, from which several doors give access to workshops in which polishing and other light work is executed. On the ground, open arches communicate with shops, in which large lathes and other heavy tools are established. This seemed all that I could wish, but, unfortunately, I could have the use of it only till the end of June, when it was wanted for the erection of the equatorial. There were, however, two inconveniences to be feared. One was that the open arches might admit irregular currents of air; the other was that when heavy turning or planing was going on, tremors were produced in the ground which might be expected to make the friction of the apparatus vary. I supposed, however, that this disturbance would equally affect the motion of the anemometers and the measures of their frictions; but this influence proved to be of far greater importance than I had anticipated. On the other hand, the vicinity of skilled workmen, ready to make any necessary repairs or any alteration suggested by my experiments, was of great importance, and not less so the help which Mr. GRUBB and his assistant, Mr. VEREKER, gave me during the whole of this work.

(2.) The vertical axle of the machine was placed in the centre of the polygon, supported by a wooden pyramidal framing. It is an iron tube $3\frac{1}{4}$ inches diameter and 10·4 feet long. It turns in a collar above and a perforated step below, supported by a transverse piece of the frame. On its top is fixed an iron box, 10 inches square and 12 inches high, open at two opposite sides. The third side supports, on a bar 40 inches long, a counterpoise disc of lead, 40 lb. weight, set edgewise. The fourth carries a horizontal arm of sheet steel rivetted together, of the section shown in Plate 67, fig. 1; at the box $5\frac{1}{4}$ inches by 3 inches; at its outer extremity 2 inches by $\frac{6}{8}$ths of an inch. In this turns the axle of the anemometer between three friction wheels at each end. These were at first of bronze, 1 inch diameter, but on May 22 the outer three were changed for others of hard steel $1\frac{1}{16}$ inch. On the top of the box is an upright, bearing a steel wire attached on one side to the counterpoise, on the other to the arm to prevent its flexure. The box also contains a brake apparatus (described at paragraph 21) and provision against centrifugal friction. The vertical axis carries near its bottom a driving pulley 24 inches in diameter, and both it and the horizontal axis are connected by circuit interruptors with a chronograph.



The centre of the horizontal axis is 14 feet from the floor, and the plane of the anemometer's arms is 9·0771 feet from the centre of rotation. Hence it follows that the cups of the instruments are 12 feet from the walls, 10 feet from the railing of the gallery, and, in the lowest case, 12 feet from the floor. The annexed sketch will show the disposition of the apparatus (except the driving machinery). A, A is the inner wall of the building; D, D a portion of the dome; G, G the gallery; Y, Y the vanes of the apparatus for measuring the air vortex subsequently described in paragraph 13.

(3.) The plan of having the plane of the anemometer's rotation vertical instead of horizontal, is liable to three objections, which will be noticed hereafter, but I was induced to adopt it on two accounts. Had the brake apparatus been at the outer end of the horizontal arm, the centrifugal friction would have been twice as great, and the action of the brake less certain; and secondly, the cylindric zone of disturbed air in which the cups move is little thicker than their diameter, while in the other case it would be nearly five feet across, and as the velocity of the air vortex decreases outwards, the uncertainty of its influence would be greatly augmented.

(4.) From the above we find that calling N the number of revolutions of the vertical axis in S seconds, the velocity of the anemometer's centre $V=\frac{N}{S}\times L^{-1}$ (1·58979) in miles per hour. The anemometers used had arms 2 feet and 1 foot long respectively. Calling A the turns in S, $v=\frac{A}{S}\times L^{-1}$ (0·93288) is the velocity of the centres of the cups in miles per hour; and half this for the shorter arms.

(5.) The revolutions of the vertical shaft and the anemometer are recorded on an electric chronograph a little modified from KRILLE's construction; which was placed in one of the rooms opening into the gallery. It has three tracing points, of which the left records N by transverse marks on the helical line which it traces on the barrel of the chronograph as it is moved uniformly by a screw. This line is drawn on peculiar paper enamelled with some white composition, which is coated over with black. The point cutting through the black exposes a white line beautifully distinct on the dark ground.* The right hand point records A similarly. The middle point was connected with Mr. H. GRUBB's regulator, and recorded S by interruptions of the helix, so that the three elements of the experiment were closely contiguous. The motion of the barrel was regulated by a governor clock, such as Mr. GRUBB applies to his equatorials, and its accuracy is very great, as is shown by the line of seconds; so great, indeed, that before long I dispensed with the seconds record, each revolution of the barrel being *exactly* a minute. There was attached, of course, a commutator, which brings each point separately into action.

(6.) The driving apparatus is carried by a vertical frame resting on the ground and attached at top to the gallery; it is an application of HUYGHEN's maintaining power, and shown in fig. 2. A B is one of the top bars of this frame; it carries the driving pulley D and the free pulleys I, C, G, H, F, and K. The endless rope ($\frac{1}{2}$ inch diameter) passes from D round these. I and K are set obliquely to the plane of the

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* This paper is supplied of excellent quality by Messrs. DE LA RUE.

others, so that the rope after leaving them and crossing, goes round the driving pulley on the vertical shaft. On the axle of D is fixed a large drum, E, which gives motion to it, being driven by a band from a PORTER'S wheel. These pulleys are 15 inches diameter, and their spindles $\frac{5}{8}$ths of an inch. To G and H weights of 60 lb. are hung to maintain the requisite tension on the driving pulleys. If an additional weight, D, be hung to G, and the drum turned so as to raise it, it is obvious that when G descends the rope C I will be drawn up, F K drawn down, and H up; therefore the vertical shaft will be turned with a uniform speed by a force half D (neglecting friction) as long as G remains suspended. It works freely. When the anemometer was removed from the end of the horizontal arm, V=52 miles with D=20 lb. I made, however, a great mistake here, from motives of economy, in using manual labour as the motor of this apparatus, instead of adopting the suggestion of Mr. GRUBB of constructing an additional piece, by which the power of his steam engine could have been applied to drive it with any required speed. It would have been far less expensive than the labour proved to be; and I could have carried my experiments to a far higher velocity. When D exceeded 30 lb., two men were required at the wheel; and in one instance, where it was 105 lb., four men were required, and with difficulty could go beyond three minutes of the exertion.

(7.) Opposite the driving frame is a platform which gives easy access to the anemometer for measuring its friction, and cleaning or oiling the various bearings. It is a trapezium 10 feet long, the longer end of which is 8 feet long and is hinged to the platform of the gallery; the shorter end, which is 2 feet long, reaches towards the anemometer. When in use it is supported by a strut resting on the ground; at other times it hangs vertically, and its influence to disturb the air vortex was found to be insensible.

(8.) Five anemometers were experimented on. No. I. is of the Kew type; its cups 9 inches diameter, its arms 24 inches long; but instead of being of thin plates moving edgewise, it was necessary to make them of steel tube 0·5 inch external diameter, as stays could not be applied to oppose the centrifugal force, and even these were scarcely strong enough.[*] Its weight =110·87 oz. In No. II. the cups are 4 inches diameter, and the arms 24 inches; the weight =72·5 oz. No. III. has cups 9 inches and arms 12 inches; weight =68·25 oz. No. IV. has cups 4 inches and arms 12 inches; weight =40·5 oz. No. V. was tried as a matter of curiosity; its cups were semicylinders with mouths 9 inches square, the planes which closed their ends set parallel to the

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[*] The equation (C) given when treating of centrifugal friction enables us to compute the outward throw due to this cause. One of the 9-inch cups weighs 12 oz., and its arm 15·7 oz. At the highest V which I obtained with them =36·29 miles this throw =124·8 oz. acting at their C G by a leverage =17·2 Now we found that 80 oz. acting on the arm by leverage =19·5 inches deflected the centres of the cups outwards 0·275 inch. This would make the real V in this case 0·091 mile greater than the estimated one. I did not, however, think it necessary to take this into account, as there are so many other and more important cases of uncertainty.

plane of their rotation; the arms (to the centres of the square mouths) 24 inches, and the weight $=129$ oz.

The other parts (beside the anemometers) which move with the horizontal shaft weigh 178·82 oz.

(9.) The method followed in experimenting was this. The machine was put in motion with as small a D as would make the anemometer revolve; when this had attained as uniform a motion as could be judged of by the eye, the electric circuit was completed by the commutator, and the action was continued generally for four minutes. During this time a person watched the chronograph to guard against its failing to record, which sometimes happened from oxidation of the contact maker, or from the points becoming blunt.* I made the observations for vortex motion, to be soon described, and Mr. VEREKER kept watch over the labourers.

When this observation was finished, a larger D was applied to give increased velocity, and so up to 76 lb., which I only passed in one instance. Even this was very severe on the men; the more so as the temperature of the place was often as high as 70°.

(10.) When this series was completed a weight was hung on the brake, and a new one taken. On account of the increased friction the anemometer would not move with the first D of the preceding series, and I adopted the plan of keeping the V's nearly equal. I laid down on a card the average length of the mark made by the point which records N in each experiment of the first series; and then set N alone to act; I altered D till its trace was nearly as long as the corresponding one on the card. Then the A point was made to record; and thus another series was completed. This was continued till the load on the brake was as large as could be used with safety to the levers in the case of No. 1; and with the others such as would admit of a sufficient value of v. The chronograph sheets, which are carefully preserved, were tabulated independently by two persons.

(11.) Before proceeding to discuss these observations, it is desirable to consider the conditions which determine the amount of an anemometer's motion. This obviously depends on the impelling force of the wind and the various resistances which oppose it. These resistances can only arise from the action of the anemometer itself on the air, and the friction of its parts; and therefore V, the velocity of the wind, is a function of v and F. The nature of this function cannot be determined *à priori* in the present state of hydrodynamics; but a general conception of its form is easily obtained. Suppose the wind makes an angle θ with the mouth of a cup (or the arm which carries it) which is revolving with such angular velocity that the centres of the cups move with the velocity v, I have shown in the paper already referred to that the velocity R with which the wind is incident on the cup $=\sqrt{V^2+v^2\mp 2Vv\sin\theta}$, the negative sign belonging to the first semicircle. The pressure of this to turn the cup$=R^2\times a_1$, a_1

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* These points should have been diamonds, as in the chronograph at the Armagh Observatory, but they could not be immediately procured.

being the pressure of an unit of wind on the cup estimated in a direction normal to the arm. But as the arm carries on the other side of the anemometer's axis another equal cup equidistant and in a reversed position, the action of the wind on its convex surface will oppose the motion with the pressure $R'^2a'_1$. The actual force therefore $=a_1(V^2+v^2-2Vv\sin\theta)-a'_1(V^2+v^2+2Vv\sin\theta)=(a_1-a'_1)(V^2+v^2)-2Vv\sin\theta(a_1+a'_1)$. This force is opposed first by the moment of friction at the centres of the cups, secondly by the resistances depending on v^2, of which the chief are the resistance to the arms and the reaction of the cups on the air which forms an air vortex in the plane of the anemometer. When these opposing forces balance each other through a revolution, the angular velocity of the instrument must be constant (except for small periodical fluctuations, the effect of which is much lessened by the moment of inertia of the arms and cups; still more so if four cups are employed). If we could find the mean values in a revolution of a_1; a'_1; $a_1 \sin\theta$; $a'_1 \sin\theta$, we could express this state of permanent motion by the equation $\alpha V^2-2\beta Vv-\gamma v^2-F=o$ (I.), in which F can be obtained by measurement. a_1 and a'_1 must be functions of the angles Ψ and Ψ' which the resultants R and R^1 make with the arm that carries the two cups, and which are given by the equation $\sin\Psi=\frac{V\sin\theta\mp v}{R \text{ or } R'}$.

(12.) But even in the case where the cups are at rest and $v=o$, we do not see our way to a determination of these functions. In the case of the concave surface, one would naturally suppose that when $\theta=180°$, the wind being parallel to the mouth of the cup can exert no pressure on it; but so far is this from being the case that a single cup will only be in equilibrio 30° beyond this position, notwithstanding the pressure on the convex surface. I cannot say how it behaves at $\theta=o$, for then the equilibrium is unstable and the cup gets into rotation. Yet more; the centre of the wind's pressure on the concave varies with θ; before 90° it is within the centre, after it outside; and its place depends on the deflection of the air stream-lines, the law of which is unknown. Equally uncertain is a'_1; but it is evidently a different function. It acts through the entire circumference; the surface which the convex exposes to the wind varies according to a different law, and the deflection of the stream-lines on it is of an entirely different character. I thought it possible that eddies might introduce terms depending on the first powers of V and v, but it will hereafter be shown that this is not sensibly the case; though the expression for Ψ implies that v should lessen α by diminishing the arc of θ through which R is effective. But as a_1 must be small at the beginning and end of the semicircle, it is possible that this influence is not important. These considerations, I think, justify me in believing the equation (I.) to be a close approximation to the general conditions of anemometer motion.

If we had a series of observations in which V', v, and F were accurately known, we could determine by minimum squares the three coefficients α, β, and γ. If on applying these to the successive values of v and F, we reproduce the values of V', the correctness of (I.) is established; if not, the march of the errors may enable us to trace whether they be casual or depend on extra powers of v; and I entertained such a hope when I entered on the experiments, but it has been but very imperfectly realised.

(13.) In the first place, we have instead of real wind the transport of the anemometer with the velocity V through air which is not quiescent, but moving in the same direction with a velocity W. Therefore we must use instead of V, $V'=V-W$. Here are two elements of uncertainty. It is not certain that a body moving through a fluid even if this be quiescent, is equally resisted with one at rest sustaining the impulse of a current fairly uniform, much less so if the fluid be in a state of disturbance; and secondly, though V and v are given with sufficient accuracy by the chronograph* it is otherwise with W. We cannot measure it in the actual track of the anemometer, and must reduce our measures to that track's centre on some hypothesis; while we may be sure that it varies in every part of the circumference described by the cups. But even in the line of its measurement it will be found very irregular and disturbed by powerful eddies; and besides these vorticose motions in the direction of V, there is another Z of very irregular character in a direction normal to this, so that the air moves in spirals not in circles, and instead of $V-W$, we should use $\sqrt{(V-W)^2+Z^2}$, but I see no possible mode of estimating the effect of Z correctly, on account of its intermittent character. WOLTMAN's fly, which was used by M. DOHRANDT to measure W, seemed unknown to our opticians; but at a latter period I was informed by a scientific friend that it was called here an air meter, and he lent me one, which I found useful. But I could scarcely have used such a one for habitual measurement without chronographic registry, and I wished for something that would show the changes of the vortex more evidently. Fig. 3 (Plate 70) shows the method I adopted. A slip of deal, $\frac{1}{4}$ inch square and 23 feet long, was suspended by a fine thread from the summit of the dome; to prevent bending, it was braced by other threads fastened 4 feet above it; to it were suspended two of those thin caoutchouc balloons which are sold for playthings for children. They were about 8 inches diameter and a little higher, and hung 4 feet below the rod, their centres on a level with that of the anemometer, and 14 inches outside it. Threads connecting them with the ends of the rod prevented their being drawn in among the cups. From this it will be seen

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* A second of time measures on the chronograph 0·208. Now the highest value which I obtained for N was one revolution in a second, for v 1·67; so that even in this last case $\frac{1}{10}$th of a unit of $A=0·012$ inch, a quantity quite visible. It must, however, be remarked that my V belongs only to the cup whose arm is perpendicular to the horizon, whose $\theta=90°$. For any other θ it $=V(1+\frac{1}{2}\frac{r^2}{R^2}\times\cos^2\theta)$. For Nos. I. and II. this is $1+0·243\times\cos^2\theta$. At its maximum the addition is $\frac{1}{41}$ of the whole; in its mean value through the semicircle half this. But this increase of V is counteracted by its obliquity to the plane of the anemometer motion, which lessens its power to turn the cups The tangent of this $=\frac{r}{R}\cos\theta$; it$=12°\ 26'$ at maximum. If we knew the forms of a and a' we could compute the effect of this obliquity; but as it is, we can only say that both these disturbances are greatest when a and a' are least, so that probably their influence may be neglected.

that the free part of the thread is 23 feet long, and its torsion force is insensible in these experiments; after being twisted during several series when left to itself it resumed its original position so slowly that ten minutes were required for about a third of a revolution. The lightness of the balloons and their large surface make them obedient to very slight currents, and extremely sensitive to any irregularity in its motion. The process of observing W is this: When the experiment has lasted for about a minute, the time b of two revolutions of one of the balloons was noted by a stop-watch (by ROBERT, of Paris) whose beats are $0\cdot2^{s}$. From the radius of the circle described by them, we find that W in miles per hour$=L^{-1}\frac{(1\cdot94335)}{b}\times$reduction to cross. This reduction is made on the assumption that the velocity in any part of the vortex is inversely as the square of the distance from the centre. This agrees as well with experiment as under the circumstances could be expected. Thus on one occasion, at the normal distance $b=83^{s}$, when the distance was increased to 30 inches it became $104\cdot6^{s}$. The inverse square gives $106\cdot2^{s}$. Hence the factor for reduction $=L^{-1}(0\cdot10586)$. These balloons were very perishable; drops of oil were projected occasionally from the bearings at high speeds, which perforated them like shot, and by the commencement of June we had exhausted the stock in the neighbouring shops, and most of those in Dublin. They were then replaced (June 7th) by vanes of tissue paper 1 foot square, stiffened at their edges by rods of deal 0·125 inch thick, and suspended from the rod by slips of deal 0·5 inch broad, 0·062 inch thick, and 4 feet long. These moved edgewise and were kept steady by lateral slight trusses of the same material and scantling. As the vortex reaches far above the rod these suspensions will not sensibly retard the vanes. Their centres are 20·25 inches from the cross, but they give b 0·003 smaller than the balloons. Their reduction to the cross $=L^{-1}(0\cdot14810)$.

(14.) This apparatus shows that the vortex current is very irregular: the balloons sometimes dart rapidly forwards, then move more slowly, and this not only when the anemometer is passing them; sometimes they are violently projected upwards and outwards, showing that in addition to the general translation of the air, it is affected by both horizontal and vertical eddies of considerable power and possibly magnitude. The vanes can only show irregularities in the circular motion, but fully confirm its lawlessness. Under such circumstances we can have no exact knowledge of the W which exists *at the cups*; the balloons give its mean value through the circumference, but it is probably greatest immediately after the cups have passed, and decreases by the eddies and the friction of the walls till they again meet it. Any fixed meter would yield the same result, and the only proper plan would be one capable of giving the value of W at the cups for every instant. A pressure gauge carried with the cross and connected with a chronograph promises best; but in air so agitated the relation between the velocity and pressure would be very doubtful.

(15.) The horizontal arm has little effect in producing the vortex; in the case already mentioned where it was whirled for several minutes without the anemometer, scarcely any motion of the balloons was observed though $V=52$. Nor does the anemometer's own rotation seem to have more effect, for $\frac{V}{W}$ does not change with a given value of V though v be reduced by brake friction to half its original amount; it decreases, though slowly, with V. Out of many comparisons, those given in the following table may suffice :—

TABLE I.

D=75 Brake.	V	$\frac{V}{W}$	D	Mean V.	Mean W.	$\frac{V}{W}$	No. of Observations.
0	29·81	12·79	75	29·53	2·213	13·34	8
6	29·49	12·66	65	27·15	2·128	12·75	8
12	29·81	12·20	57	25·40	2·016	12·60	8
18	29·81	12·71	46	21·66	1·825	11·87	7
24	29·97	13·18	39	19·71	1·689	11·67	7
30	29·17	12·64	28	15·19	1·370	11·08	7
36	29·25	12·96	23	12·57	1·296	9·67	5
			20	10·40	1·039	10·01	4
			18	8·705	0·923	9·43	4

The three first columns belong to those single observations of the second set of No. I., in which the driving weight was 75, and allowing for the irregularities which I have described, may be considered to show that $\frac{V}{W}$ is constant. The remaining columns relate to all in the first set of No. I., where the driving power is the same without regard to brake friction; here the decrease of the ratio is obvious, though it might not have been evident from single observations. For instance, the first of the eight ratios belonging to D=75 is 11·89, the fifth is 14·30.

(16.) I obtained some additional information about the vortex by the air meter already mentioned. It is a small windmill of eight vanes (five would have been better), 2·8 inches diameter, connected by a stop and wheelwork with a series of indices registering its revolutions to a million. It is very sensitive, though less so than the balloons or vanes. A revolution is stated to be a foot; but there was in its case a table of corrections which seemed to me so strange that I did not use them. Attaching it to the horizontal arm without the anemometer, and whirling for 8^m with $V=17$, I found a division $=0·867$ foot. On placing the meter on the platform with the necessary precautions, with its axis of rotation in the plane of the arm and in the direction of W at 14 inches from the cross, I got in 2^m 212 divisions with $D=35$. This on the above scale is 1·53 foot per second, the balloons at the same time giving 1·57 feet. It is, however, needless to turn the divisions into feet, as they serve for comparison. When the meter was placed to measure the radial current it gave 70—a third of the other—but with this unexpected peculiarity: that during the two minutes it sometimes moved very rapidly and sometimes almost stopped. In another trial with the same D at 28 inches distance the tangential velocity=162, the radial 60; with D=22, distance=20 inches, tangential 122, radial=82 with the same irregularity. It follows from this that the vortex motion is not, as I had supposed, in a regular spiral, but in a track of very complicated character, which it is not practicable to define. At the same time I tried to verify with this instrument the law of inverse squares, which cannot easily be done with the balloons, as it was very troublesome to shift them. A scale was fixed on the platform, so that the meter could be exactly placed at any distance from the cross. Four distances were taken for each value of D, and the machine was whirled for 5^m before beginning to measure, and kept in motion till the four were finished. The irregularities are considerable, partly owing to the small size of the vanes, for even in real wind the current is far from uniform, narrow veins of great intensity being contiguous to others comparatively feeble. The results are given below with their reduction to the cross by the factor $\frac{(108+d)^2}{(108)^2}$.

TABLE II.

D=25, V=15·55.				D=30, V=17·82.			D=35, V=19·12.		
Dist.	Meter.	Reduced.	Do. −M.	Meter.	Reduced.	Do −M.	Meter.	Reduced.	Do. −M.
12·25	233·0	238·9	+1·1	274·0	339·7	−34·6	291·0	360·7	−1·4
20·25	−38·5 194·5	274·3	−13·5	−57·5 216·5	307·3	+2·2	−40·5 250·5	353·2	−9·9
28·25	−15·0 179·5	285·7	−2·1	−24·5 192·0	305·6	+0·5	−1·5 243·0	386·8	+24·7
36·25	−10·0 169·5	302·4	+14·6	−42·5 149·5	267·9	−37·2	−48·0 195·0	347·9	−14·2
		287·8	0·00		305·1	−17·3		362·1	−0·2

The irregularities of the differences show that the errors of the formula are merely casual.

(17.) Another question occurred to me: Does W go on increasing for any considerable length of time? To ascertain this a set were recorded during ten successive minutes as soon as the anemometer got into full motion; the *b*'s are separated by the time of one revolution; the circumstances were very favourable, and the results are a good specimen.

TABLE III.

No.	N.	A.	*b.*	V.	*v.*	W.
I.	28	45	78·8	18·15	6·43	1·421
II.	28	46		18·15	6·57	
III.	28·17	45·5	71·4	18·15	6·50	1·566
IV.	28·17	45·25	66·4	18·26	6·46	1·689
V.	28	45		18·15	6·43	
VI.	28	45	66·4	18·15	6·43	1·689
VII.	28	44·5		18·15	6·36	
VIII.	28	45	68·2	18·15	6·43	1·643
IX.	27·5	44·8	66·4	17·82	6·40	1·689
X.	27·5	44		17·82	6·28	

W becomes sensibly constant after four minutes, and this time was allowed in subsequent experiments.

(18.) On the whole, the estimation of W is a weak, perhaps the weakest, part of this investigation; and what I have observed respecting this air vortex makes me doubt whether the conclusions deduced from such experiments can be fully applicable to real wind. Still, it may be hoped that they will give useful information.

(19.) Friction is an important agent in anemometric motion, and one in which I found more difficulty in the measuring than I had anticipated. It was fourfold: (1°) that due to the instrument's weight; (2°) that caused by the application of the brake; (3°) that caused by the lateral pressure of the axle on the upper bearing, caused partly by the action of the wind, partly by the force which, as in the gyroscope, is required to change the plane of the anemometer's motion; and (4°) that due to centrifugal force. There are three degrees of this resistance; the highest is that which occurs when a machine is started from rest; the intermediate is that which acts during continuous motion, which is the one to be considered here, and the least which prevails when the parts that rub are put into a state of vibration by tapping or jarring, which may be sometimes only half the first. Friction occasionally varies from the effect of temperature or chemical change on the oil used to lubricate the bearings, or else of dust finding its way into them (as sometimes happened).

(20.) Nos. 1 and 2 were measured thus. A cast-iron disc, weighing 103·17 oz., was fixed on the shaft in place of the anemometer; on its circumference a groove was cut whose bottom was 1·0 foot diameter, in which a fine silk thread was wound; to this, weights were appended till when the disc was barely moved they continued the motion. This weight, divided by four, is considered to represent the moment of friction at the centre of the cups after a correction for the increase of friction due to its own pressure.

1. The normal friction f is larger than in ordinary instruments on account of the long shaft, its horizontal position, and the weight of the brake apparatus :[*] with the disc in place the moving parts weigh 282 oz. The measurements at Rathmines gave values from 100 to 120 grains; but I adopt one which I obtained at Armagh by a better process: that used by ATWOOD in determining the constants of his celebrated machine. If a weight W be hung to the disc, $W-f$ accelerates a mass $W+M$; M being the moment of inertia of the moving parts reduced to the circumference of the disc, and we have by the law of uniformly accelerated motion $g\times\left(\frac{W-f}{W+M}\right)=\frac{2S}{T^2}$, S being the space through which W descends in the time T. We get S most easily by taking a given number of the disc's revolutions. M was got by suspending the disc and its appendages in the sector tower of the Observatory with a bifilar suspension whose lines were 62·5 inches long and 3·01 inches apart. A graduation fixed on the disc enabled me to note the arcs of vibration; 300 complete vibrations were taken for each trial, and by the usual formula x^2 the square of the distance of the centre of gyration was found. $\frac{\text{This} \times W}{36''}$ gave by ten sets $M=25{,}773·6$ grains. Hence we get f far more consistently than in the former method.[†] I obtained 72 observations with W from $\frac{5}{8}$th of an oz. to $1\frac{1}{4}$ oz., and divided them into three groups to see if f varied with the velocity. The first 24 gave $f=114·18$; $v=1·33'$. The second gave $f=110·41$; $v=2·13'$. The third $f=108·20$; $v=2·87$. There is a decrease, but I think it is mere error of observation, for the effect of an error in noting the time acts inversely as T^3;[‡] so I take the simple mean$=110·93$. To obtain from this the f for each of the anemometers, 2 lb. were hung on the axle, and ten observations gave

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[*] In its present position with its axes vertical the normal friction is only 22 grains.

[†] As the rotation of the disc is liable to be disturbed at its commencement by any casual irregularity of the friction, it is better to reject this part and take two spaces immediately consecutive. In this case it is easily shown that $g\frac{(W-f)}{W+M}=\frac{2S't-2St'}{tt'\,(t+t')}$, or (if $S'=S$)$=\frac{2S\,(t-t')}{tt'\,(t+t')}$. Both S and t are given by the chronograph, of which, however, I could not then avail myself. This method has over that which I used at Rathmines the advantage of giving the friction during actual motion over a considerable space. If when the descent of W has given a rapid angular motion to the disc it be detached from it, the disc will continue to revolve till stopped by the friction. This is equivalent to supposing $W=0$ in the equation; and if f be so small that it makes twenty or thirty revolutions, it is ascertained with great precision. It is much to be regretted that this process did not occur to me at Rathmines, for it would probably have very much improved my results. Had I then possessed my present knowledge, I would have made an arrangement to give *the mean friction during each experiment*, which could have been effected by a modification of the means (hereafter described) used to measure a alpha.

[‡] Some observations made with the anemometer's axis vertical show that f does not change with v, at least within the limits of 29·6 and 7·1 miles.

increase of $f = 9{\cdot}7$ grains for the added pressure $= 0{\cdot}294$ grains for every ounce, or 4·71 for every pound. Hence for the different anemometers—

No. I.	$f = 113{\cdot}19$.
No. II.	„ 101·27.
No. III.	„ 201·32.
No. IV.	„ 184·98.
No. V.	„ 118·53.

(21.) 2. *Brake friction.*—This is produced by means shown in Plate 70, fig. 4. A is the box mentioned (2), to the bottom of which is screwed the strong brass frame b. In this turns the lever D, with its arbor E. The longer arm of this lever is connected with the piece F, slotted to give room for the shaft and one of its three friction wheels, as shown in the figure, and attached below to the wire W, which passes down through the vertical axis and its perforated step, and carries a scale-dish below. The shorter arm of D is connected by the link G to the right angled lever HH′, whose arbor is at L. If now a weight be placed in the dish it draws down F, and thereby D, and G raises H, makes H′ press the piece M, which turns with the arbor N, and presses the rubber R which it carries against the friction-disc C, six inches in diameter. A duplicate of this mechanism is placed opposite (but not shown, to avoid confusion), and being also actuated by the wire W, presses a similar rubber against the corresponding part of C. K is one of the counterpoises to balance the scale-dish, which, however, was much too heavy, as I had no idea what weight might be required, and it produced unnecessary friction. The rubbers are of stout cloth, 1·25 by 0·8 inch, and show no signs of wear. The leverage of the brake is $\frac{3{\cdot}1}{1} \times \frac{6{\cdot}5}{1{\cdot}7} = 8{\cdot}03$. By this elegant arrangement (for which I am indebted to Mr. Grubb, F.R.S.) the friction, while it can be varied at pleasure, causes neither vertical or lateral pressure on the supports of the shaft. When, therefore, the brake is made to act, it merely adds to f another friction, which can be similarly measured. The measures, however, of this friction were much more discordant than those of f: two differing in one instance 234 grains. This irregularity is probably caused by the tremors of the ground (already referred to) acting on the brake-weight; when thrown upwards it would relax its pressure momentarily, and permit the measuring disc to revolve with a less pull than what really represented the friction.

When I measured this by Atwood's process at Armagh, the results were very regular, but far larger than those obtained at Rathmines, except on one occasion when Mr. H. Grubb took a set very early in the morning before there was work going on. This agreed well with *them*. As, however, all the anemometer experiments were affected by this vibration, I deemed it best to use the Rathmines frictions.

But I found an unexpected fact: that the coefficient of *this* friction is not constant, but decreases with the pressure; this is probably owing to the elastic nature of the rubbers, which becoming condensed by the pressure tend to act like a hard body. On laying down the curve of this coefficient, it looked so like an equilateral hyperbola with coordinates parallel to its asymptotes, that I tried the Armagh values of it by the equation $\frac{\mathrm{F}}{\mathrm{B}}=\frac{y}{\mathrm{B}+u}+x$, B being the load on the brake in ounces -200 grains, the force required to bring the rubbers into contact; each value of $\frac{\mathrm{F}}{\mathrm{B}}$ gives an equation of condition, combining which we get values of the constants. Substituting these in each, we get values of $\frac{\mathrm{F}}{\mathrm{B}}$ whose errors enable us to approximate still more by the equation $\frac{d\mathrm{F}}{\mathrm{B}}=dx+\frac{dy}{\mathrm{B}+u}-\frac{du}{(\mathrm{B}+u)^2}$. Thus we obtain $y=315{\cdot}52$; $x=77{\cdot}814$; $u=6{\cdot}662$. These give when $\mathrm{B}=0$, $\frac{\mathrm{F}}{\mathrm{B}}=125{\cdot}28$; when $\mathrm{B}=\infty$, $77{\cdot}81$; and afford values of F differing in most cases from the observed ones far less than the probable errors of the latter. I have therefore used these values in reducing the observations. The following table gives these: their differences from the observed ones, the number of observations, and $\frac{\mathrm{F}}{\mathrm{B}}$ for each value of B.

TABLE IV.

No.	On Brake.	Computed Friction.	$o-c$.	No. of Observations.	$\frac{\text{Friction}}{\text{B}}$
	oz.				
1	3	284·75	+ 2·3	7	113·03
2	6	574·3	+13·1	6	105·85
3	9	841·7	−18·2	4	96·42
4	12	1098·1	−14·0	7	93·51
5	15	1347·6	+11·2	5	93·45
6	18	1593·5	−15·9	5	89·95
7	24	2026·9	+13·1	4	86·66
8	30	2556·0	− 8·5	5	86·24
9	36	3031·1	+ 7·5	5	85·50

3. A lateral friction is produced by the wind pressing the anemometer's axis against its outer bearing. Omitting the consideration of the arms, this pressure will be the sum of the mean pressures on the cups during a revolution, and the same reasoning as in the case of I. shows that it $=\epsilon\mathrm{V}'^2-2\kappa\mathrm{V}'v+\epsilon v^2$. If, therefore, the constants of I. be determined by the observations, the effect of this friction will merely be to diminish α and β and to increase γ; ϵ and κ are larger than the other, and κ much less than ϵ, so that probably $2\kappa\mathrm{V}'v-\epsilon v^2$ is small in comparison of the first term, and the pressure P is simply $=\epsilon\mathrm{V}'^2$. This is not merely confirmed by these experiments, but a good measure of it is obtained. In these the force which turns the vertical axle is $\frac{1}{2}\mathrm{D}$—the frictional resistance of the driving apparatus: this when the motion has become uniform, acting at the end of the horizontal arm=air's resistance=P at the point of

bearing. Now if we examine Plate 67, fig. 2, it is obvious that the spindle of the pulley G is pressed by the weight $W+D$; that of C by the same $-$friction of G, and so on to I. From this the tensions of the cords GC, CI, and of U, the cord which leaves I for the driving pulley can be determined. The same thing can be done for U′, FK, and HF. Now tension of $U-$tension of $U'-$friction of vertical axle is the force which drives the axle; and if this be worked out, supposing f equal in each pulley, and so small that powers above its square may be neglected, we have $U-U'=\frac{1}{2}D-(2W+D)\times f$. When $D=12$ the axle sometimes moves, but seldom; I think 11 is the limit, and as $2W=120$ we have $f_1=\frac{5\cdot5}{131}$, and therefore $P=\left\{\frac{1}{2}D-(2W+)\times\frac{5\cdot5}{131}\right\}\frac{1\cdot025}{9\cdot078}$, and we can try if $\frac{P}{V'^2}$ is constant. This proves to be the case as is seen in the following table, which gives the results with Nos. I. and II.

TABLE V.

No. I.

D	P	ϵ'.	$P-\epsilon V'^2$.	No. of Observations.
22·3	0·583	0·004278	−0·017	3
28·5	0·903	0·004464	+0·015	4
33·8	1·174	0·004418	+0·004	5
44	1·700	0·004531	+0·048	6
55	2·267	0·004412	+0·006	7
64	2·730	0·004479	+0·047	7
74·3	3·260	0·004380	−0·018	7
105	4·843	0·004239	−0·186	1

No. II.

D	P	ϵ.	$P-\epsilon' V^2$.	No. of Observations.
25·75	0·760	0·001332	−0·051	4
28·8	0·917	0·001370	−0·002	5
35·8	1·277	0·001383	−0·006	6
40	1·507	0·001332	−0·062	6
48	1·906	0·001464	+0·014	7
56	2·318	0·001441	+0·078	1

The agreement is good; for the 9-inch cups $\epsilon=0\cdot004400$; for the 4-inch$=0\cdot001387$; and I think it likely that they would answer well even for real wind.

(22.) The observations for determining α (*vide* par. 27) show that the effect of v may be neglected, as in them $v=0$. I take three from No. I.

D	V′	P	$P-\epsilon V'^2$.
20	9·49	0·464	0·067
29	14·36	0·993	0·047
52	21·50	2·112	0·079

It will be observed that for the smaller cups ϵ is relatively larger than in the proportion of their areas. This is, in great measure, owing to the arms, 0·5 inch diameter and 24 inches long, bearing so much greater proportion to them. From the preceding it follows that $f''=\epsilon V'^2\times 4{\cdot}71$.

As to what I call gyroscopic friction, for want of a better name, the pressure which produces it $=(A-C)(\omega_{,}\ \omega_{,,,})$; A and C being the moments of inertia round the principal axes, and $\omega_{,}\ \omega_{,,,}$ the angular motions round them. C was determined, but it would be rather awkward to get A. As, however, the angular velocities are as V and v, the effect of this friction will be merely to lessen β.

(23.) 4. In the rotation of an anemometer at the end of a revolving arm, the centrifugal force produces an outward pressure which must be resisted by some stop, and produces there a friction f'''. This pressure is given by the formula (C) $P'''=\frac{M'\times G}{g\times R^2}\times V^2$ (not V'^2), where M′ is the weight of the parts which revolve round the axis of the anemometer, G the distance of the centre of gravity of M′ from the vertical axis, and R the length of the horizontal arm. This pressure was in some of these experiments considerable; in No. 133 it was 52 lb.; on another occasion it broke a steel arm of No. 111, $\frac{3}{8}$ths of an inch thick. As the resulting friction varies nearly as V'^2, it might, like f'', have been included in α; but as I hoped that the results of these experiments might be available for real wind (where f''' has no place), I thought best to measure it and add it to f. I determined it thus: A strong upright was fixed in the gallery, and secured by a strut to the platform; to this was fixed a pulley 1·73 inch diameter, as nearly as the eye could judge in a line with the seam of the horizontal arm, which was my nearest guide to the direction of the axis of the shaft. Over this passed a fine iron wire attached to the centre of the shaft, to which weights were suspended which pulled it in the direction of its length. The friction thus produced was measured by weights placed in the cups, and the normal f subtracted from this. The tensions were corrected for the friction of the pulley, which when this line crossed it at right angles was $\frac{1}{20}$th of the load. The only other mode of measuring this friction was the setting the arm vertical with the 12-inch disc below, placing weights on this and proceeding as in the case of f. This, however, would have required the total dismounting of the apparatus and constructing a proper stand, for which time could not be spared.

With $P'''=10$ lb., $F=180$ grains; with $P'''=21$ lb., $F=280$, each a mean of three trials. These give $\frac{F}{P'''}$ in the first instance $=7{\cdot}032$, in the second $=8{\cdot}361$. It seems from this that here the coefficient of friction *increases* with the pressure. I see no mechanical reason for this, but must accept the result, for the difference is clearly marked. The outward pressure was resisted by a single roller bearing on one side of a plate fixed to the shaft ; this must have tended to make the latter press more on its bearings, and as it acted on the plate 1·6 inch from its centre, the friction was needlessly great. Therefore on May 18th it was replaced by two opposite ones, larger, carried by the shaft, and bearing on the back plate of the frame 0·5 inch from the centre of the shaft. This reduced the friction by 0·6 of its first amount. The measures were now repeated. With $P=56$ lb., $F=277·5$ grains ; with $P=70$ lb., $F=392$, each also a mean of three. Here, however, f must not be subtracted ; for with the two rollers the pull raises the shaft sensibly from the outer bearing. These give for the first $\frac{F}{P'''}$ 5·216, for the second 5·895. The simplest mode of representing these is the formula $F=P'''x+P'''^2y$. These give for the first set (which were used till No. 79) $x=5·8232$; $y=0·1272$, and for the second $x=2·5010$; $y=0·0509$.

All the constants of the equation (C) except G have been already given ; I give it here, and with it the factors for P''' for each instrument.

No. I.	$G=5·4605$	$P'''=V^2\times L^{-1}$ (8·56260).
No. II.	5·0232	L^{-1} (8·46338).
No. III.	5·0482	L^{-1} (8·53793).
No. IV.	5·0099	L^{-1} (8·40158).
No. V.	5·5965	L^{-1} (8·60018).

(24.) I suppose this variation of the friction is somehow connected with the rollers, but both Coulomb's experiments and railway experience indicate that rolling friction is simply as the pressure.

(25.) On the whole, I consider that these friction measures, though perhaps not so uncertain as those of W, are yet sufficiently so to increase materially the difficulty of drawing accurate conclusions from these experiments. I expected that increasing m by additional friction would give a wider range to the coefficients of the equations of condition; and this is so. But experience shows that the advantage thus gained is neutralised by uncertainty when the value of v is small, for then the latter is greatly affected by the irregularities of friction to which I have already alluded. This probably arises from the momentum of the apparatus not being sufficient to overcome any casual increase of friction. But it is also possible that as the friction when a body is started from rest is greater than when it is in motion, the passage from one state to the other may be gradual through a certain small range of v.

(26.) Lastly, I shall describe the means which I employed to determine directly the coefficient α of equation (I.) and the results which they gave. The outer end of a strong clock-spring was attached to an arm of the anemometer set horizontal, the inner end to the horizontal arm of the machine : a circle divided to 100 parts was fixed on the shaft and read by an index on the arm. I hoped the tension of this spring would be so nearly as the angle of its torsion that mere reading of the circle would give it; but this was not quite the case, and I had to form a scale for it. The instrument was turned through a quadrant, and held while weights were hung on the outer edge of the cup till they balanced the spring. As the friction caused some uncertainty, the cup was a little raised and allowed to descend gently, and its point of rest read; it was then depressed and allowed to rise, the mean of the two readings being considered the true point of equilibrium. This was repeated for the second quadrant, and so on for three complete revolutions. The numbers so obtained were reduced to the centres of the cups, and corrected for the angular deviation from the horizontal position. The same spring was used for No. II. and No. III. with separate scales; but for No. I. it was necessary to combine two springs. As the equilibrium points were scarcely ever at the quadrants, they were reduced to them by interpolation, and tables formed with first and second differences, which easily gave T the tension in grains corresponding to a given θ.*

(27.) Now things being thus arranged, if the whirling machine be put in action the anemometer will turn and tend the spring till its elastic force balances the pressure $\alpha V'^2$; when this has occurred I place on the brake a weight of 6 lb., which produces a friction far surpassing the tension; the chronograph is then made to record N, generally for three minutes. The machine is then stopped, the brake holds the anemometer immovable, and θ is read off on the circle to a tenth of a division. But the tensions thus obtained are too large: the anemometer's motion is *accelerated* up to the point where $\alpha V'^2 = T + F$; there it has acquired a velocity which carries it on beyond this. The force which brings it back is now $T - \alpha V'^2 - F$, and it will rest when $\alpha V'^2 = T - F$.† Even the α thus obtained must be a trifle too large, for V′ ought to include, as a component, the radial vortex-motion Z. I subjoin the measures which I have taken in the following tables. Of the headings, D, S, N, b, V, W, and f''' have been explained; C is the reading of the circle, T the corresponding tension in grains, θ the angle made by the highest of the cups with the horizon.

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* It seemed unnecessary to give these tables.

† This supposes that the moment due to the final velocity is greater than 2F, which is a minor limit of α. I have used it as giving results nearest to those obtained by minimum squares. If the moment does not exceed F, $\alpha = \frac{T+F}{V'^2}$.

TABLE VI.

No. I. f=113·3 ; two springs ; June 20, Bar. =29·77 ; Therm. =73·5.[*]

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	a.
I.	17	180	31·25	0	6·75	0	5·4	16·5	564	59°·4	9·749
II.	18	,,	39·25	0	8·48	0	10·2	21·8	638·6	78·5	7·145
III.	19	,,	42·5	214	9·18	0·61	12·5	38·5	1035·5	48·6	12·359
IV.	20	,,	46·75	214	10·10	0·61	14·6	40	1068·6	54·0	10·422
V.	21	,,	50·6	198·6	10·93	0·66	17·13	42	1155·4	61·2	9·917
VI.	22	,,	55·7	161·6	12·03	0·81	20·9	66·5	1616·7	55·8	11·793
VII.	23	,,	58·5	162·4	12·64	0·80	23·2	66·5	1616·7	55·8	10·394
VIII.	25	,,	66·5	128·8	14·37	1·01	28·6	88·7	2171·6	53·2	12·148
IX.	27	,,	68	116·6	14·69	1·12	31·8	101·2	2650·9	85·9	14·868
X.	29	,,	72·5	100	15·66	1·30	36·4	102·2	2694·7	82 1	12·327
XI.	32	,,	79	88·6	17·03	1·47	43·55	115·1	3236·1	54·4	12·697
XII.	35	,,	82·5	87·6	17·82	1·485	48·1	129·5	3712·3	75·8	13·276
XIII.	38	,,	89·3	85·2	19·29	1·53	57·15	145	4041·6	72	12·302
XIV.	41	,,	93·5	86·1	20·21	1·51	64·15	169	4543·0	68·4	12·461
XV.	44	,,	97	94·8	20·96	1·37	69·2	190	5156·2	54	12·978
XVI.	48	,,	100·5	72·3	21·71	1·80	74·2	213	6012·6	46·8	14·669
XVII.	52	,,	108·8	45·1	23·50	2·00	88·79	215	6090·9	54	12·714

The mean of the 17=11·896. They differ more than might be expected, considering that V and T are pretty certain. The discrepancies evidently are connected with the fluctuations of W and F, and are a sort of measure of their uncertainty. It is also evident that they follow no law which might indicate the presence of any power of V′ in the equation (I.), except the squares; the values of θ also show the effect of the disturbing influences. If, as is probable, the mean value of θ is that at which a is a maximum, it is here 62°·3, and the a of No. V. should be the largest, the other should decrease as their θ's recede from that value ; but nothing of the sort is observable.

TABLE VII.

No. II. f=101·3 ; June 15, Bar. =29·63 ; Therm. =62.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	a.
I.	14	180	47	204·6	10·15	0·64	11·6	20·7	245·3	72°·5	1·456
II.	15	,,	55·2	214·3	11·48	0·61	15·0	30	358·7	72·0	2·045
III.	18	,,	72	152·4	15·555	0·85	28·15	50	612·3	90·0	2·229
IV.	21	,,	83·2	115	16·88	1·13	33·4	53·5	658·2	77·4	2·104
V.	24	,,	95·8	91·4	20·15	1·42	48·9	96	962·0	75·6	2·310
VI.	27	,,	106	86·8	22·90	1·70	64·7	121·5	1363·3	77·4	2·657
bis.								128	1428·7	79·2	2·803
VII.	30	,,	114·75	81	24·79	1·61	77·2	129	1460·8	75·6	2·379
bis.								144·5	1596·4	70·2	2·631
VIII.	33	,,	121·5	80	26·25	1·63	87·8	146	1607·2	75·6	2·333
IX.	36	,,	138·5	64·6	29·92	2·01	118·5	195	2093·7	72·0	2·400
X.	39	,,	147	57·8	31·76	2·25	140·3	196	2103·7	75·6	2·132
XI.	45	,,	158·5	63·6	34·34	2·05	164·5	269	3003·0	68·4	2·634

The mean of the 13=2·316. Here also there is no appearance of the equation

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[*] The coefficients α, β, and γ depend on the air's density. This must be allowed for in reducing the equations.

containing any power of V but the square. If we reduce the mean α of No. I. and No. III. in the ratio of the areas $\frac{16}{81}$ and the air's density, we get 2·477, so nearly that actually found that we may fairly assume the α to be proportional to the areas of the cups.

TABLE VIII.

No. III. f=201·3 ; June 19, Bar. 29·83 ; Therm. 67.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	α.
I.	18	180	52·5	120·4	11·34	1·08	17·4	67·25	1326·6	62°·1	10·496
II.	21	,,	58·6	118·4	12·675	1·10	22·0	92	1918·2	61·2	12·625
III.	24	,,	68	101	14·69	1·29	29·9	107·5	2293·9	63·0	11·466
IV.	27	,,	74·4	82·2	16·07	1·58	36·2	142	3118·1	64·2	13·385
V.	30	,,	81·3	93·2	17·56	1·40	43·8	158	3477·5	61·2	12·063
VI.	33	,,	84·5	68·8	18·255	1·89	47·6	167	3650·2	61·2	12·393
VII.	36	,,	91·6	63·4	19·70	2·05	56·2	188	4178·2	45·2	12·571
VIII.	39	,,	99	58·6	21·39	2·22	67·4	201	4509·4	89·4	11·521
IX.	42	,,	103·7	59·6	22·40	2·18	74·7	221	5074·5	75·6	11·714
X.	45	,,	107·5	64·2	23·06	2·03	79·7	248	5482·3	82·8	11·843
XI.	48	,,	115	51·4	24·84	2·53	94·4	341	6828·0	57·6	13·098

The mean of the eleven =12·107 ; that of No. I. reduced to the same air density =12·383. Making reasonable allowance for errors, I think we may infer that the α is not changed by reducing the arms of the anemometer to half their length.

(28.) I tried similar measures with No. IV., but had very indifferent success. The main cause of this was the bad quality of a supply of oil which had just been sent in for lubricating the machinery, and which probably prevented the excursion of the anemometer beyond the point of balanced tension. From the high temperature, also, it was necessary to have all the windows open. Eleven were taken, of which from the second to the seventh, C changed only from 23 to 25·5, though V varied from 18·1 to 28·5. C then passed at once to 94, and continued increasing. The extreme hypothesis would make $\alpha=\frac{\text{F}+\text{T}}{\text{V}'^2}$; it is precarious, but I give those that are not palpably wrong for what they are worth.

TABLE IX.

No. IV. f=185·0 ; June 21, Bar. 29·66 ; Therm. 75·0.

No.	D.	S.	N.	b.	V.	W.	f'''.	C.	T.	θ.	α.
I.	15	180	66	361	14·26	0·36	20·2	20	364·8	72°·0	2·940
II.	17	,,	84	153	18·15	0·85	33·6	23	426·9	82·8	2·147
VIII.	33	,,	141	117·4	30·46	1·11	105·3	94	1967·7	68·4	2·614
IX.	34	,,	150	70·8	32·41	1·84	121·5	96	2017·5	75·6	2·480
X.	39	,,	159	91·6	34·35	1·42	129·4	121	2594·8	89·4	2·677
XI.	42	,,	159	72·3	34·35	1·80	129·4	97	2042·8	79·2	2·168

The mean of the six=2·502. The a of No. II. reduced to the present density =2·288. The irregularity of W is notable, as also that XI. with a larger D has only the same V as X. As it was at this period necessary to prepare for the erection of the Vienna equatorial, I could not repeat these experiments, but the results obtained with Nos. I. and III. leave no room for doubting that a similar agreement would be found here with No. II.

(29.) No. V. was not tried this way. The less exact method of gradually increasing the brake friction till the anemometer stopped, gave by two observations a=10·400 on June 2, Bar. 30·06, Therm. 71.

(30.) After these preliminary details I proceed to state the experimental results, beginning with those of No. 1. The first 79 of these were obtained while the centrifugal pressure was opposed by a single friction roller, and the second set (80 to 123) were taken after two were applied.

Most of the headings of the tables have been already explained; $m=\frac{V-W}{v}$ the ratio of the wind's velocity to that of the anemometer; ρ is the factor to reduce the air's density when a was measured to that at each observation.

TABLE X.

No. I. f=113·2; May 8, Bar. 30·37; Therm. 52.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
1	15	240	33	39	290	5·35	1·39	0·45	6·5	0·06616	3·519
2	16	180	32·6	34·8	242	7·04	1·66	0·54	11	,,	3·935
3	18	180	41·3	52	173·3	8·92	2·475	0·74	17·2	,,	3·301
4	20	180	48·71	65	147·3	10·52	3·09	0·89	22·9	,,	3·115
5	24	180	60·4	85·17	122·3	13·05	4·05	1·095	39·2	,,	2·948
6	28	180	69	103·7	104	14·91	4·94	1·26	47·1	,,	2·7655
7	33	240	103·3	160·2	96·2	16·74	5·72	1·36	77·35	,,	2·689
8	39	240	119	191·5	81·8	19·28	6·84	1·59	101·6	,,	2·587
9	46	240	133·9	216·2	74	21·695	7·72	1·765	138·3	,,	2·582
10	52	180	106·33	181·5	67·5	22·97	8·64	1·93	158·6	,,	2·435
11	58	180	113	198	63·5	24·41	9·425	2·05	184·7	,,	2·372
12	66	180	123·5	216·9	58	26·68	10·32	2·25	229·95	,,	2·366
13	75	240	178·5	310	53·6	28·92	11·07	2·43	296·7	,,	2·393

B=3 oz.; $f+B'$=398·0.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
14	18	240	42·5	19·5	159·8	6·89	0·70	0·815	10·5	0·06616	9·872
15	20	240	56·58	51 67	101·6	9·17	1·845	1·28	18·9	,,	4·272
16	$20\frac{3}{4}$	240	59·9	64·12	110·8	9·71	2·18	1·18	21·4	,,	3·955
17	$23\frac{1}{4}$	240	71·67	82	95·6	11·61	2·93	1·36	32·0	,,	3·502
18	25	240	84·8	114·5	91	13 74	4·09	1·43	45·9	,,	3·011
19	28	240	93·5	131 4	98·8	15·15	4·69	1·32	57·8	,,	2·948
20	33·2	180	80·3	111·8	84·4	17·35	5 62	1·515	79·9	,,	2·8135
21	39	180	91	142·5	81·8	19 66	6·78	1·64	107 5	,,	2·656
22	46	180	100·9	166·8	74	21·80	7·93	1·83	139 9	,,	2·519
23	52	180	108	180	67·5	23·33	8·57	1·95	165·7	,,	2 496
24	58	240	151 5	263·9	63·5	25·09	9·42	1·97	219·5	,,	2·454
25	66	240	169·7	297	58	27·49	10·60	2·14	258·8	,,	2·391
26	75	240	184	332·3	53·6	29·81	11·51	2·20	323·4	,,	2·400

B=6 oz ; $f+B'=687·6$; May 30, Bar.=30·20 ; Therm. 61.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''	Log. ρ.	m.
27	17·5	240	56·45	24·85	130	9·15	0·89	1·005	19·4	0·03873	9·179
28	18	,,	60·9	35·5	115·4	9·87	1·27	1·13	22·2	,,	6·894
29	20	,,	69 6	57·5	110	11·28	2·05	1·185	30	,,	4·916
30	25	,,	88·1	98·4	95·4	14·27	3·51	1·37	52·3	,,	3·674
31	27	,,	108	127	86·4	17·50	4·53	1·51	70·8	,,	3·527
32	38	,,	120·1	173·3	75·2	19·47	6·19	1·73	95·5	,,	2·8665
33	45	300	168	253	73·4	21·78	7·23	1·78	139·5	,,	2·768
34	52	240	148	228·9	75	23·98	8·17	1·74	178	,,	2·722
35	58	,,	158·5	254·5	64·2	25·68	9·09	2·03	214·4	,,	2·603
36	65	,,	168·5	272	61·8	27·30	9·71	2·11	276·8	,,	2·594
37	75	,,	182·5	302	58·4	29·525	10·78	2·235	316·8	,,	2·531

B=9 oz ; $f+B'=955·0$; May 15, Bar.=30·16 ; Therm. 58.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
38	19·5	180	49·25	9·5	133	10·64	0·45	0·98	26·3	0·04636	21·579
39	24	240	74·75	39·9	102·6	12·11	1·42	1·27	35	,,	7·688
40	28	,,	92·9	85·4	90·8	15·05	3·05	1·44	56·9	,,	4·465
41	31	,,	99·25	111	85·2	16·08	3·96	1·53	66	,,	3·6715
42	42	,,	126	175·3	70·6	20·42	6·26	1·85	118·3	,,	2·968
43	46	,,	136	192·3	77·8	22·04	6·865	1·68	143·9	,,	2·966
44	55	,,	148	227·5	65·4	23·98	8·12	1·995	178	,,	2·707
45	59	180	124·5	195·5	67·2	26·89	9·31	1·941	243·5	,,	2·682*
46	65	240	166·0	265·5	60·8	26·90	9·48	2·15	273·9	,,	2·611
47	75	,,	182·5	298·5	57·2	29·57	10·66	2·28	315·9	,,	2·561

B=12 oz. ; $f+B'=1211·3$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
48	25	240	82	37·6	102·6	13·29	1·34	1·27	46·9	0·05971	8·950
49	30	,,	98·4	86·2	95·6	15·95	3·08	1·365	65·9	,,	4·741
50	32	180	76·9	70·7	86·6	16 61	3·365	1·51	72·6	,,	4·488
51	40	240	120·5	140·7	74·8	19·525	4·84	1·74	105·6	,,	3·670
52	46	,,	131·7	166·2	68·4	21·34	5·93	1·91	132	,,	3·376
53	54	,,	147	203·5	68·8	23·82	7·265	1·92	174·9	,,	3·014
54	58	,,	155·1	216	65·8	25·13	7·71	1·98	202	,,	3·003
55	65	,,	166·5	249·8	66	26·98	8·92	1·98	243·5	,,	2·804
56	75	,,	183·3	281·4	62·8	29·70	10·05	2·08	319·8	,,	2·750

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* At the end of this it was noted that one of the cups was disfigured by something striking it; 45 must therefore be rejected: it is given here to show how little v was affected by a considerable deformation of the hemisphere. I am not certain but the accident may have occurred earlier, the cup was immediately restored to shape, but the refixing it put it a little out of balance, which was not corrected till the 16th. Some dirt also had got to the outer friction wheels, so that Nos. 46 and 47 and 73 to 79 were not of equal weight with the others.

$B=15$ oz. ; $f+B'=1460·9$; May 12, Bar. $=30·12$; Therm. 53.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
57	28	240	92·8	34·2	93	15 04	1·23	1·40	56·7	0·05971	11·095
58	31	,,	98·4	57·9	85·2	15·95	2·07	1·53	65·3	,,	6·975
59	39	,,	119·4	124·5	76·8	19·35	4·445	1·70	103·3	,,	3 971
60	45	,,	133·75	160	69	21·67	5·71	1·89	137·8	,,	3·463
61	52	,,	148·25	193	70·6	24·02	6·96	1·85	178·8	,,	3·185
62	56	,,	152·8	207·4	64	24·76	7·405	2·04	194·1	,,	3·068
63	64	,,	165·45	234	60·6	26·805	8·35	2·53	241·3	,,	2·951
64	75	,,	181·2	267·5	60·2	29·36	9·55	2·17	309·4	,,	2·8475

$B=18$ oz. ; $f+B'=1707·9$; May 13, Bar. $=30·34$; Therm. 55.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
65	28	180	72·7	33·3	97	14·00	1·585	1·345	48	0·05725	7·9825
66	31	240	100	56·2	91·9	16·20	2·01	1·42	67·8	,,	7·367
67	39	,,	120	105·5	81·4	19·44	3·77	1·60	104·5	,,	4·737
68	45	,,	131·5	140·75	68·8	21·31	5·025	1 92	131	,,	3·8575
69	54	,,	147·75	187	67·1	24 26	6.68	1·94	183·8	,,	3·343
70	58	,,	156·25	202·75	64·6	25·32	7·24	2 02	206·2	,,	3·219
71	65	,,	166·75	226	60	27·02	8.07	2·17	246·5	,,	3·079
72	75	,,	183	265·75	59·6	29·65	9.49	2·19	318	,,	2·895

$B=32$ oz. ; $f+B'=2827·9$; May 15, Bar. $=30·16$; Therm. 58.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
73	32	240	102·4	36	96	16·59	1·285	1·359	71	0·04636	11·853
74	41	,,	122·4	80	72·2	19·83	2·85	1·81	109·8	,,	6·311
75	56	,,	136	117	66·4	22·03	4·18	1·96	143·8	,,	4·804
76	?	,,	158·8	161	62·8	25·73	5·85	2·09	215·2	,,	4·037
77	60	,,	160	180	62·2	25·92	6·43	2·10	220	,,	3·708
78	66	,,	169	200	63·2	27·38	7·14	2·07	254·4	,,	3·545
79	75	,,	185	244	61·4	29 975	8·71	2·125	328	,,	3·197

A change was now made in the friction rollers, in consequence of which it seemed desirable to take another series of No. 1.

$B=0$; $f=113·2$; May 24, Bar. 29·702 ; Therm. 60.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
80	15	240	37	38	294·4	5·99	1·36	0·45	5·8	0·03423	4·088
81	18	,,	61	81·9	148·2	9·88	2·92	0·88	9·9	,,	3·3805
82	22	,,	79·5	116·5	107·2	12·88	4·16	1·22	16·8	,,	2·806
83	28	,,	98	151	83·8	15·88	5·50	1·56	27·4	,,	2·605
84	35	,,	112	185	83·8	18·15	6·60	1·56	37	,,	2·512
85	45	,,	133·5	225	69·8	21·63	8·03	1·87	57·4	,,	2·461
86	56	,,	153	265	59·2	24·79	9·46	2·20	81·4	,,	2·387
87	65	,,	169·2	294·8	55·6	27·41	10·52	2·345	106·9	,,	2·383
88	75	,,	184	324	56	29·81	11·84	2·33	134·5	,,	2·322
89	85	,,	200	354	54·8	32·41	12·64	2·38	171	,,	2·375
90	105	,,	224	399·6	53·4	36·29	14·27	2·44	238	0·05728	2·373

$B=6$ oz. ; $f+B'=687{\cdot}6$; May 25, Bar. 29·90 ; Therm. 56·1.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
91	17	240	55·9	22·7	173·6	9·06	0·81	0·75	7·8	0·05625	10·251
92	22	,,	80	71·75	111	12·96	2·56	1·17	17	,,	4·602
93	28	,,	100	114·25	87·8	16·20	4·08	1·485	27·9	,,	3·609
94	33	,,	110	137·6	86·4	17·82	4·91	1·51	35·4	,,	3·3205
95	43	,,	132·8	187	72·2	21·52	6·68	1·81	56·7	,,	2·952
96	54	,,	153·5	234	63·4	24·87	8·35	2·06	82·1	,,	2·731
97	63	,,	166	264	56·6	26·90	9·425	2·30	101·5	,,	2·609
98	74	,,	182	294	56	29·49	10·50	2·33	131·0	,,	2·588

$B=12$ oz. ; $f+B'=1211{\cdot}3$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
99	23	240	76	26	174	12·31	0·93	0·75	15·4	0·05625	12·460
100	29	,,	97·7	63	92·4	15·83	2·25	1·41	27·2	,,	6·411
101	34	,,	109	92	83	17·66	3·28	1·57	34·7	,,	4·899
102	44	,,	136	151	73·8	19·44	5·39	1·77	43·9	,,	3·278
103	55	,,	153	204	62·2	24·79	7·28	2·09	81·4	,,	3·117
104	64	,,	171	236	58·4	27·71	8·425	2·235	110·1	,,	3·023
105	74	,,	184	272	53·4	29·81	9·71	2·44	134·6	,,	2·8185

$B=18$ oz. ; $f+B'=1707{\cdot}9$; May 26, Bar. 29·84 ; Therm. 58·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
106	29	240	94	35	85·6	15·23	1·25	1·52	24·8	0·04113	10·970
107	33	,,	108	52	85·6	17·50	1·61	1·52	34·4	,,	9·941
108	44	,,	136	116	73	22·04	4·14	1·79	60·3	,,	4·890
109	55	,,	154	163	63·2	24·95	5·82	2·06	82·8	,,	3·933
110	64	,,	168	204	60·4	27·22	7·28	2·16	105	,,	3·442
111	74	,,	184	242	55·6	29·81	8·64	2·32	134·5	,,	3·182

$B=24$ oz. ; $f+B'=2140{\cdot}1$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
112	34	240	111	42	91·6	17·985	1·50	1·425	36·2	0·05625	11·090
113	44	,,	136	84·2	72	22·04	3·01	1·81	60·3	,,	6·728
114	55	,,	156	140	63·4	25·28	5·00	2·06	85·8	,,	4·646
115	64	,,	169	179	61	27·38	6·39	2·14	91·5	,,	3·950
116	74	,,	185	220	57·6	29·975	7·85	2·27	136·6	,,	3·529

$B=30$ oz. ; $f+B'=2669{\cdot}3$; May 26, Bar. 29·84 ; Therm. 58·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
117	44	240	131	40	73·2	21·23	1·43	1·78	54·8	0·04113	13·617
118	55	,,	152	84	62	24·63	3·00	2·10	79·9	,,	7·5105
119	64	,,	165·5	129·2	62·8	26·815	4·61	2·075	100·7	,,	5·634
120	74	,,	180	175	56·6	29·165	6·25	2·30	126·9	,,	4·299

B=36 oz. ; f+B′=3144·4.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
121	55	240	148	70·4	67·4	23·98	2·51	1·92	79·5	0·04113	8·774
122	64	,,	165	112	63	26·73	4·00	2·17	99·9	,,	6·155
123	75	,,	180·5	152	57·8	29·25	5·43	2·26	128·4	,,	4·974

TABLE XI.

No. II. f=101·3 ; June 6, Bar. 29·695 ; Therm. 62.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
124	12½	420	113·4	55·4	241·6	10·50	1·13	0·54	8·2	0·00095	8·813
125	15	240	92	71	184	14·91	2·535	0·82	18·1	,,	5·518
126	18	,,	113·0	99·5	156·2	18·31	3·55	0·97	31·3	,,	4·910
127	22	,,	133	131	115·2	22·52	4·75	1·31	47·7	,,	4·415
128	25	,,	152	156	102·2	24·628	5·57	1·48	61·7	,,	4·157
129	28	,,	164	166·0	104·4	26·57	5·93	1·45	72·3	,,	4·266
130	35	,,	195·2	204	82	31·27	7·28	1·84	112·0	,,	4·090
131	40	,,	217	237	74	35·16	8·46	2·045	155·4	,,	3·914
132	48	,,	240·8	276·7	68·4	39·02	9·84	2·24	210·1	,,	3·739
133	56	180	196	232·2	67·2	42·34	11·05	2·25	268·7	,,	3·627

B=1 oz. ; f+B′=167·0 ; June 7, Bar. 29·895 ; Therm. 63.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
134	18½	180	106	80	138·6	17·175	2·86	0·94	25·2	9·99883	5·685
135	23	,,	135·5	125¼	111·2	21·955	4·47	1·04	44·8	,,	4·678
136	26	,,	152·6	148¼	102·2	24·725	5·30	1·27	59·9	,,	4·432
137	29	,,	158	157	94·4	25·60	5·605	1·38	65·65	,,	4·322
138	36	,,	185¾	192¼	75	30·10	6·83	1·73	100·8	,,	4·154
139	41	,,	156	168·5	65	34·03	8·02	2·00	141·4	,,	3·992
140	48	120	115·5	129·5	64·2	37·43	9·24	2·02	236·7	,,	3·833

B=2 oz. ; f+B′=280·4.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
141	23	240	138	110¾	104	22·36	3·95	1·25	49·75	9·99883	5·339
142	26	,,	161·3	146·3	81·2	26·13	5·22	1·60	69·2	,,	4·697
143	29	,,	166	155	87	26·90	5·53	1·495	74·75	,,	4·554
144	36	,,	205	214·8	72·6	33·215	7·67	1·795	132·05	,,	4·097
145	41	,,	223·2	241	62·6	36·16	8·60	2·08	168·6	,,	3·9615
146	48	180	186	213·5	56·4	40·18	10·16	2·30	233·1	,,	3·727

B=3 oz. ; f+B′=398·0.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	f'''.	Log. ρ.	*m*.
147	26	246	164·5	130¼	82	26·65	4·44	1·59	73·0	9·99883	5·6315
148	29	,,	175	149	74	28·35	5·32	1·76	85·7	,,	5·000
149	36	,,	206·2	200	70·4	33·41	7·14	1·85	134·3	,,	4·420
150	41	,,	225·5	233·5	62·6	36·54	8·34	2·08	173·7	,,	4·136
151	48	180	187	203·7	51·6	40·40	9·70	2·52	232·8	,,	3·9065

$B=4$ oz. ; $f+B'=486{\cdot}2$; June 8, Bar. 29·745 ; Therm. 59·5.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*‴.	Log. ρ.	*m*.
152	29	240	175·9	132	81·6	28·50	4·71	1·59	86·9	9·99156	5·709
153	36	„	203	187·3	60·2	32·88	6·69	2·16	128·4	„	4·595
154	41	„	223	214	58	36·13	7·64	2·24	167·8	„	4·436
155	48	„	245·9	253	55·8	39·835	9·03	2·33	224·9	„	4·042

$B=5$ oz. ; $f+B'=582{\cdot}5$.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*‴.	Log. ρ.	*m*.
156	36	240	202·4	163·4	61·4	32·79	5·85	2·12	127·3	9·99156	5·246
157	41	„	221	199·8	63·6	35·80	7·14	2·045	163·7	„	4·732
158	48	180	182	181·5	58·2	39·32	8·64	2·235	215·0	„	4·292

No. III. was originally constructed with arms of steel plate $\frac{1}{8}$th of an inch thick, as I thought that with a length of 12 inches there would not be much danger of flexure. Sixteen observations were made with it in this state; but during the seventeenth, with D=75, the screw which attached one of the arms to the centre piece snapped off, though 0·32 inch thick. The arms were now made of steel tube similar to No. I. On June 8 an observation, 164, was made with D=35, which gave exactly the V of 163; and a *v* only 0·071 less. At the time I thought this difference was merely casual, and did not repeat these sixteen observations; but it is more probable that the difference is real, in part at least, and must be kept in mind when discussing the observations. These are given in Table XII., No. III.

Table XII.

No. III $f=201{\cdot}3$; May 30, Bar. 29·94 ; Therm. 67·9.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*‴.	Log. ρ.	*m*.
159	14·5	240	35¾	60	283·2	5·79	1·07	0·48	5·9	9·99927	4·964
160	18	„	64·9	179·5	111·4	10·515	3·20	1·23	20·4	„	2·896
161	22	„	83	246	85·4	13·45	4·39	1·59	34·6	„	2·699
162	28	„	101	312	69·7	16·36	5·57	1·95	54·5	„	2·589
163	35	„	120	392	66·2	19·44	7·00	2·05	81·6	„	2·4855
164	35	„	120	388	66·4	19·44	6·93	1·89	81·9	„	2·534
165	45	„	145·7	482·8	56	23·61	8·62	2·42	130·7	„	2·461
166	56	180	124	424	56	26·79	10·09	2·42	184	„	2·415
167	75	240	195·2	676	61	31·63	12·07	2·23	290·8	„	2·4365

$B=3$ oz. ; $f+B'=770{\cdot}8$.

No.	D.	S.	N.	A.	*b*.	V.	*v*.	W.	*f*‴.	Log. ρ.	*m*.
168	17·5	240	65·3	44	103·4	10·58	0·77	1·34	20·7	9·99927	11·963
169	22	„	84	154·9	93	13·61	2·765	1·46	35·7	„	4·394
170	28	„	102	249·6	81·2	16·53	4·445	1·67	56	„	3·324
171	37	„	123·5	351·5	76	20·01	6·27	1·79	88	„	2·905
172	47	„	150·8	472·4	77·2	24·43	8·43	1·76	143·4	„	2·689
173	57	„	165·6	535	59·6	26·83	9·55	2·28	184·9	„	2·571
174	66	„	182·6	608·3	66·2	29·59	10·86	2·05	246	„	2·519

B=6 oz. ; $f+B'=1349{\cdot}9$; June 3, Bar. 29·56 ; Therm. 61·5.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
175	22	240	83·5	66·3	118	13·53	1·18	1·28	35·1	0·01060	10·245
176	28	,,	102·2	165	91·2	16·56	2·945	1·66	56·25	,,	5·060
177	36	,,	124	279	84·8	20·09	4·98	1·78	88·8	,,	3·676
178	46	,,	146	393·6	65·2	23·66	7·03	2 39	134·7	,,	3·028
179	56	,,	164	481·4	66	26·57	8 59	2·29	180	,,	2·826
180	66	,,	183	570·7	59·4	29·65	10·20	2·55	245·4	,,	2·6545
181	75	,,	198·5	631·3	58·2	32·16	11·27	2 60	304·1	,,	2·623

B=9 oz. ; $f+B'=1884{\cdot}7$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
182	28	240	100·5	45	99·2	16·28	0·82	1·52	54·7	0·01060	18·376
183	36	,,	123·5	168·5	90·2	20·01	3·01	1·68	88	,,	6 096
184	47	,,	148	337·5	72	23·98	6·02	2·10	136·6	,,	3·633
185	57	,,	167	436	66·4	27·06	7·78	2·27	189·3	,,	3·185
186	66	,,	183	517·7	62·8	29·65	9·24	2·41	245·4	,,	2·948
187	75	,,	197	585·2	56·2	31·92	10·445	2·69	297·5	,,	2·798

B=12 oz. ; $f+B'=2397{\cdot}5$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
188	36	240	126	142·3	94·4	20·42	2·54	1·65	92·4	0·01060	7·397
189	47	,,	150·3	310·4	74	24·35	5·54	2·04	142·1	,,	4·0265
190	57	,,	166·5	369	69·4	26·98	6·59	2·18	187·8	,,	3·765
191	66	,,	177	431	63	28·68	7·69	2·40	225·5	,,	3·416
192	76	,,	198	530	57	32·08	9·46	2·65	302·1	,,	3·111

B=15 oz. ; $f+B'=2896{\cdot}5$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
193	47	240	146	195	76·4	23·66	3·48	1·98	131·5	0·01060	6·228
194	57	,,	164	332·9	68 2	26·57	5·94	2·22	180	,,	4·100
195	66	,,	179	408	61·8	29·00	7·28	2·45	232	,,	3·646
196	76	,,	196	496	60·8	31·76	8·85	2 49	294	,,	3·306

B=18 oz. ; $f+B'=3390{\cdot}5$.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
197	57	240	164·4	274·3	67·6	26·64	1·90	2·24	181·3	0·01060	4·984
198	66	,,	181·3	384	62·2	29·375	6·85	2·43	239·5	,,	3·931
199	76	180	147	339·5	56·6	31·76	8·08	2·67	293	,,	3·599

The results with No. IV. are less reliable than any of the others; the shorter arms and smaller cups having less power to overcome the casual irregularities of friction.

TABLE XIII.

No. IV. f=185·0 ; June 21, Bar. 29·66 ; Therm. 75.*

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
200	14	240	82·7	104	184	13·40	1·86	0·71	24·3	9·96688	6·836
201	16	,,	106·4	181·8	143	17·22	3·245	0·91	42·8	,,	5·025
202	20	,,	121·8	242·3	152	19·73	4·325	0·86	58·4	,,	4·364
203	18	,,	136·8	292·6	167·6	22·16	5·22	0·78	76·9	,,	4·095
204*	22	,,	142 8	314	111	23·14	5·60	1·17	85·1	9·97611	3·916
205	24	,,	149·3	339	111	24·19	6·05	1·17	95·2	9·96688	3·831
206	21	,,	155·5	359·8	106·1	25·19	6·42	1·22	104·6	9·97149	3·733
207*	26	,,	166·6	392·7	96·2	26·99	7·01	1·35	125·8	9·97611	3·581
208*	28	,,	175·6	425·5	82·9	28·45	7·59	1·58	143·4	9·97149	3·539
209	28	,,	181·9	439	86	29 47	7·835	1·51	157·5	9·96688	3·568
210	30	,,	194·2	485·8	78	31 465	8·67	1·61	187·8	,,	3·437

B=1·5 oz. ; f+B′=432·0 ; June 22, Bar. 29·95 ; Therm. 73.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
211	16	240	120	212·8	178	19·485	3·80	0·73	56·6	9·97611	4·938
212	18	,,	134·9	274	167·6	21·86	4·89	0·78	74·5	,,	4·290
213	20	,,	151	333	153·3	24·47	5·94	0·85	97·7	,,	3·9775
214	22	,,	162·2	370·9	93·3	26·28	6·62	1·395	117·5	,,	3·761
215	24	,,	178	424	82·4	28·84	7·57	1·58	148·7	,,	3·603
216	25·5	,,	184	444·2	82	29·81	7·89	1·59	163·4	,,	3·576
217	27·5	,,	196	495	78·8	31·76	8·835	1·65	192·8	,,	3·408
218	29·5	,,	199	506·7	69·6	32·24	9·04	1·87	201·7	,,	3·359

B=3 oz. ; f+B′=754·5.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
219	18	240	125·7	99·3	133	20·365	1·77	0·98	63	9·97611	10·939
220	20	,,	147·6	217	120	23·915	3·87	1·08	92·5	,,	5·8945
221	22	,,	160	274	97·2	25·92	4·89	1·34	113·5	,,	5·027
222	24	,,	172	328	92·6	27·87	5·85	1·405	135·9	,,	4·520
223	26	,,	187	379·6	63·2	30·30	6·775	2·06	168·9	,,	4·169
224	29	,,	205·7	456	79·2	33·83	8·14	1·64	227	,,	3·893

B=4·5 oz. ; f+B′=1052·0 ; June 23, Bar. 29·68 ; Therm. 72.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
225	23	240	176	238	80	28·52	4·25	1·63	144·3	9·97782	6·330
226	25	,,	188	304	72·6	30·46	5·43	1·795	168·8	,,	5·283
227	27	,,	199	342·8	67·8	32·24	6·12	1·92	200·7	,,	4·956
228	29	,,	208·5	394	64·2	33·785	7·03	2·03	226	,,	4·516

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* Those marked * were observed on June 22. There is a great confusion in the D's. This may have arisen from irregular friction of the vertical axes. See the remarks preceding Table IX.

B=6 oz. ; f+B′=1333·6 ; June 22.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
229	23	240	176·2	108	77	27·58	1·60	1·69	132·7	9·97611	16·140
230	26·5	,,	192	224	71	31·11	4·00	1·83	182	,,	7·323
231	29·5	,,	202	267	69·8	32·89	4·77	1·86	209	,,	6·511

TABLE XIV.

No. V.—Cylinder Cups. f=118·5 ; May 31, Bar. 30·165 ; Therm. 67·9 ; Standard for ρ, Bar. 30·05 ; Therm. 71.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
232	15	240	40	38	196·2	6·48	1·36	0·77	4·3	0·00946	4·209
233	22	,,	65·2	77·3	112·2	11·05	2·76	1·36	13·3	,,	3·512
234	28	,,	82·3	102	73·6	13·335	3·64	2·05	19·6	,,	3·098
235	35	,,	96·8	124·5	70·1	15·68	4·445	2·16	29·4	,,	3·043
236	45	,,	114·3	151·5	72	18·52	5·41	2·10	53·1	,,	3·0345
237	56	,,	129·3	173	65	20·95	6·18	2·33	59	,,	3·0155
238	65	,,	141	190	57·4	22·85	6·78	2·64	73·3	,,	2·979
239	75	,,	152	208	57	24·63	7·43	2·65	89·7	,,	2·9595

B=6 oz. ; f+B′=692·8 ; June 1, Bar. 30·17 ; Therm. 70.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
240	25	240	79	44·9	87·8	12·80	1·60	1·72	17·7	0·00442	6·911
241	37	,,	102	97	87	16·53	3·46	1·74	33·5	,,	4·271
242	46	,,	117·2	124·9	65	18·99	4·46	2·33	46	,,	3·737
243	57	,,	132	150	61·6	21·39	5·355	2·46	62·2	,,	3·535
244	66	,,	143	169	53·3	23·17	6·03	2·81	76·1	,,	3·375
245	76	,,	155·5	189·2	50·1	25·195	6·75	3·02	95·5	,,	3·283

B=12 oz. ; f+B′=1216·6.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
246	35	240	98	41·5	72·8	15·88	1·48	2·08	30·3	0·00442	9·316
247	46	,,	117·4	89·3	63·8	19·02	3·19	2·40	46·2	,,	5·215
248	57	,,	132	121	59·8	21·39	4·355	2·53	62·2	,,	4·330
249	66	,,	144	144	56·8	23·33	5·14	2·64	77·5	,,	4·024
250	76	,,	157	164·5	51·6	25·46	5·87	2·93	98·3	,,	3·836

B=18 oz. ; f+B′=1715·2.

No.	D.	S.	N.	A.	b.	V.	v.	W.	f'''.	Log. ρ.	m.
251	46	240	119	49	64·4	19·28	1·75	2·34	47·8	0·00442	9·685
252	56	,,	132	98	59·6	21·39	3·23	2·54	52·2	,,	5·387
253	66	,,	144	125	53·6	23·33	4·46	2·82	77·6	,,	4·596
254	76	,,	156·3	146·6	55·2	25·32	5·23	2·74	96·8	,,	4·315

$B = 24$ oz. ; $f + B' = 2145 \cdot 4$.

No.	D.	S.	N.	A.	*b.*	V.	*v.*	W.	*f'''.*	Log. *ρ.*	*m.*
255	56	240	131	69	61	21·23	2·46	2·46	60·9	0·00442	7·618
256	66	”	144	105	55·4	23·33	3·75	2·73	77·6	”	5·763
257	76	”	156·8	130·5	53	25·405	4·66	2·85	97·7	”	4·837

On examining these tables it is seen that m, the ratio of the wind's velocity to that of the anemometer, is different in each of the four instruments, and varies in each. In No. I. the extreme values are 21·58 and 2·32; in No. II. 8·81 and 3·63; in No. III. 18·00 and 2·41; in No. IV. 16·14 and 3·44.[*] It decreases with the increased size of the cups and length of the arms, and with increased v it increases with F. On examining its rate of decrease with an increasing v it is seen to be of an asymptotic character, such that it may be expected to remain finite even when v is quasi infinite. All these conditions are satisfied by equation (I.), the positive root of which gives $\frac{V}{v} = \frac{\beta}{a} + \sqrt{\frac{\beta^2}{a^2} + \frac{\gamma}{a} + \frac{F}{\rho a v^2}} = x + \sqrt{z + \frac{F}{\rho a v^2}}$ (III.).

If, as is probable, β and γ are proportional to α, the variation of m depends upon $\frac{F}{a v^2}$ alone; and its limiting value $= x + \sqrt{z}$. It follows from this that the variation of m will be less in proportion as F is less, the cups larger, and the arms longer.

(31.) We can now proceed to determine the constants α, β, and γ, as already indicated, by combining the equations given by the observations by minimum squares. The present case, however, is not favourable for the employment of this method, which supposes that the error of each equation, besides being small, depends solely on errors of the constants, and that their coefficients are exact. This is by no means true here. V' and v are both liable to errors which are variable, and F to variable ones which may be of considerable amount, and also to casual ones even larger. Still, the result so obtained is probably better than what would be obtained by dividing the equations into three groups and proceeding by elimination.

(32.) I put the equations into the form (II.),

$$\alpha - 2\beta \times \frac{v}{V'^{1}} - \gamma \times \frac{v^2}{V'^2} = \frac{F}{\rho V'^2}\text{; or, } \alpha - 2\beta\xi - \gamma\xi^2 = \eta \text{ (IV.),}$$

both to avoid the large numbers that would occur in dealing with the original form (I.) and to diminish the influence of errors of V'. The measures of α, given in Tables VI., VII., and VIII., show that (I.) contains no term of V'; and in the first instance I tried one ζv, but found that it led to values of V' so much astray that the presence of it is inadmissible.

(33.) As it was possible that α, &c., might be functions of v, I divided the equations

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[*] These numbers explain my assuming in my original paper that the limiting value of m is 3. The anemometer with which I experimented had 3-inch cups, 6-inch arms, and considerable friction.

of No. III. (with which I commenced operations) into three groups; the first those in which $v<5$; the second those where it is between 5 and 9; the third those above 9. The final equations are:—

$$\begin{array}{lrl}
1.\ \alpha\times 13\ \ \ \ \ \ \ -2\beta\times\ 2{\cdot}501-\gamma\times 0{\cdot}591=\ 71{\cdot}641 & & \alpha=\ \ 12{\cdot}117\\
\ \ \ \ \alpha\times\ 2{\cdot}501-2\beta\times\ 0{\cdot}591-\gamma\times 0{\cdot}160=\ 11{\cdot}226 & \text{giving} & \beta=\ \ 24{\cdot}436\\
\ \ \ \ \alpha\times\ 0{\cdot}591-2\beta\times\ 0{\cdot}160-\gamma\times 0{\cdot}047=\ \ \ 2{\cdot}217 & & \gamma=-59{\cdot}672
\end{array}$$

$$\begin{array}{lrl}
2.\ \alpha\times 13\ \ \ \ \ \ \ -2\beta\times\ 3{\cdot}831-\gamma\times 1{\cdot}170=\ 49{\cdot}175 & & \alpha=\ \ 9{\cdot}593\\
\ \ \ \ \alpha\times\ 3{\cdot}831-2\beta\times\ 1{\cdot}170-\gamma\times 0{\cdot}368=\ 13{\cdot}451 & & \beta=\ \ 6{\cdot}568\\
\ \ \ \ \alpha\times\ 1{\cdot}170-2\beta\times\ 0{\cdot}368-\gamma\times 0{\cdot}021=\ \ \ 3{\cdot}782 & & \gamma=\ \ 21{\cdot}554
\end{array}$$

$$\begin{array}{lrl}
3.\ \alpha\times 14\ \ \ \ \ \ \ -2\beta\times\ 5{\cdot}077-\gamma\times 1{\cdot}864=\ 29{\cdot}319 & & \alpha=\ \ 11{\cdot}975\\
\ \ \ \ \alpha\times\ 5{\cdot}077-2\beta\times\ 1{\cdot}864-\gamma\times 0{\cdot}692=\ 10{\cdot}037 & & \beta=\ \ 14{\cdot}287\\
\ \ \ \ \alpha\times\ 1{\cdot}864-2\beta\times\ 0{\cdot}692-\gamma\times 0{\cdot}259=\ \ \ 3{\cdot}485 & & \gamma=-\ 3{\cdot}617
\end{array}$$

$$\begin{array}{lrl}
\text{All. }\alpha\times 40\ \ \ \ \ \ \ -2\beta\times 11{\cdot}409-\gamma\times 3{\cdot}625=150{\cdot}135 & & \alpha=\ \ 9{\cdot}472\\
\ \ \ \ \alpha\times 11{\cdot}409-2\beta\times\ 3{\cdot}625-\gamma\times 1{\cdot}220=\ 34{\cdot}714 & & \beta=\ \ 8{\cdot}469\\
\ \ \ \ \alpha\times\ 3{\cdot}625-2\beta\times\ 1{\cdot}220-\gamma\times 0{\cdot}427=\ \ \ 9{\cdot}484 & & \gamma=\ \ 9{\cdot}814
\end{array}$$

There is here no evidence of dependence on v; but there is very great discordance. The values of α are most consistent, and do not differ much from my measures of it, being (except in one number) a little less, as I had expected. β is more aberrant; but the range of γ is extravagant: its value in 1 is unreal, for it cannot have a negative value greater than α. In the case of this set, one cause of error is obvious. When v is small, the moment of the anemometer is not sufficient to master these casual irregularities of friction, of which I have already spoken; but at higher speeds their effect is much less sensible. Another is the smallness of the coefficients of β and γ compared to those of α. In 1 the coefficients of β and γ compared to those of α are 0·38 and 0·045 that of α; in 2 are 0·59 and 0·09; in 3 are 0·72 and 0·13. We may therefore expect α to be better determined than β, and β than γ.

To make this clearer by an example. If we determine the α and γ of the first group, keeping the independent terms of the three equation as symbols, we have—

$$\alpha=\mathrm{H}\times 2{\cdot}46-\mathrm{H}'\times 26{\cdot}01+\mathrm{H}''\times 57{\cdot}59,$$
$$\gamma=-\mathrm{H}\times 55{\cdot}79+\mathrm{H}'\times 659{\cdot}19-\mathrm{H}''\times 1564{\cdot}41,$$

from which it is evident that errors in H will affect γ very largely in comparison of α. If we suppose the probable errors of the frictions in the observations to equal those given by my friction measures at Rathmines, we can compute those of α and γ due to *this* cause of error. In three of the thirteen observations the PE of $\mathrm{F}=\pm 5{\cdot}4$; in three, $\pm 35{\cdot}4$; in three, $\pm 9{\cdot}7$; in two, $\pm 17{\cdot}8$; in one, $\pm 47{\cdot}0$; and in the last, $\pm 3{\cdot}2$.

The errors must have the same sign in each H. Hence I find PE of $\alpha = \pm 1{\cdot}30$; PE of $\gamma = \mp 39{\cdot}06$; thirty times that of α. The errors of W must also be injurious; the more so as they affect every term of the three equations, but it is impossible to measure them, and if known the calculation of their influence would be very complicated.

(34.) We cannot test surely the accuracy of these constants by trying which of them gives the best value of V′, for it is to be noted that they, and especially γ, may be changed to a considerable extent without ceasing to satisfy, at least approximately, the original equations.* Supposing F unchanged, we have for one of them $m^2\Delta\alpha - 2\Delta\beta \times m - \Delta\gamma = 0$. Combining this with a second one we obtain $\Delta\alpha\left(\frac{m+m'}{2}\right) = \Delta\beta$; $\Delta\gamma = -\Delta\alpha \times mm'$ (V.); and the constants so increased will satisfy these two equations. But it will be found that, except v is very small, $\frac{m+m'}{2}$ and mm' do not differ much for any other pair; and therefore if we take their means for an entire set we shall be able to find the β and γ belonging to a small change of α. Twenty pair from No. I., and as many from No. III., give $\Delta\beta = \Delta\alpha \times 3{\cdot}293$; $\Delta\gamma = -\Delta\alpha \times 11{\cdot}463$.†

(35.) As γ seems the chief difficulty, if it could be found *à priori* the others would be much better determined; but no means of doing this has occurred to me. By causing No. I. to revolve in quiescent air, convexes foremost, it was found that the resistance $= v^2 \times 28$; but this coefficient is far greater than what it would be in a current of air. When moving down that current, whose velocity is always more than $2v$, the convexes experience hardly any pressure, when against it, the pressure is largely included in the expression of $\alpha'.(V^2 + v^2)$. Therefore γ must be much less than $28 - \alpha$. We may, however, consider α as tolerably determined by the process described in paragraph 27.

(36.) In this state of uncertainty I consulted Professor STOKES, and his reply was so instructive that (with his permission) I annex it in an Appendix.

In it he suggested that as the equation (IV.) has only two variables ξ and η, it could be plotted on a plane surface; that such plotting might give valuable information, not only as to the existence of the errors which I suppose to affect the V, &c., but as to the equation (I.) really representing the conditions of the anemometer

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* This is well shown in the Appendix.

† The same may be done without changing α by making $\Delta\gamma = -2\Delta\beta \times$ mean m. I tried this in rather an extreme case on the constants given in paragraph 38, changing them so that $y_1 = -x_1^2$. I got a value for x_1 identical with that obtained on this hypothesis from thirty-three of the equations in Table XII. It may be remarked that this supposition gives a formula analogous to M. DOHRANDT's $V' = vx_1 + \sqrt{\frac{F}{a}}$. This, of course, gives good results for many of the observations, but is astray both for high and low values of v. Its maximum $\Delta V' = +5{\cdot}56$; its minimum $= \div 2{\cdot}37$; while those of Table XVI. are $+2{\cdot}68$ and $-1{\cdot}17$. Its probable error is twice that of (III.).

motion, and possibly some information as to the value of constants. In the parabola of equation (IV.) its ordinate at the origin of $\xi=\alpha$; its tangent there $=-2\beta$; where the curve meets the axis of ξ the $\xi=\lambda$ is the limiting value of $\frac{v}{\mathrm{V}'}$, and the tangent there $=-2(\beta+\lambda\gamma)$.

Plate 68 shows the plotting of No. III., in which ξ is on a scale twenty times that of η to prevent crowding; the dots are marked with numbers expressing the series to which they belong. The irregularity of their distribution is notable: least so at the higher values of ξ, where for a considerable distance from the limit their general direction is nearly a right line, which would be represented by $\eta=\alpha'-2\beta'\xi$. A slight curvature downwards may, however, be traced, indicating a γ of small magnitude. Near the origin of ξ the dots ramble so much that no curve can be drawn through them with any certainty; but their predominant tendency is in favour of a downward curvature. Four of them are so far above the right line as to explain fully the enormous negative value of γ derived from the first set of equations. It is evident that this right line $\eta=\alpha'-2\beta'\xi$ or its primitive $\alpha'\mathrm{V}'^2-2\beta'\mathrm{V}'v=\mathrm{F}$ must very nearly satisfy the observations; and, at least for the higher speeds, would suffice to give V′ in terms of v and F. This supposes that the coefficient of that part of the resistance which depends on v^2 is $=0$, a supposition by no means improbable.

(37.) Assuming $\gamma=0$, the second and third of each of these equations—

1.	2.	3.	All.
$\alpha'=9{\cdot}999$	$\alpha'=11{\cdot}308$	$\alpha'=11{\cdot}550$	$\alpha'=9{\cdot}989$
$\beta'=11{\cdot}645$	$\beta'=12{\cdot}768$	$\beta'=13{\cdot}037$	$\beta'=10{\cdot}928$

These are much more consistent than the results when γ is also sought. It is obvious that nine or ten of the aberrant dots can in no wise contribute to a correct determination of α and β, and therefore that observations when ξ is low and η large had better be omitted in the minimum squares. And lastly, it may be inferred from this plotting that even if we had accurate values of α, β, and γ, we cannot expect to get very correct values of V′.

(38.) These considerations induce me to assume my measure of α, or rather 0·9 of it to be the true one, and to substitute it in the second equation.

This reduction of it is arbitrary, but I have given reasons for thinking my measures a little too large, and the general tendency of No. III. and No. I. is to give it even smaller. If on comparing V′ with that given by the constants thus obtained, it is found systematically erroneous, they may be corrected by (V.), combined with

$$\frac{d\mathrm{V}}{v}=\frac{-m^2d\alpha+2md\beta+d\gamma}{2(\alpha m-\beta)} \quad \text{(VI.)}.$$

I also reject the observations where $v<5$ as unlikely to give reliable results. I find from $v=5$ to $v=9$; $\alpha=10{\cdot}896$; $\beta=11{\cdot}360$; $\gamma=5{\cdot}176$.*

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* The first and third are deduced from values of α' and β' by the formula V.; I consider them better than the direct value.

$v=5$ to $v=9$,	$\alpha=10·896$;	$\beta=11·360$;	$\gamma=5·176$
All above 9 .	10·896	11·227	5·291
All above 5 .	10·896	11·830	2·437
All	10·896	13·104	−6·505
	10·896	11·880	+1·600

The values of $\frac{\beta}{a}=x=1·0903$; $\frac{\gamma}{a}=y=0·1468$; and $x^2+y=z=1·3356$.

(39.) The observations with No. I. constitute two series: the first (1·79) taken while there was only one roller to sustain the centrifugal pressure; the other (80·123) after two were applied.

Plate 66 shows the plotting of the two: those of the first being represented by dots, those of the second by small circles. There is the same irregular distribution, and it is still more difficult to trace the curvature. It is to be remarked that the system of small circles falls *below* that of dots, which implies that the friction for it must have been greater than I assumed; but for both the average curve is nearly the right line.

In applying minimum squares, my previous experience made me omit those where v was very small. I obtained—

First set 73. $\alpha=12·299$; $\beta=17·488$; $\gamma=-15·372$; $\alpha'=11·350$; $\beta'=13·401$.
Second set 39. $\alpha=\ 8·721$; $\beta=12·974$; $\gamma=-14·397$; $\alpha'=\ 7·753$; $\beta'=\ 8·990$.
Total 112. $\alpha=11·183$; $\beta=16·954$; $\gamma=-19·862$; $\alpha'=\ 9·880$; $\beta'=11·538$.

The second, as might be expected from the plotting, is not to be relied on; and in all of them the negative value of γ is too great. It will be seen in the subsequent values of $\Delta V'$ that those for May 26 are too small, so as to confirm my suspicion that the F which I used was too small. This may have been owing to the engine having been stopped during part of the time, or else doing lighter work. I have already mentioned that a set of measures taken by Mr. H. Grubb while the engine was not acting gave larger results than those in Table IV. For B=36 the excess was 406·4; the $\Delta V'$ for 123 would correspond to an excess of 291. Under the circumstances this set by itself is useless; but as the earlier part of it seems correct, it may be retained in the total. But I give a double weight to the first. Using $\frac{9}{10}$ measured α as before.

First set 73	$\alpha=10·706$;	$\beta=12·328$;	$\gamma=0·524$
,, reduced by (V) . .	,,	12·269 ;	0·937
,, omitting $v<3$ . .	,,	12·178 ;	0·936
Total 112	,,	13·193 ;	−2·991
	10·706	12·492	−0·148

Hence $x=1·1648$; $y=-0·0138$; $z=1·3414$.

In both cases β seems tolerably certain; but it is not so for γ, especially in No. III.

The confusion of the summits of the curves (where the curvature is most influenced by γ) may account for this.

(40.) The near equality of β and γ for Nos. I. and III. seems to show that in these two anemometers they are nearly proportional to α. There should be a little excess in the first on account of the greater resistance of the arms, which are nearly twice as long, and instead of being (as they should be in working anemometers) of thin plates with sharp edges, are of $\frac{1}{2}$-inch tube. This must increase both β and γ in No. I. more than in No. III. Still the difference can be but very small, and I think that probably the mean values of them will satisfy both sets, within the limit of error of observation.

This may be tried by computing V′ by equation (III.) with the x and z deduced from their mean. We have on this assumption $x=1\cdot1282$; $z=1\cdot3400$.

ΔV′ the observed—the calculated V′ is given in Table XV.

TABLE XV.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
1	−0·18	34	+0·33	65	−1·26	97	+0·40
2	+0·93	35	−0·32	66	+0·23	98	+0·66
3	+0·77	36	−0·22	67	+0·65		
4	+1·28	37	−0·42	68	+0·11	99	+0·42
5	+1·45			69	+0·13	100	+1·47
6	+1·34	38	+0·06	70	+0·07	101	+1·55
7	+1·25	39	−0·03	71	+0·03	102	−0·45
8	+1·04	40	+0·30	72	−0·18	103	−0·10
9	+1·12	41	−0·25			104	+1·67
10	+0·25	42	−0·44	73	−1·79	105	+0·11
11	−0·23	43	+0·15	74	+1·44		
12	−0·27	44	−0·72	75	−1·17	106	+0·05
13	+0·55	45	−0·28	76	−0·34	107	+1·82
		46	−0·90	77	−1·08	108	+2·38
14	+0·08	47	−0·83	78	−1·25	109	+2·24
15	−0·38			79	−1·13	110	+1·82
16	−0·29	48	+0·25			111	+1·61
17	+0·11	49	+0·26	80	+0·43		
18	+0·04	50	+0·36	81	+1·01	112	+1·10
19	+0·37	51	+0·52	82	+1·10	113	+2·60
20	+0·47	52	+0·16	83	+1·07	114	+2·52
21	+0·23	53	+0·15	84	+0·99	115	+2·29
22	−0·29	54	+0·50	85	+0·67	116	+2·12
23	−0·34	55	−0·10	86	+0·17		
24	−0·61	56	+0·02	87	+0·26	117	+1·11
25	−0·94			88	−0·46	118	+3·45
26	−0·75	57	+1·05	89	+0·34	119	+2·79
		58	+0·94	90	+0·39	120	+2·77
27	−0·64	59	+0·24				
28	−0·65	60	+0·15	91	−0·22	121	+2·41
29	−0·49	61	+0·23	92	+0·80	122	+2·86
30	+0·01	62	−0·07	93	+1·11	123	+2·94
31	+1·30	63	−0·38	94	+1·19		
32	−0·12	64	+0·10	95	+1·20		
33	−0·02			96	+0·41		

The end of each series of a set is marked by a single line, and that of each of the two sets by a double one. It must be remembered that each set begins with a low value of v which increases in each term; in each also there is a different brake friction. The values of $\Delta V'$ are not very close; but this was to be expected from the uncertainty of W and F; and as the aberrant ones are not regularly distributed they are evidently due to casual errors. Thus in the first series of the first set, six have large positive values; but this is not the case in the corresponding terms of the other series, in which they are small and mostly negative, except the last, in which are four large negative and two positive.

In the second set the three first series are of the same character as the previous one; but there was evidently some disturbing influence at work in the last four. I have already mentioned what I believe this to have been.

The brake friction of the fourth occurs in the last but one of the first set, and the last of No. III., without producing such anomaly, so it is not due to error in the measure of the friction. The results for No. III. are given along with those obtained by supposing $\gamma=0$ in

TABLE XVI.

No.	$\Delta V'$.	Do. $\gamma=0$.	No.	$\Delta V'$.	Do. $\gamma=0$.
159	−0·43	−0·43	181	−0·64	−0·68
160	−0·17	−0·11			
161	−0·27	−0·18	182	+2·68	+2·68
162	+0·08	+0·20	183	+1·49	+1·20
163	−0·08	+0·09	184	−0·09	−0·00
164	+0·33	. .	185	−0·32	−0·20
165	−0·06	+0·28	186	−0·61	−0·45
166	−0·11	+0·14	187	−1·08	−0·69
167	+0·27	+0·58			
			188	+0·67	+0·90
168	−0·20	−0·29	189	−0·33	−0·07
169	+0·23	−0·25	190	+0·33	+0·63
170	−0·25	−0·30	191	−0·27	−0·02
171	−0·25	−0·22	192	−0·27	+0·06
172	−0·75	+0·20			
173	−0·72	−0·56	193	+0·78	+0·70
174	−0·59	−0·03	194	−0·35	−0·25
			195	−0·40	−0·31
175	−0·31	−0·30	196	+0·39	−0·36
176	−0·36	−0·33			
177	+1·30	−0·99	197	−0·49	−0·45
178	−0·77	−0·66	198	−0·49	−0·41
179	−1·03	−0·63	199	−0·46	−0·35
180	−1·17	−0·98			

Here also V′ is sufficiently well represented; but it is to be noted that $\gamma=0$ gives rather better results than the other. The probable error $=\pm 0{\cdot}384$, the maximum $=+2{\cdot}68$, the minimum $=-0{\cdot}98$. For the other the probable error $=\pm 0{\cdot}455$, the maximum also $=+2{\cdot}68$, the minimum $=-1{\cdot}17$. I did not think it necessary to compute the entire of No. I. on this hypothesis, but ten on which I tried it showed similar advantage.

(41.) I think these results warrant us to believe—first, that if the constants were properly determined, the equation (III.) with or without γ would give V′ with sufficient accuracy for all practical purposes; and secondly, that for 9-inch cups the same constants avail for arms of 24 and 12 inches. It also deserves notice that the substitution of $\frac{1}{2}$-inch tube for knife-edged arms in No. III., makes very little difference in the value of V′, as is seen by comparing 164 with 163.

(42.) By using the differential formulæ (V.) and (VI.) to correct the constants, the mean value of $\Delta V'$ might be considerably reduced; but I think this unnecessary, and I hope to get more accurate values of them by another process.

The plotting of No. II. is shown in Plate 67. It is far more confused than the previous two, and shows that the disturbing agents were still more powerful. It is not possible to detect the curvatures, and we can only infer that a right line can be drawn giving the average direction of the system, while the breadth of that system shows that we may expect the range of $\Delta V'$ to be considerable. This was to be expected, as the impelling force $\alpha\ (V'^2+v^2)$ is only one-fifth of its value in I., while the frictions are nearly the same, therefore any irregularity on the latter will be felt more powerfully; the same result will also be produced by the lesser momentum of No. II., which has less power to equalise fluctuation of motion. That such fluctuation exists is evident from examination of the chronograph sheets on which the time of each revolution of the anemometer is recorded. For instance, in 125 the maximum time of one revolution during the four minutes of the experiment was $4{\cdot}58^s$, giving $v=1{\cdot}87$; the minimum time $2{\cdot}39^s$, $v=3{\cdot}58$; and there were many intermediate values. For higher speeds the difference was not so great: in 132 the maximum time was $1{\cdot}06^s$, $v=8{\cdot}16$; minimum $0{\cdot}72^s$, $v=11{\cdot}97$. With 9-inch cups these variations are scarcely perceptible in the high speeds, but notable when v is very small. Thus in 182 maximum time is $6{\cdot}80^s$, $v=0{\cdot}62$; minimum $4{\cdot}50^s$, $v=0{\cdot}95$. In such cases it is evident that the mean v cannot give the mean V′, as m is not constant. If the arms of the anemometer had been of sharp plate, it is not improbable that the x and z of Nos. I. and III. might have availed here also; but as their section is 0·87 of the area of the 4-inch cups, γ, and in still greater degree β, bear a much greater proportion to α.

The character of the plotting prepared me for bad results from minimum squares: 34 gave $\alpha=3{\cdot}100$; $\beta=9{\cdot}6607$; $\gamma=-31{\cdot}140$, α cannot be larger than that given by my measures; and the γ would give V′ imaginary for v above 8. The result was not improved by omitting three of the smallest v's. Assuming $\gamma=0$, I had $\alpha'=1{\cdot}681$; $\beta'=2{\cdot}602$; both evidently too small. I then tried, as in previous cases, nine-tenths of my measured α, and had by six various combinations, $\alpha=2{\cdot}084$; $\beta=4{\cdot}4065$; $\gamma=-7{\cdot}626$. Still, as formerly remarked, γ is too large, and avoiding it entirely I got $\alpha'=2{\cdot}084$; $\beta'=3{\cdot}489$. Then taking twelve distributed over the entire set, I computed the values of $\Delta V'$ for them, and from these found by (VI.) $y=-0{\cdot}8146$; $x=1{\cdot}7390$; $z=2{\cdot}209$. The results are given in

Table XVII.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
124	+0·54	134	+0·75	144	−0·04	152	+0·63
125	+1·24	135	+0·37	145	−0·82	153	−0·66
126	+1·40	136	+1·17	146	−1·09	154	−0·62
127	+1·94	137	+1·00			155	−1·19
128	+1·35	138	+1·26				
129	+2·18	139	+1·33	147	−0·54		
130	+1·95	140	−0·22	148	+0·87	156	+0·38
131	+1·81			149	−0·15	157	−1·40
132	+0·63	141	+0·33	150	−0·59	158	−1·51
133	−0·27	142	+1·10	151	−1·43		
		143	+0·04				

Here also the entire set is nearly as well represented with $\gamma=0$ as by the entire equation. The probable error of Table XVI. $=\pm 0·630$. The maximum value of ΔV′ in it $=+2·18$; the minimum $=-1·51$. In the other case the probable error $=\pm 0·637$; the maximum $=+1·66$; the minimum $=-2·18124$. It was observed on a different day from the rest of that series, and its difference from its neighbours seems to imply an excess of friction in the latter. The table might evidently be improved by farther approximation.

(43.) The plotting of No. IV., Plate 69, resembles the other except in one notable circumstance. The dots of the first series (200 to 210) are lower than the rest, although their general direction is nearly parallel to that of the others. This indicates that the friction during their course was greater than on the following days. This may have been caused by bad oil, as already mentioned, and a rough measure of f taken then gave it $=250$ grains instead of 185, which I computed it to be from the disc measures. The viscidity of the oil would increase f; but if, asis probable, it was also applied to the pivots of the brake, it would lessen the pressure of the rubbers and the brake friction. Both these effects seem to have taken place to such an extent that I find it impossible to derive from the observations any constants which will represent the entire set. It was scarcely to be hoped that minimum squares could give a good result. I tried them, omitting the first 11, and had $\alpha=3·682$; $\beta=7·541$; $\gamma=-18·172$. These are inadmissible; α is too far above my measures, and γ far too negative. Assuming $\gamma=0$ the result is no better. Taking, as in the others, $\alpha=0·9$, the measured one for II., the result is still worse. It seemed however possible that though α and β were both astray, their ratio might approximate to the truth. This gives $x=1·0240$; $x^2=z=1·049$. I computed by these ΔV′, intending to correct them by the formula (VI.). The second series was well represented; in the first the V′ was too small; in the rest too great, and it was not possible by any correction of the constants to represent the whole. The constants just given for No. II. failed utterly; but to my surprise those used for Nos. I. and III. were not inferior to the preceding. I have little doubt that with the true frictions they would give correct results. Aberrant as both are, I give the results in—

Table XVIII.

No.	ΔV′	Do. I. and III.	No.	ΔV′.	Do. I. and III.
200	+0·22	−0·02	217	+0·92	+0·47
201	+2·46	+1·15	218	+0·95	+0·55
202	+2·38	+1·72			
203	+3·24	+2·46	219	−2·83	−2·99
204	+3·20	+2·28	220	−2·21	−2·66
205	+3·29	+2·29	221	−1·96	−2·58
206	+3·56	+2·49	222	−1·37	−2·00
207	+4·01	+2·76	223	−1·38	−2·35
208	+4·16	+2·67	224	−0·44	−1·64
209	+4·40	+3·04			
210	+4·50	+2·98	225	−2·57	−3·08
			226	−2·47	−2·18
211	+0·47	−0·23	227	−1·01	−2·80
212	−0·70	−1·39	228	−2·08	−2·95
213	+0·09	−0·80			
214	+0·12	−0·79	229	−3·02	−3·17
215	+0·70	−0·51	230	−2·81	−2·96
216	+1·00	−0·27	231	−2·42	−3·01

If, as I suppose, the viscid oil increased f' and f'''' and lessened the brake frictions, there must be a certain part of the set where these opposite effects will balance each other; and this seems to occur in the second series. The increase of F which would nullify ΔV′ for 210 in the second system $=139$; that for $231=-225$; both quite possible.

(44.) No. V. cylinder cups is in marked contrast to No. IV. Its plotting (Plate 70) is more regular than any of the others. Minimum squares give $\alpha=9\cdot152$; $\beta=13\cdot416$; $\gamma=-3\cdot481$. The α is a little less than what I got by two doubtful measures, so I retain it, and have $x=1\cdot4659$; $z=1\cdot769$. The results of these are given in—

Table XIX.

No.	ΔV′.	No.	ΔV′.	No.	ΔV′.	No.	ΔV′.
232	−0·81	240	−0·24	247	−0·49	254	−0·57
233	+0·39	241	−0·27	248	−0·56		
234	−0·19	242	−0·60	249	−0·51		
235	−0·03	243	−0·42	250	−0·33	255	−0·63
236	+0·12	244	−0·62			256	−1·06
237	+0·26	245	−0·59			257	−1·03
238	+0·18			251	+0·70		
239	+0·14			252	−0·36		
		246	−1·57	253	−1·16		

These might be considerably improved by a small increase of α; for instance, if I had taken it $=0\cdot9\times10\cdot4$, the error for 257 would have become $-0\cdot66$.

(45.) I was surprised at finding the cylinders so much more consistent than the hemispheres. I attribute it to two causes: 1, the cylinders are 18 oz. heavier than the 9-inch hemispheres, and their greater momentum tends to neutralise any fluctuations of friction or of the vortex's current; 2, any radial currents will probably have more power to increase the resistance by acting on the convex hemispheres than they can have on the planes which terminate the cylinders. It must be observed that in this form of anemometer the lateral friction is greater than with the hemispheres.

$D=75$ gave $V'=22\cdot5$, while with No. 1 it was $27\cdot6$.

I shall now briefly recapitulate the conclusions which, in my opinion, may be drawn from these experiments.

(46.) The equation (III.) represents the observations from $V'=5$ to $V'=40$ and through a wide range of friction. The discordances follow no law which might indicate a relation between V' and v different from that expressed by (III.); they are evidently such as would be produced by casual disturbances of W and F.

(47.) If the equation (I.) contains any other functions of V' and v than squares and products, their effect is so small that it is masked by these disturbances.

(48.) The coefficient of the impelling forces α appears to be as the area of the cups, and to be the same for radii of 24 and 12 inches.

(49.) There is reason to believe that β and γ are proportional to α, or that x and z are constant in all anemometers whose arms are of sufficient length to prevent the wake of one cup from interfering with the cup which follows it. For Nos. I. and III. this is highly probable; it is not disproved by No. IV., which is not worse represented by it than by any other system of constants. No. II. requires a larger x and z than the others, but I think I have assigned a sufficient cause for this. If this opinion be well founded, the difference between the indications of different anemometers would depend only on the fraction $\frac{F}{\alpha}$, and the limiting value of $\frac{V'}{v}$ given by the constants of paragraph 39 would $=2\cdot286$ instead of 3, which I originally assigned to it. More exact observations may show that x and z are not rigorously constant, and diminished section of the arms may alter this limit; but probably the changes will not be of much practical importance.

(50.) If these observations were repeated in a quiescent locality, if the frictions were measured at each experiment and by the means described in the note to paragraph 20, more accurate results would certainly be obtained; but there would remain the inevitable difficulty arising from the impossibility of rightly estimating the effect of W. Any measures of it which can be taken give only an average of the whole circumference, and one outside the anemometer's track, while the experiments with the balloons and air-meter show that it varies notably in various parts of the circumference, and with the distance from the centre. There is also an outward current of considerable magnitude, but very irregular. The rotation of the anemometer itself, whether vertical or horizontal, produces a secondary vortex, which must modify the primary one; to what degree we cannot say. The effect of these variations of W is shown by the changes of v in individual revolutions, while V is perfectly uniform.

(51.) For this reason, combined with the difficulty of obtaining satisfactory measures of the centrifugal friction, I have given up the idea of repeating these observations, and I propose instead to try the method described in the end of my paper.

I have established on the flat roof of the dwelling house here the anemometer No. I., with its axis vertical and 22 feet due east of that of the Kew instrument already there. Both of them are about 16 feet above the roof. The chronograph and its battery of six LECLANCHÉ's cells are placed in the wooden house which supports it, and are connected with it and the other instrument by insulated wire and contact makers. From the altered position of the axis it is necessary to apply the brake weights by means of a right-angled lever; the counterpoises already described have been replaced by springs to prevent the chance of accidental disturbance of their position. The axle now rests on a toe of hard steel, 0·25 inch in diameter, turning in oil; and to prevent it from cutting under the pressure of the axle and its appendages, the rollers formerly used to lighten the centrifugal friction have been altered by Mr. H. GRUBB so as to relieve the toe of any required portion of the weight. This arrangement works so well that the friction is little more than in the Kew one, though its weight is 1·6 times as great. The arms are now of plate steel $\frac{1}{8}$th of an inch thick, with a sharp edge, strengthened by wire stays, and with means of placing both the 9-inch and 4-inch cups at 24, 12, and 8 inches from the centre.

(52.) The mode of experimenting which I intend to follow is this. Considering the Kew as the standard S, and the other E, and arranging one of the chronograph points to register the v of S, the other of E, apply a brake friction to the latter which will diminish v. When this has lasted long enough to make it tolerably certain that each instrument has been acted on by the same amount of wind, apply a different brake friction to E and take another set. We have hence three equations, but four unknown quantities, V, α, β, and γ; and since α appears as a coefficient of V^2, it cannot be obtained separately. But if, as I believe, it is given with tolerable precision by the measures given in Table VI., the others may be determined.

The formula (VI.) enables us to estimate for the Kew anemometer the error in V caused by $\Delta\alpha=1$. When $v=1$ it is $-0·20$, 0·059 of the computed V; for $v=20$ it is $-3·30$, 0·072 of V. The highest v I ever observed was in a squall in March, 1849, when during five minutes it was 41·2; this would give $\Delta V=-8·14$, 0·072 of 112·80. It is possible that more exact measures of α might be obtained by moving the measuring apparatus in a right line, and in an enclosed space of sufficient dimensions, but the requisite conditions for such an experiment will not be easily realised.

(53.) If these constants give for any other pair taken with a different wind $V_s=V_E$ they are right; if not, the formula (VI.) will show us whether the error arises from using a wrong α, or whether the constants vary with v.

(54.) It is not easy to tell beforehand what difficulties may beset this mode of investigation. The most obvious one is the irregularity of wind which may be expected to vary from one anemometer to the other, and also at each during an experiment. If it blow in the direction W.E., or *vice versâ*, there is danger that the eddy caused by the windward one may reach the others; if it be S.N., different streams of the current may fall on each. The extent of these disturbances may be studied by making the friction of E equal to that of S; and it will also show what length of time is required to make the average V the same in both. The changes of V during the experiment may, I think, be eliminated by sorting the two v into sets, of which the individuals are all in the same proportion, and comparing *them* separately. This can easily be done by measuring the intervals on the chronograph sheets.

How far these precautions will avail can only be ascertained by trial, but I hope that it may be given to me to make this trial to its full and decisive extent.

Appendix.

The object of the experiments being to determine the relation between the velocity of actual wind supposed uniform (the air also being at, or reduced to, a normal density), the velocity of the cups and the friction, I assume in the first instance as correct, the values of those two quantites given by the experiments with the whirling machine, and proceed to consider the relation.

Let V' be the velocity with which the air passes the anemometer, that is, in the case of the actual experiments, the velocity of the centre of the anemometer itself corrected for the velocity of the wind produced by it; let v be the velocity of the centre of the cups, F the moment of the total friction. Then supposing the density of the air normal for a given anemometer, v will depend only on V' and F, that is, there will be a functional relation between the three variables V', v, F, leaving two of them independent.

In investigating experimentally the relation between two variables, it is often very useful to plot the results of experiment, as the general character of the relation sought, and the allowance to be made for errors of observation can thus be estimated. The relation between three variables would be expressed graphically by a surface instead of a curve, and it is troublesome to model a surface. If, however, we can find a relation between the variables which is satisfied, *provided* some other relation is satisfied, we can thereby reduce the number of independent variables from two to one, and employ ordinary plotting in investigating the relation between the variables. In fact, the relation sought is reduced from one of the form $V'=\phi(v, F)$ to one of the form $f(V', v, F)=\psi\{f_1(V', v, F)\}$, where ϕ, ψ denote unknown, and f, f_1 known functions.

In the present case, since by hypothesis the anemometer is in a permanent state, the moment of the friction is equal to the *total* impelling force of the air, *i.e.*, the total pressure arising from the motion of the air, without distinction of impelling or retarding, but reckoning the latter as a negative impelling force. Now, in cases of rapid motion like that of the air passing the cups of the anemometer, it is well ascertained that the resistance varies as the square of the velocity, all other circumstances being the same. Hence, with a given anemometer, when only the scale of the velocities changes, *i.e.*, when V varies as v, the moment of the total impelling force may be expected to vary as the square of the velocity. When the density changes it may be expected also to vary as the density. Hence we may expect that when v varies as V′, then F varies as $\rho V'^2$, or in other words that

$$F=\rho V'^2\phi\left(\frac{v}{V'}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where ϕ denotes some function the form of which is not at present under consideration.

Let $\frac{v}{V'}=\xi$, $\frac{F}{\rho V'^2}=\eta$, and for each observation let the point whose coordinates are ξ, η be laid down on paper. If $F\rho^{-1}$ were merely some arbitrary function of V′ and v, the points so laid down would be spread out over the paper, but if equation (1) be true they will lie in a definite curve.

The actual experiments were executed in series, in each of which only one independent variable was changed, so that if the experiments were infinitely numerous and infinitely exact the locus of the point whose coordinates are ξ, η would be a definite continuous curve. And the test of the truth of (1) is that the curves belonging to the different series shall coincide, instead of being arranged in some order of sequence.

Plate 68 shows the result of plotting the observations taken with anemometer No. III. On inspecting the figure it will be seen that the different series fit very well into one another. Departures there are no doubt in the individual observations from a mean curve, but these appear to be casual, not methodical and depending upon the order of the series.

The result of the observations then is confirmatory of the fundamental supposition made hitherto, that when the friction is so arranged that the velocity of the air passing the anemometer bears a given ratio to the velocity of the cups, the moment of the total impelling force varies as the square of either velocity.

Assuming then the truth of equation (1),[*] we have next to inquire what is the form of the function ϕ?

---

[*] It formed no part of the object of the experiments to determine the relation of the impelling force to ρ, which merely comes in as a small correction for reducing observations made on different days to a common standard. It is the dependence of F on V′ and v that is contemplated in the text.

A complete hydrodynamical solution of the problem is altogether beyond our power. On the other hand, the irregularities of the observations prevent us from going, by observation alone, more than a certain way towards the determination. We must, therefore, endeavour to combine as best may be the indications of mechanical theory with the results of experiments.

In his paper "On the Cup Anemometer," in the Transactions of the Royal Irish Academy, Dr. ROBINSON has shown that (supposing the density constant, say $=1$) the relation between the moment of the impelling force and the moment of the friction is either accurately or approximately of the form

$$F=\alpha V'^2-2\beta V'v-\gamma v^2, \qquad (2)$$

which would give for the locus of the point whose coordinates are ξ, η the parobola

$$\eta=\alpha-2\beta\xi-\gamma\xi^2. \qquad (3)$$

If now we turn to the plotting of the observations, we see that the best smooth curve to represent the observations, free from sinuosities which the observations do not warrant us in supposing real, is either accurately or approximately a straight line,

$$\eta=\alpha'-2\beta'\xi; \qquad (4)$$

in fact, so nearly does a straight line represent the observations that it is not easy to say to which side the concavity of the line, if curved it be, should lie. On the whole there appears to be a slight indication of a gentle concavity *towards* the origin.

It may be remarked in passing that the formula (4) which the experiments show to be at least very approximately true, leads to a very simple expression for v in terms of V', namely—

$$v=aV'-\frac{b}{V'},$$

where a and b are constants.

The figure shows that there cannot be much doubt as to the distance from the origin at which the curve intersects the axis of ξ, nor as to the direction of the curve at that point; and generally that the curve is well determined in its right hand half, though it becomes more uncertain towards the left. If λ be the value of ξ at the point of intersection, and $-t$ the tangent of the inclination at that point, the equation of the curve, assumed to be a parabola, will be

$$\eta=t(\lambda-\xi)-C(\lambda-\xi)^2; \qquad (5)$$

or again, if we suppose known two points (p, h) and (q, k) lying in the well-determined part of the curve, its equation will be

$$\eta=(q-p)^{-1}\{h(q-\xi)+k(\xi-p)-C'(\xi-p)(\xi-q)\}; \qquad (6)$$

and as $(\lambda-\xi)^2$ or $(\xi-p)$ $(\xi-q)$ will be small throughout the well-determined part of the curve, the constant C or C′ will admit of considerable latitude of variation without much affecting the satisfaction of the observations. Conversely, if we attempt to determine the constant C or C′ from the observations, in addition to the two elements λ, t, or h, k, the determination will be extremely precarious. And if we arrange the formula (5) or (6) according to powers of ξ, so as to throw it into the form (3), the precariousness of the determination of C or C′ will more or less affect all the three constants α, β, γ.

Accordingly, if we take this formula, and attempt to determine the three constants α, β, γ, from the observations, it may be that by different processes we shall arrive at results differing considerably, not only as regards γ, but even, though to a less degree, as regards α and β. It is not until we use the values of α, β, γ, so obtained for the determination of two out of the three elements of the parabola which are well or fairly determined by the observations, that we perceive the accordance which underlies the apparent discrepancy.

If the simple formula (4) so nearly fits the observations, it is by no means *merely* as an empirical formula of interpolation presenting two arbitrary constants whereby an approximate accordance may be brought about, or in the way that a small arc of an arbitrary curve may be approximately represented by a straight line. The observations were also plotted by taking for coordinates $V':v$ and $F:v^2$ instead of $v:V'$ and $F:V'^2$, and in this case the curvature of the curve was very decided. Accordingly, though the observations may be satisfied by the first two terms of the formula (2) almost as well as by the three, that is by no means true of the last two, though in both cases alike we have two arbitrary constants at our disposal.

Explanation of the Plates.

In these plates the results of the experiments made with the five anemometers are graphically represented. In the plottings the abscissa is throughout $20v \div V'$. In Nos. I. and III. the ordinate represents $(f+B'+f''') \div \rho V'^2$; in Nos. II., IV., and V. it represents the same divided by the ratio of the area of the mouth of the cups to that of the mouth of the cups in Nos. I. or III.; that is, it represents the above expression multiplied by $81 \div 16$ for Nos. II. and IV., or by $\pi \div 4$ for No. V.

In the plottings, 0·75 inch is taken as the unit.

The reference numbers represent the order of the series, 1 meaning no weight on the brake, 2 the lowest weight, and so on.

In the experiments with anemometer No. I., the full dots represent the first set, experiments Nos. 1 to 79; No. 45 being omitted for a reason already stated. The small circles refer to the second set, experiments Nos. 80 to 123.

In all the plottings, except that for No. IV., a parabola or straight line, or both a parabola and a straight line, are laid down for comparison. The elements chosen for these are as follows :—

For No. I., $\alpha=11{\cdot}183$, $\beta=16{\cdot}954$, $\gamma=-19{\cdot}862$, $\alpha'=9{\cdot}880$, $\beta'=11{\cdot}538$.

For No. II., $\alpha'=2{\cdot}084$, $\beta'=3{\cdot}489$.

For No. III., $\alpha=9{\cdot}472$, $\beta=8{\cdot}469$, $\gamma=9{\cdot}814$, $\alpha'=9{\cdot}989$, $\beta'=10{\cdot}928$.

For No. V., $\alpha=9{\cdot}152$, $\beta=13{\cdot}416$, $\gamma=-3{\cdot}481$.

The ordinates of the straight line and parabola were of course reduced in the same ratios as those of the dots, giving for the reduced constants in their equations—

For No. II., $\alpha'=10{\cdot}550$, $\beta'=17{\cdot}663$.

For No. V., $\alpha=7{\cdot}188$, $\beta=10{\cdot}537$, $\gamma=-2{\cdot}734$.

PLATE 67.

Fig. 1. Section of the horizontal arm (paragraph 2).

Fig. 2. The driving apparatus (paragraph 6).

PLATE 70.

Fig. 3. Apparatus for measuring the vortex current (paragraph 13).

Fig. 4. Brake apparatus (paragraph 21).

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TO THE

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FOR THE YEAR 1878.

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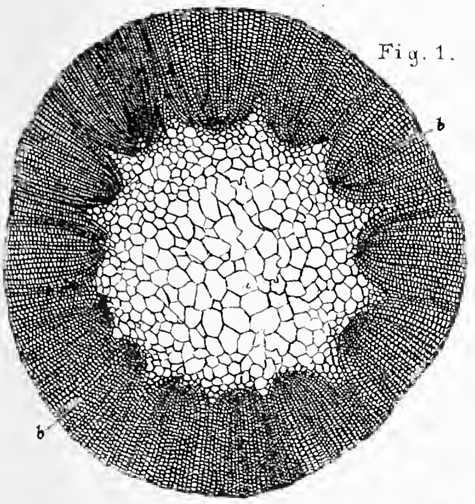
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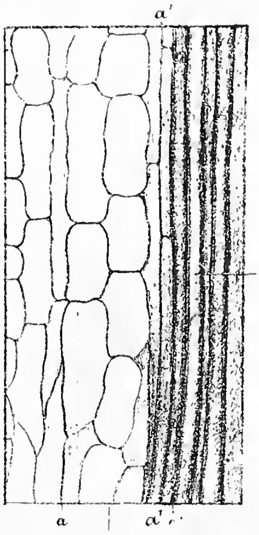
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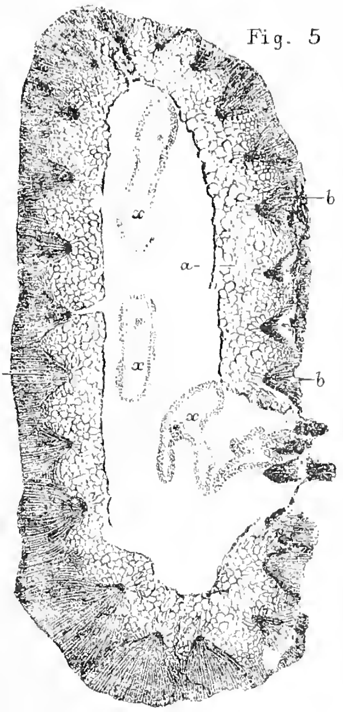
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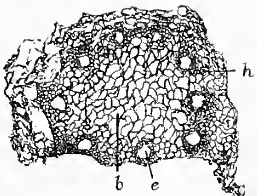
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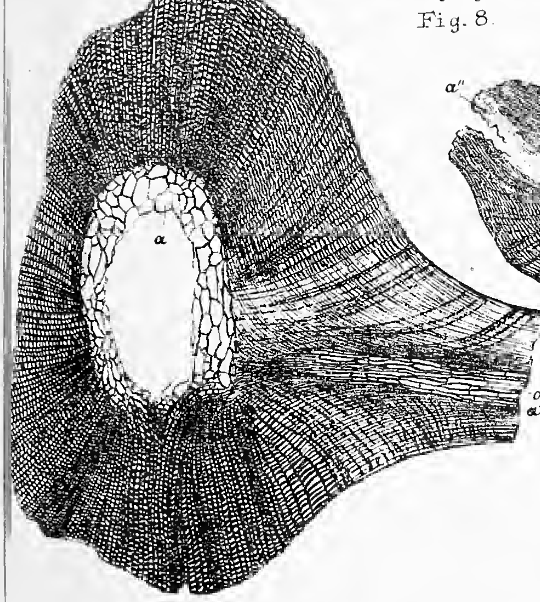


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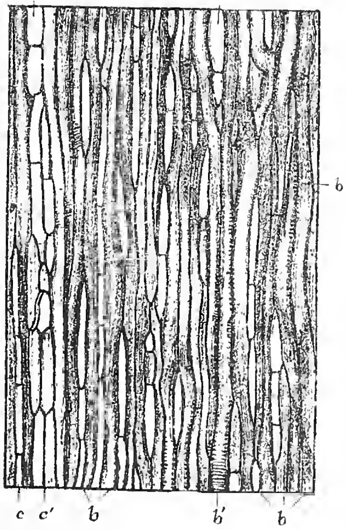
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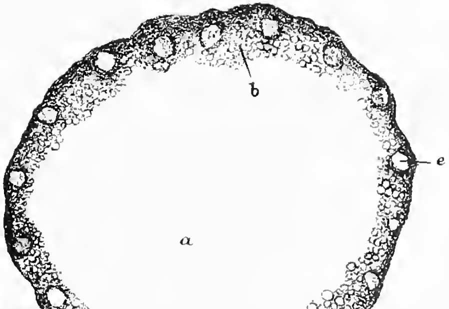
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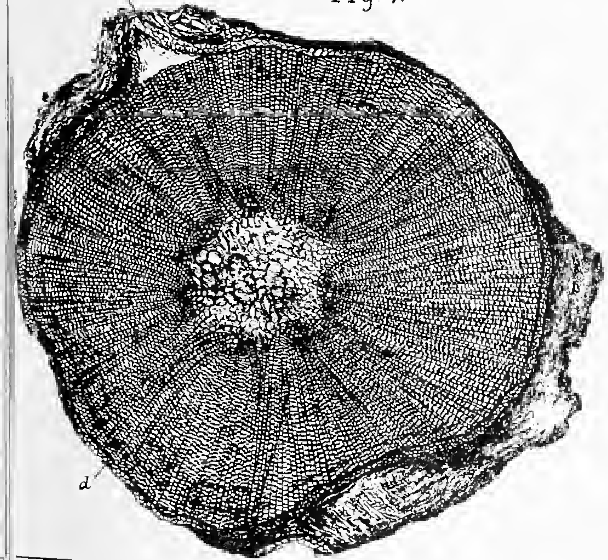
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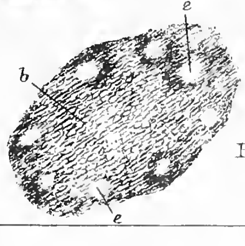
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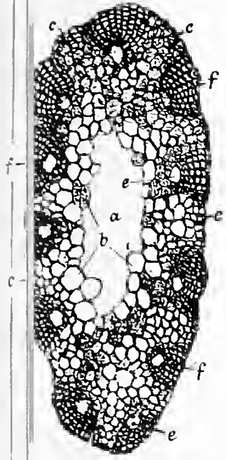

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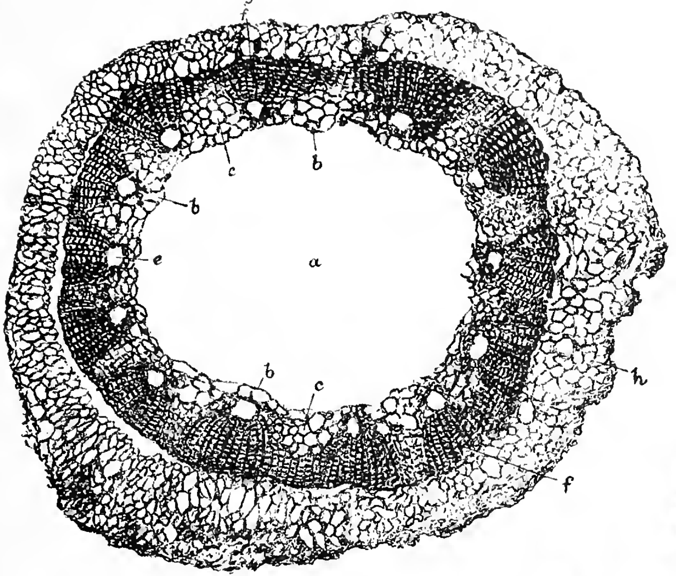

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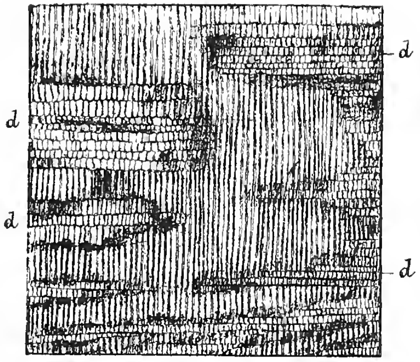

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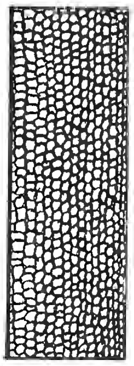

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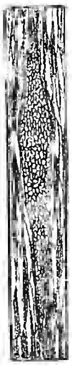

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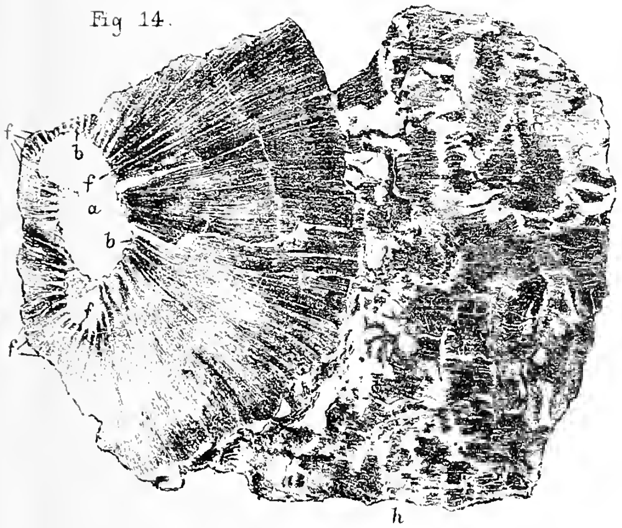

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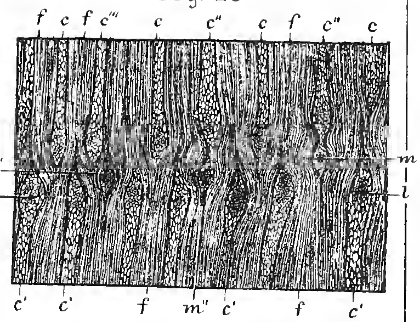

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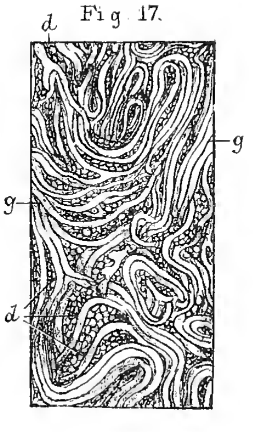

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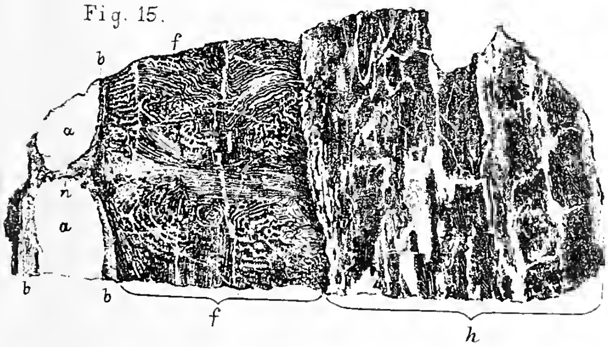

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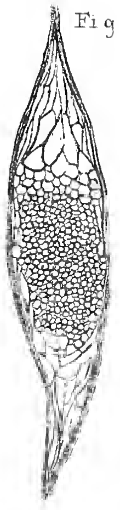

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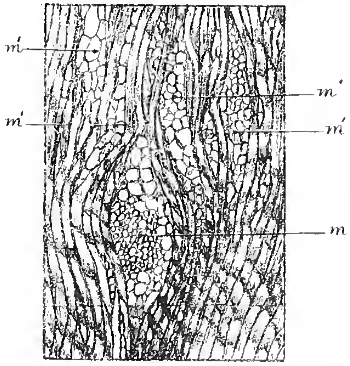

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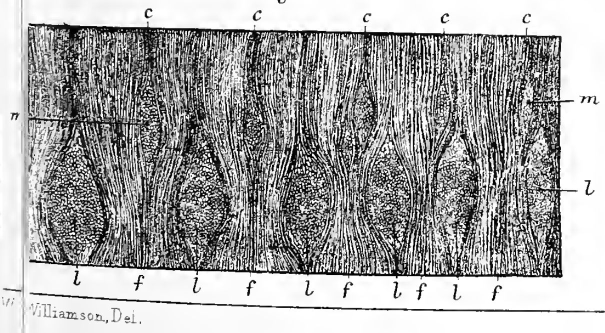

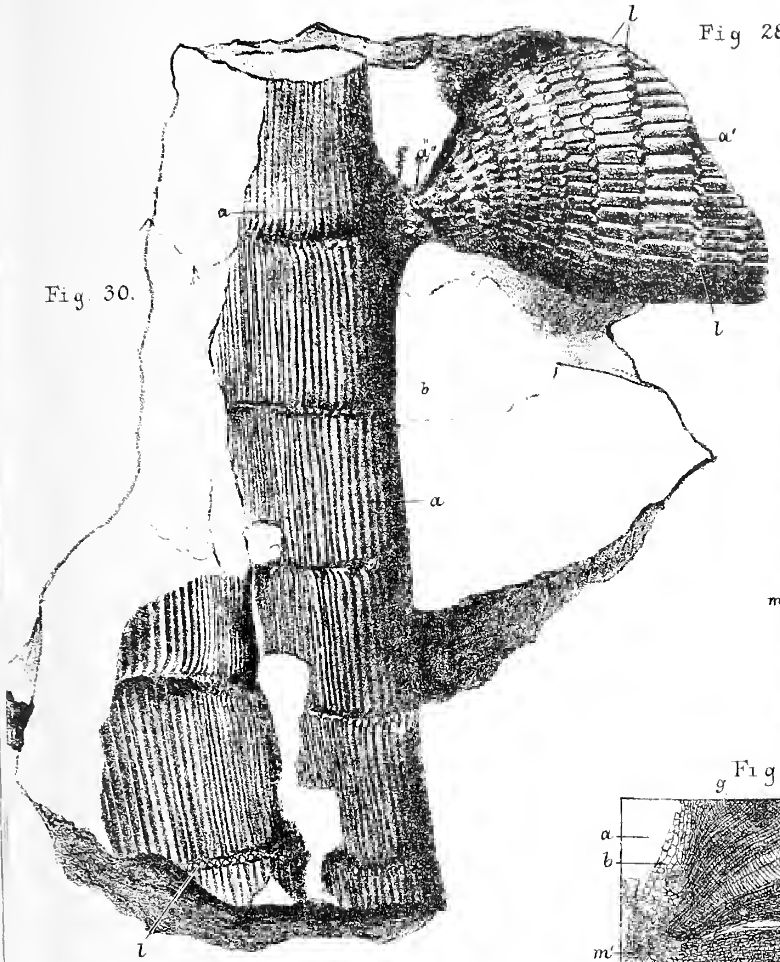
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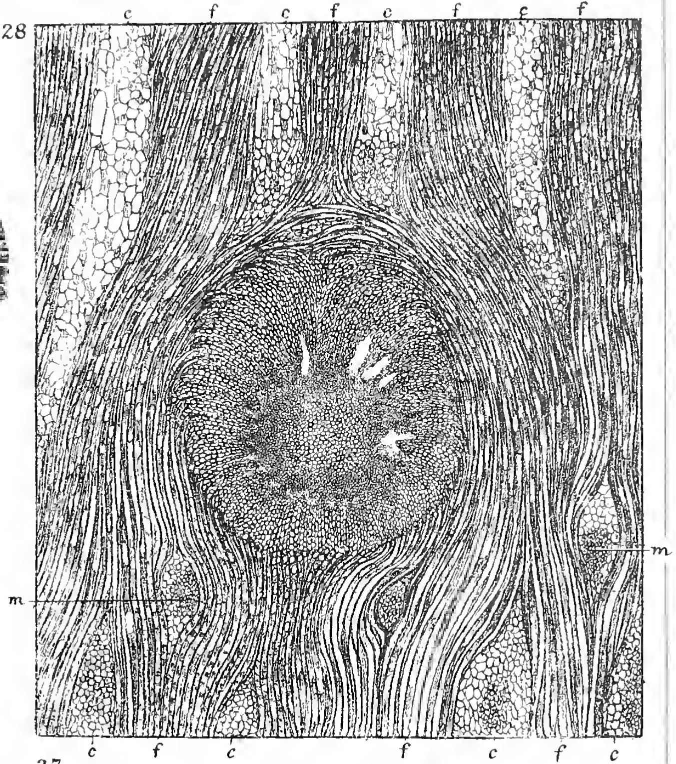
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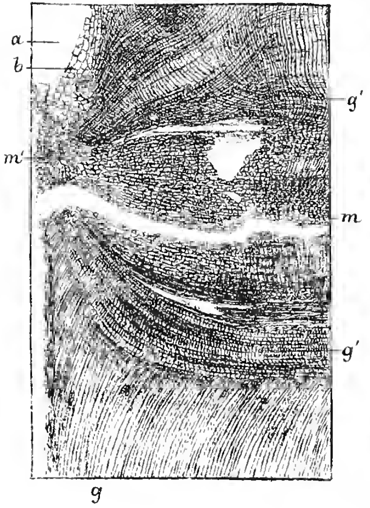
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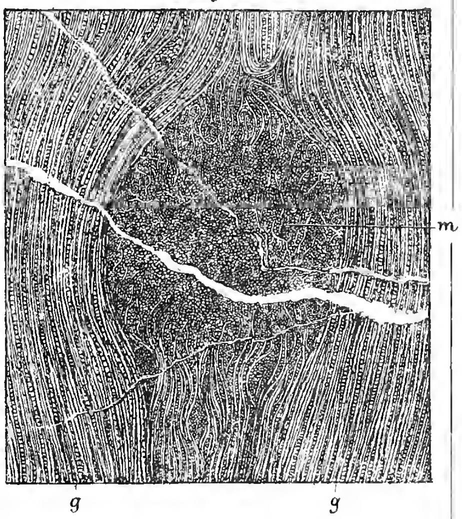
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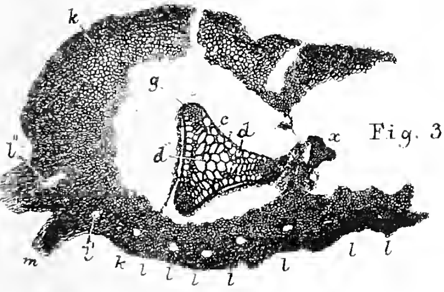
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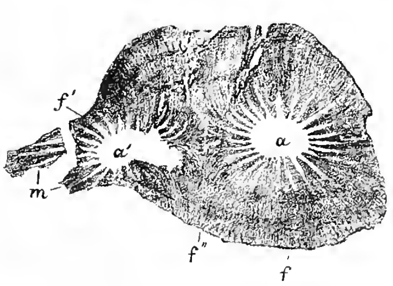
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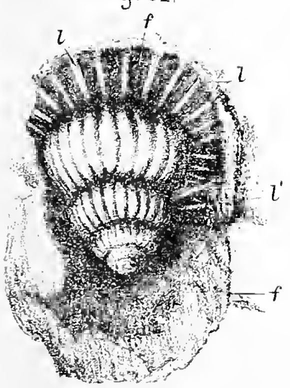
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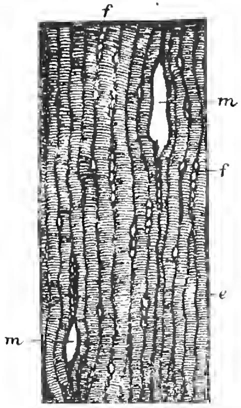
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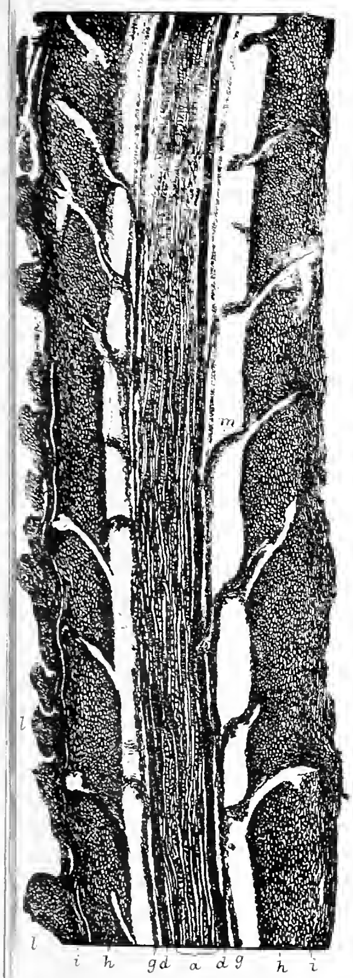

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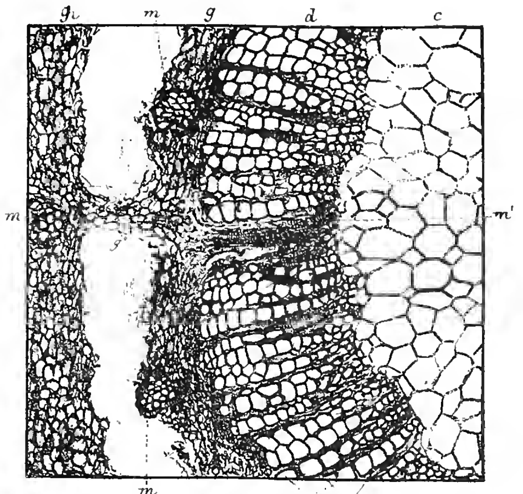

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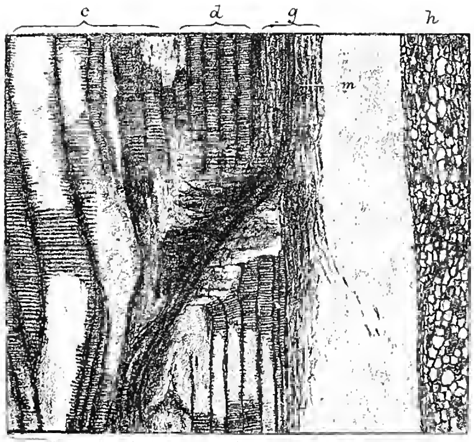

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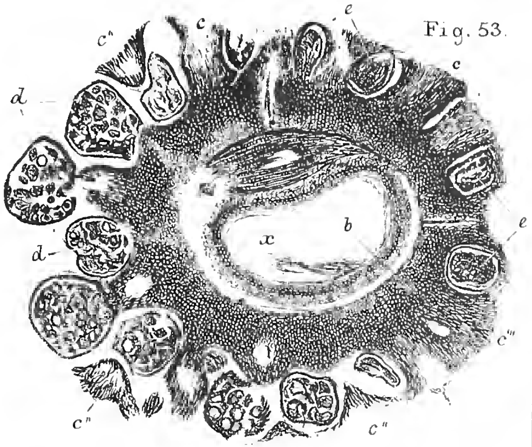

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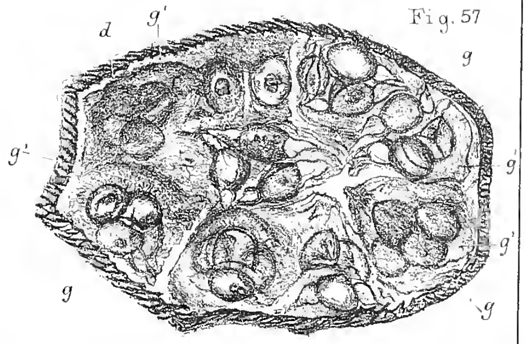
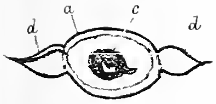


Fig. 57



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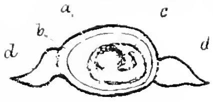

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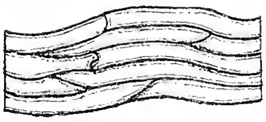

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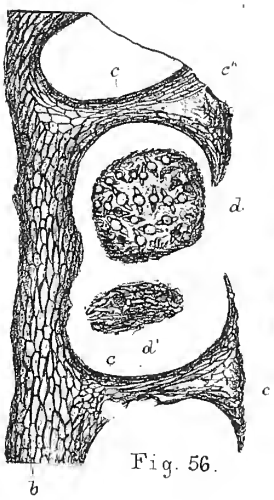

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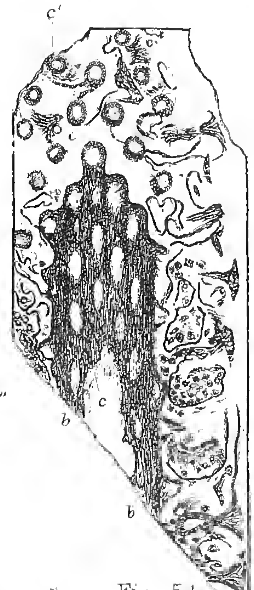

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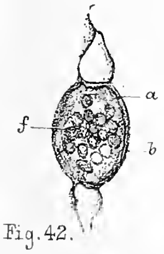
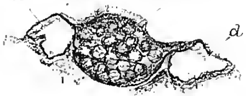


Fig. 42.

Fig. 41



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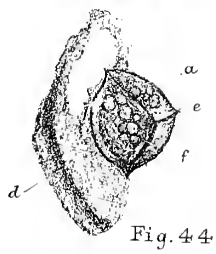

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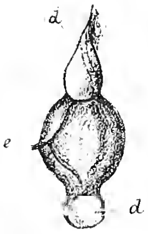

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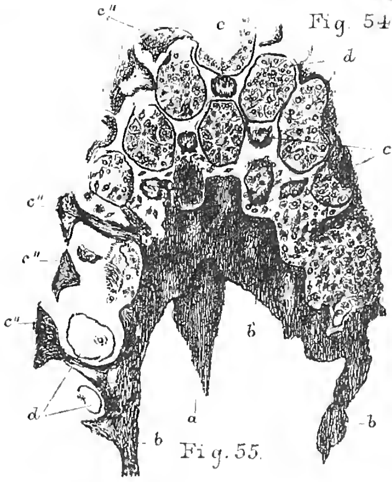

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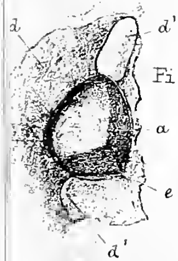

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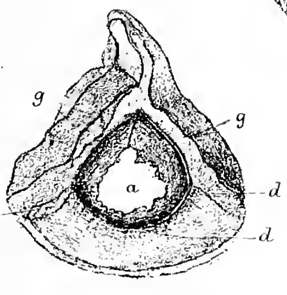


Fig. 46.

Fig. 50.

Fig. 51.

Fig. 52.

g.48.

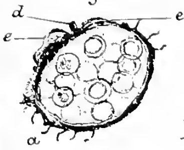
Fig. 49.

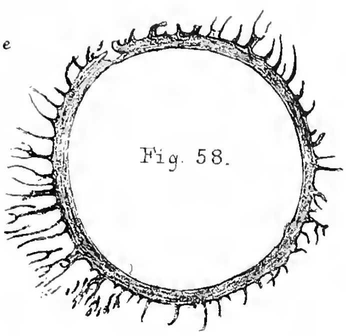
. Williamson, Del.

Whiteman & Bass, Photo-Lith.


Fig. 59.

Fig. 66.

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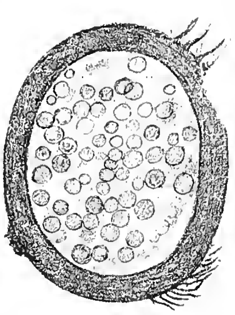


Fig. 66 A.


Fig. 71.

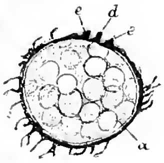
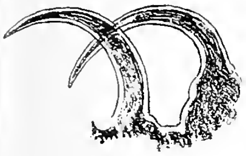


Fig. 65.


Fig. 74


Fig. 73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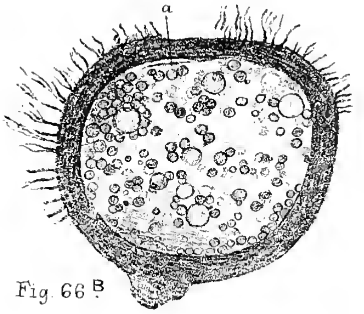
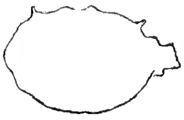


Fig. 66 B.


Fig. 63.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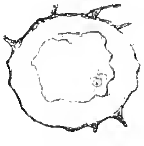
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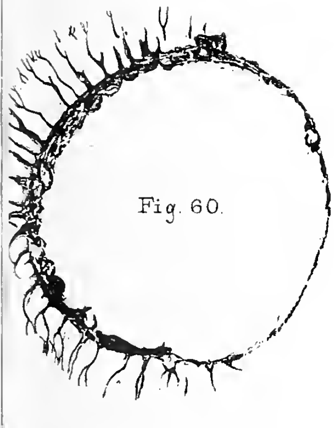
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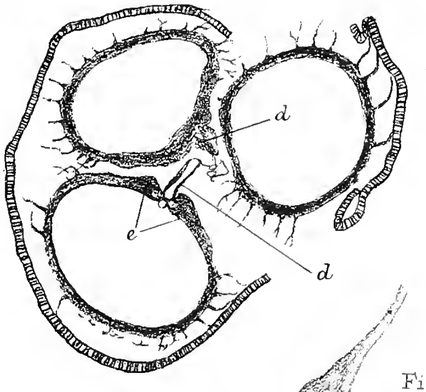

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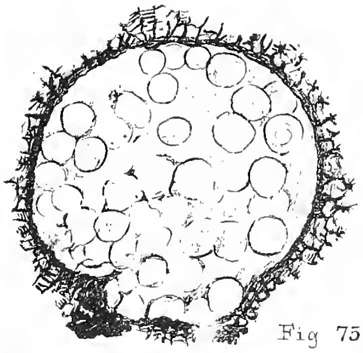
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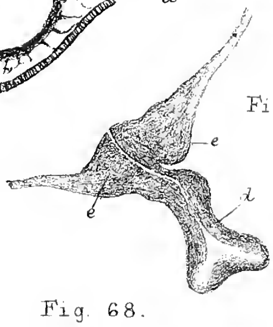


Fig. 64*

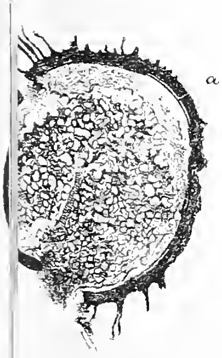


Fig. 66 C.

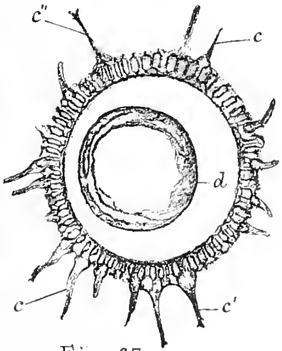
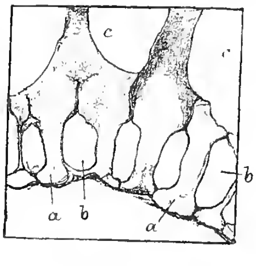
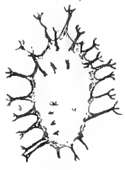


Fig. 67.

Fig. 68.

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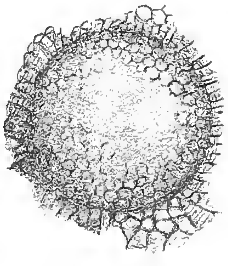
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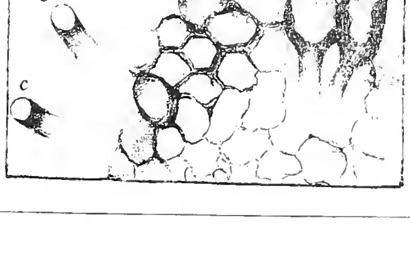
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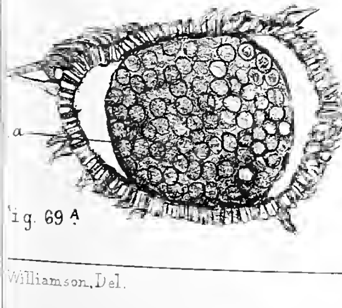


Fig. 69 A.

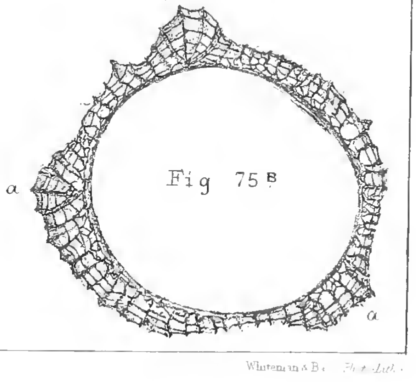
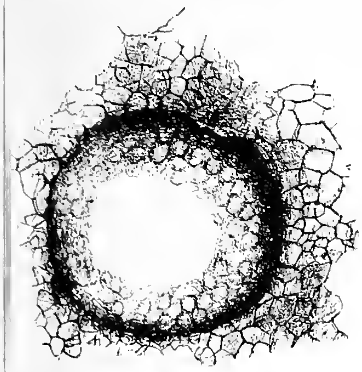


Fig. 75 B.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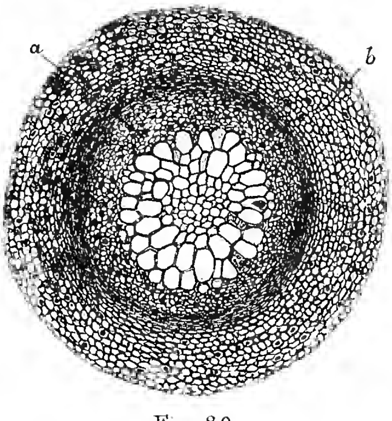


Fig. 80

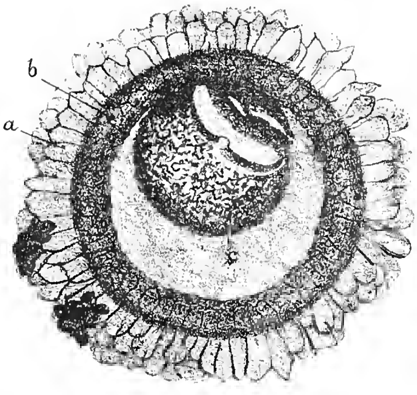
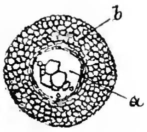


Fig. 76.A


Fig. 77.


Fig. 83.



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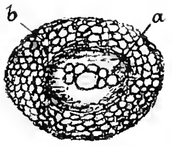

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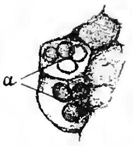
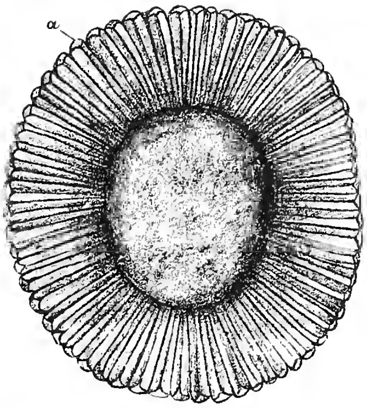


Fig. 75.D


Fig. 78.A



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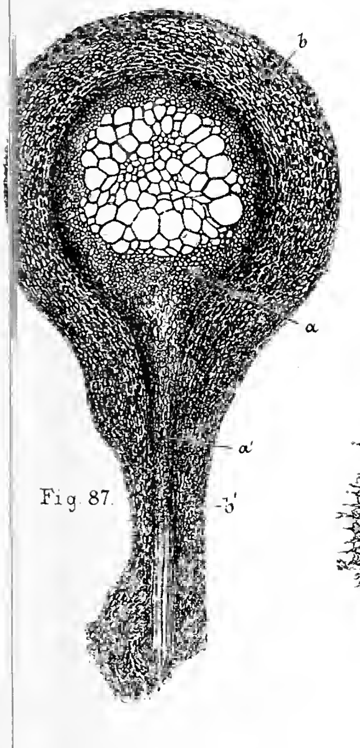

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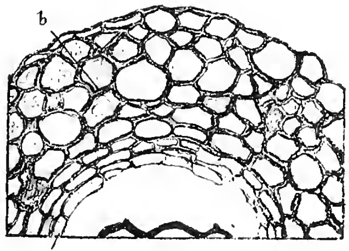

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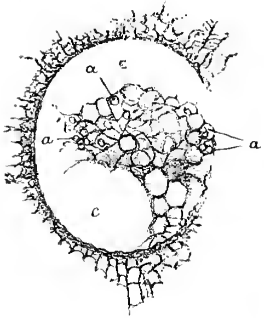
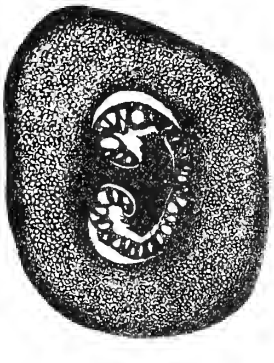


Fig. 75.C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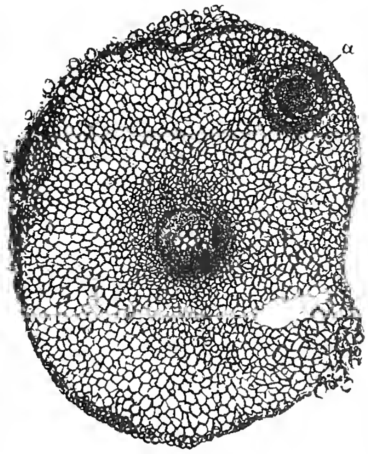
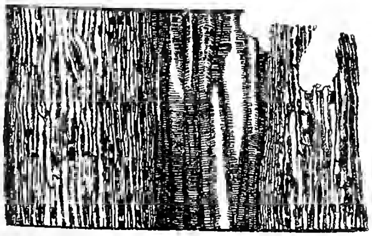


Fig. 88.

Fig. 86.



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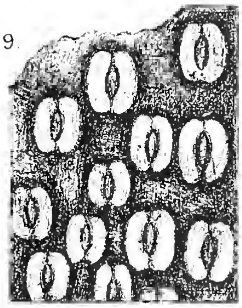

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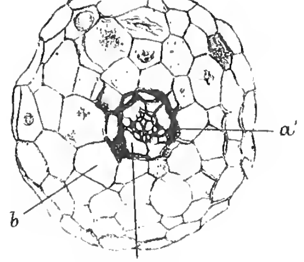

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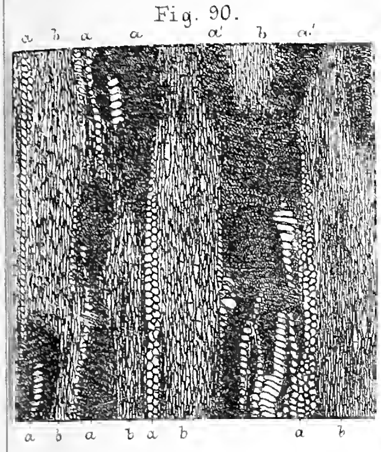

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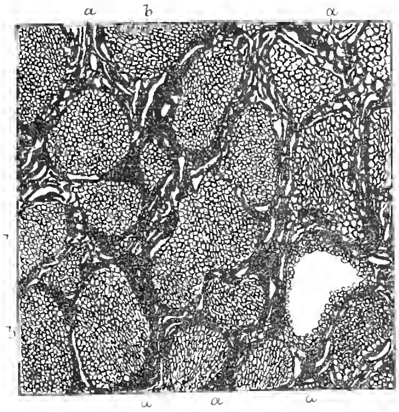

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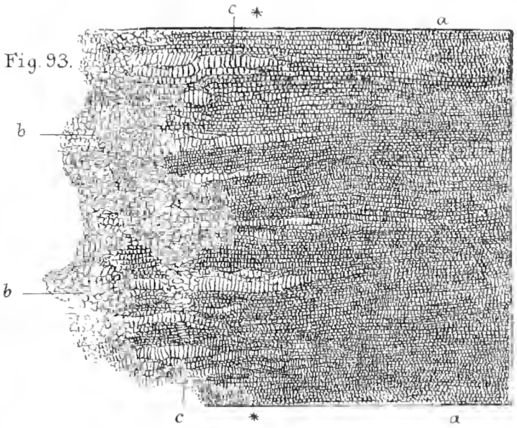

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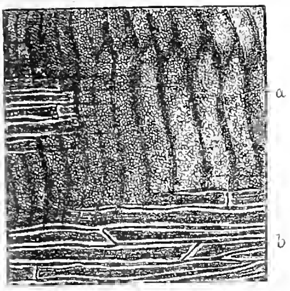

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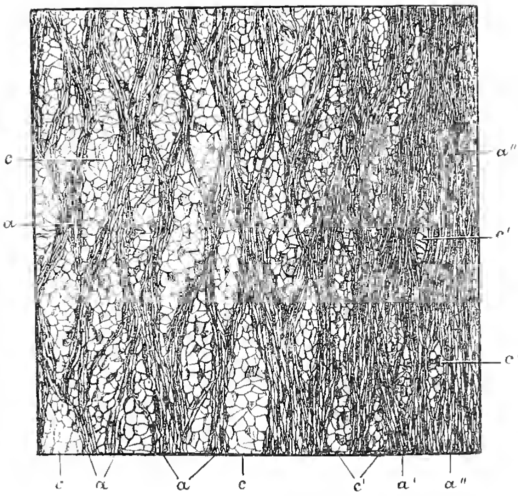

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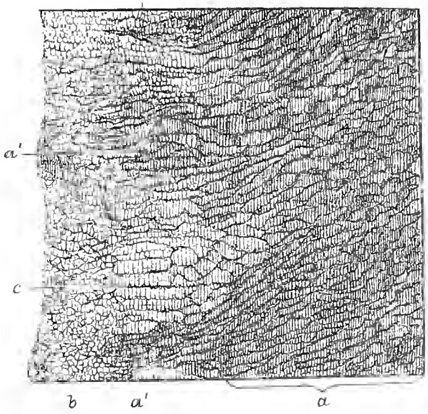

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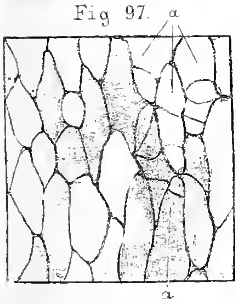

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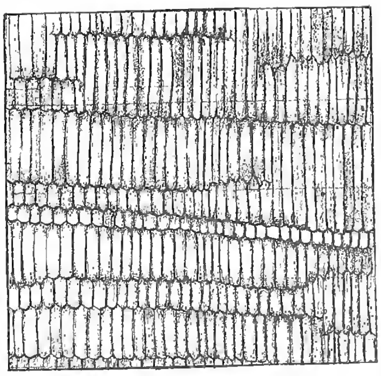

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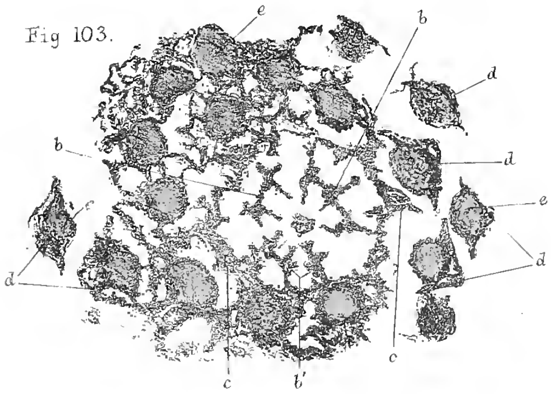

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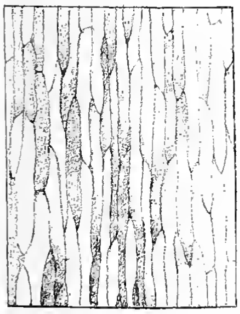

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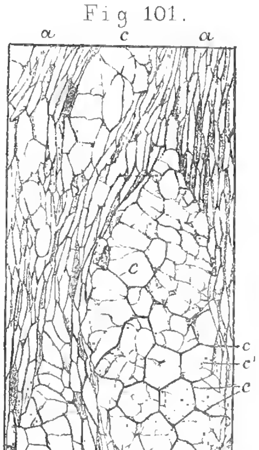

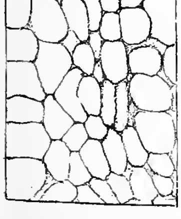

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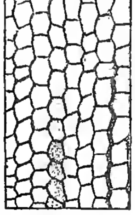

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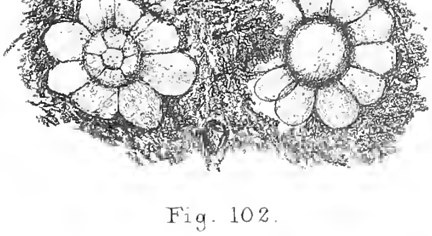

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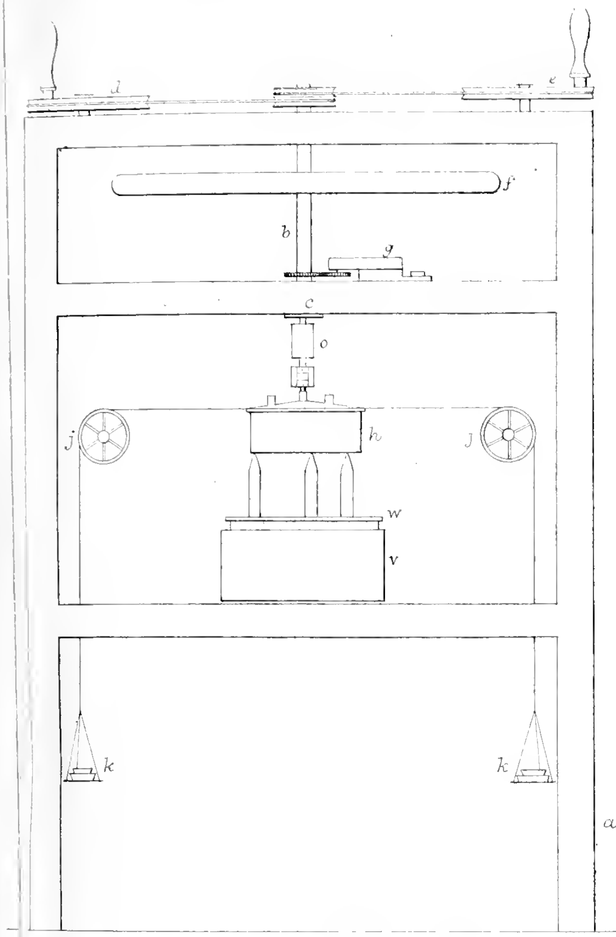


Fig. 1

Scale $\frac{1}{18}$

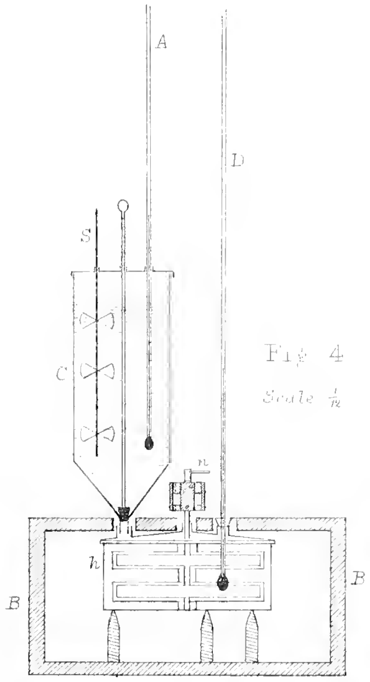


Fig. 4

Scale $\frac{1}{12}$

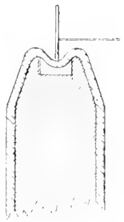


Fig. 5

Scale $\frac{2}{3}$

ig. 2.

cale $\frac{1}{6}$

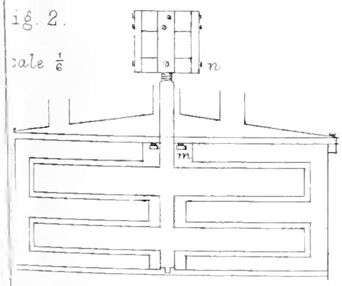


Fig. 3

Scale $\frac{1}{6}$

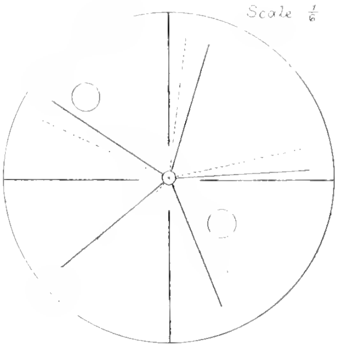


Fig. 6

Scale $\frac{1}{12}$

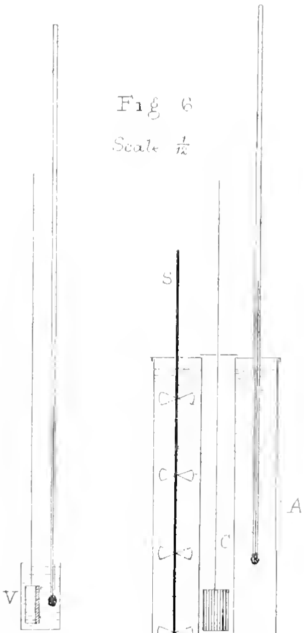
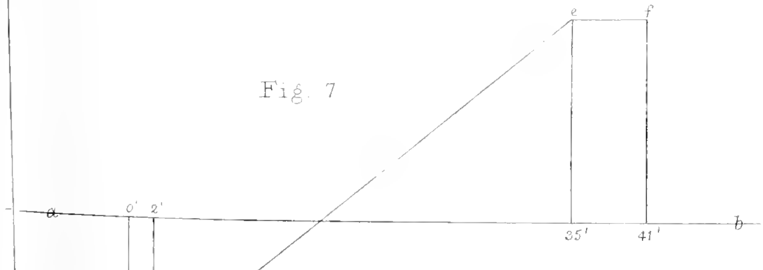


Fig. 7



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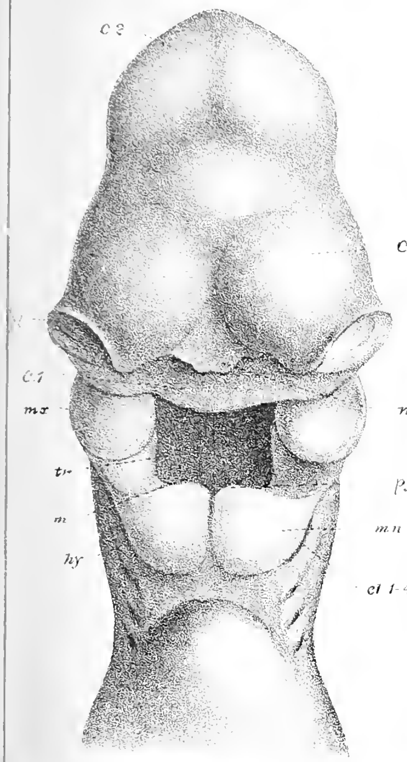

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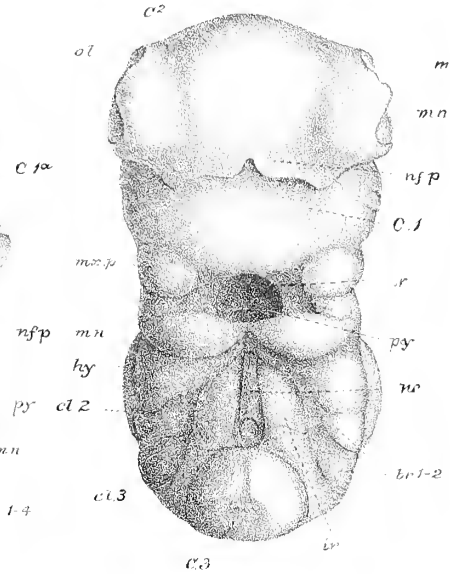
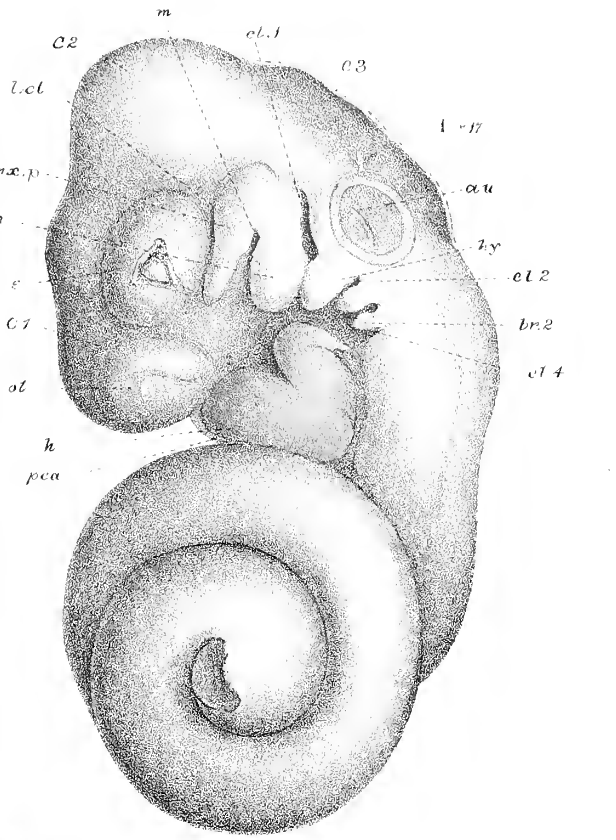
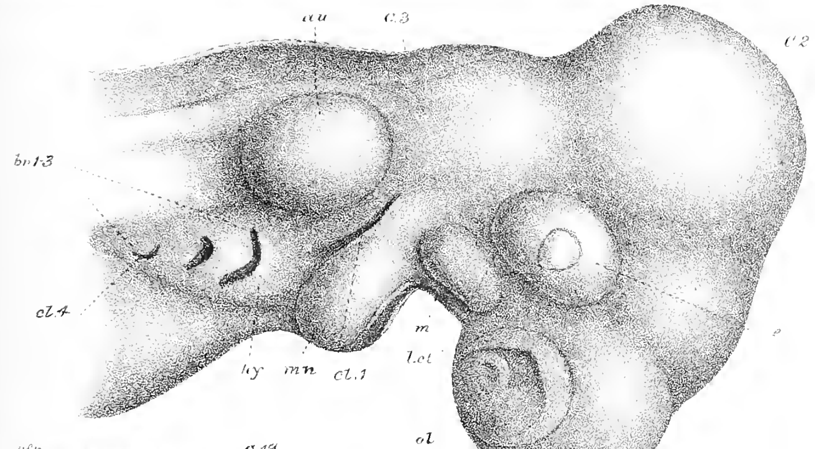


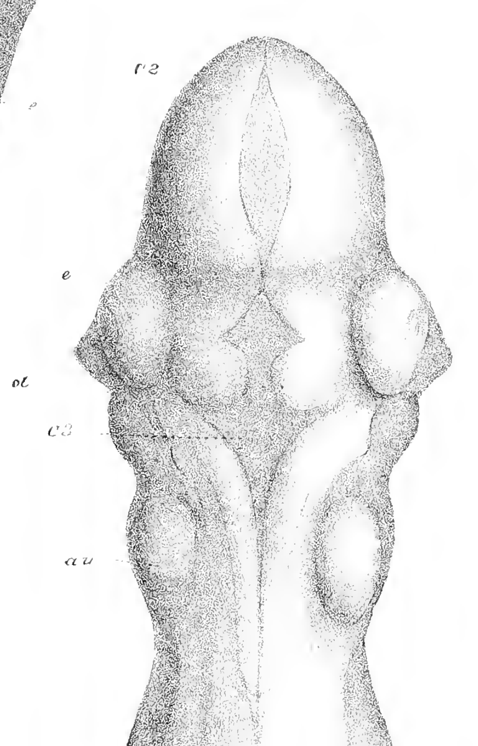
1 × 17



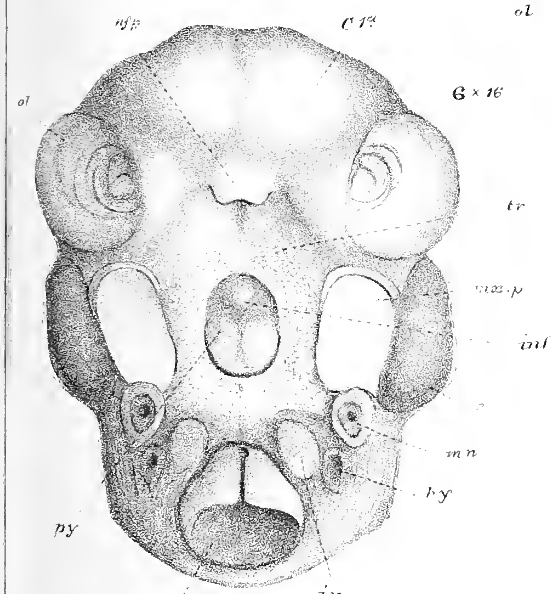
3 × 16



5 × 16



6 × 16

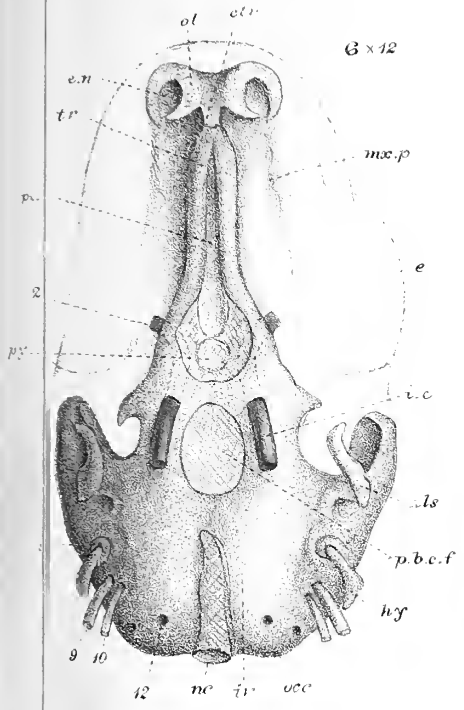


W.K.P. del. ad nat.
G.H. Ford lith.

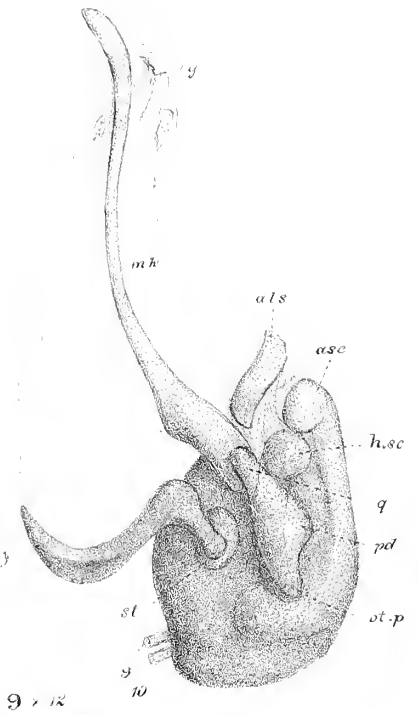
W West & Co imp.

Common Snake.

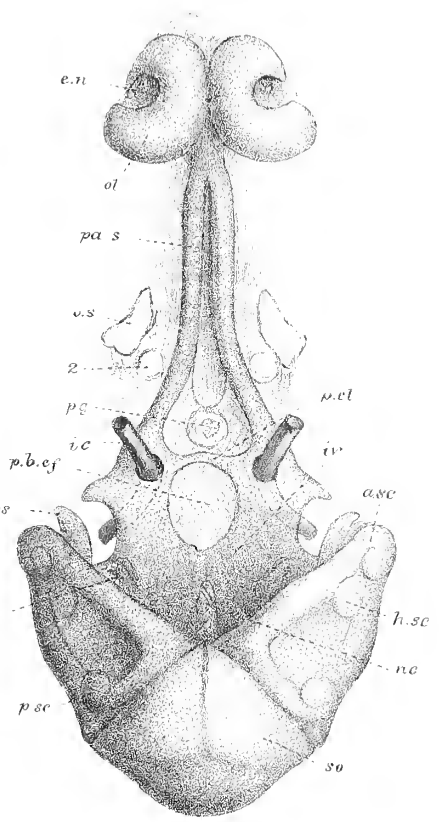
6 × 12



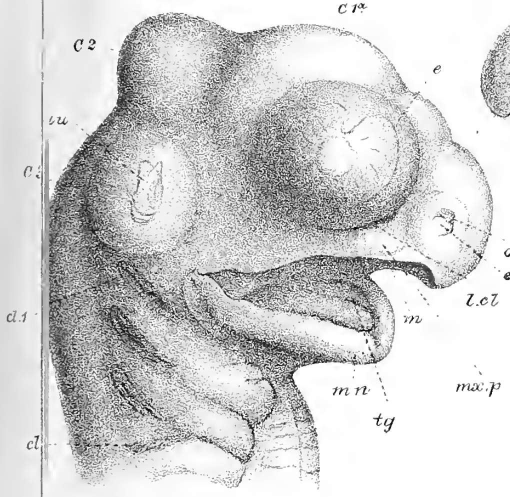
7 × 12



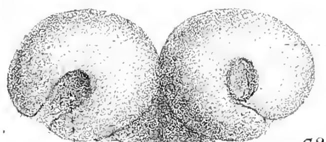
8 × 12



1 × 8



9 × 12



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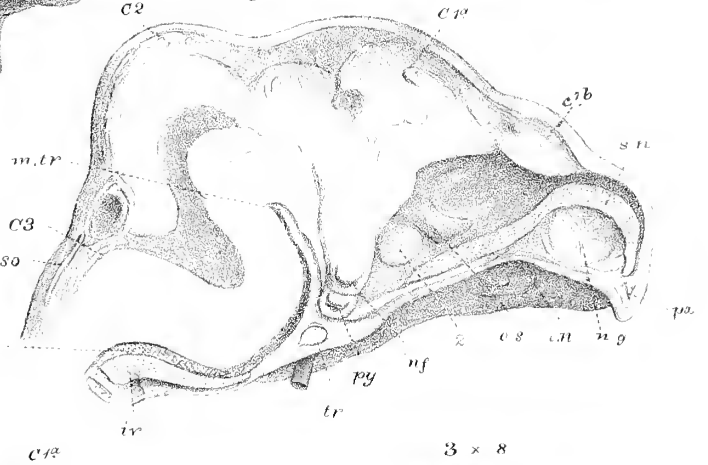
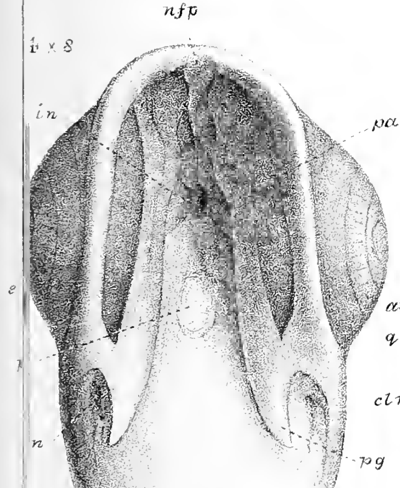


4 × 8



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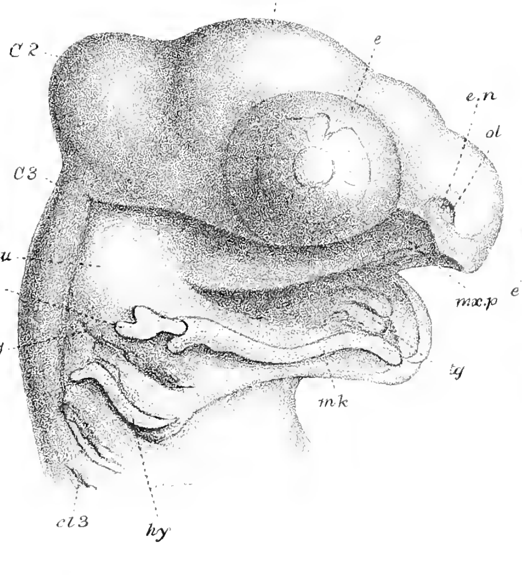
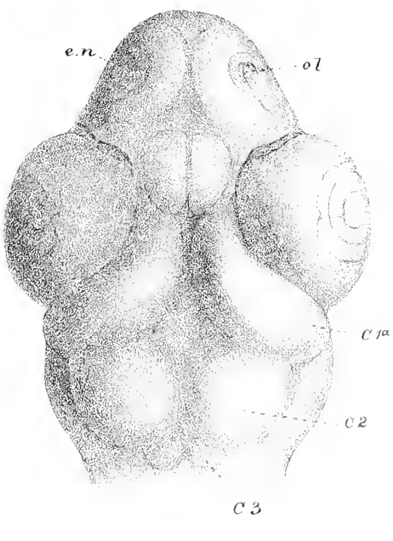


3 × 8

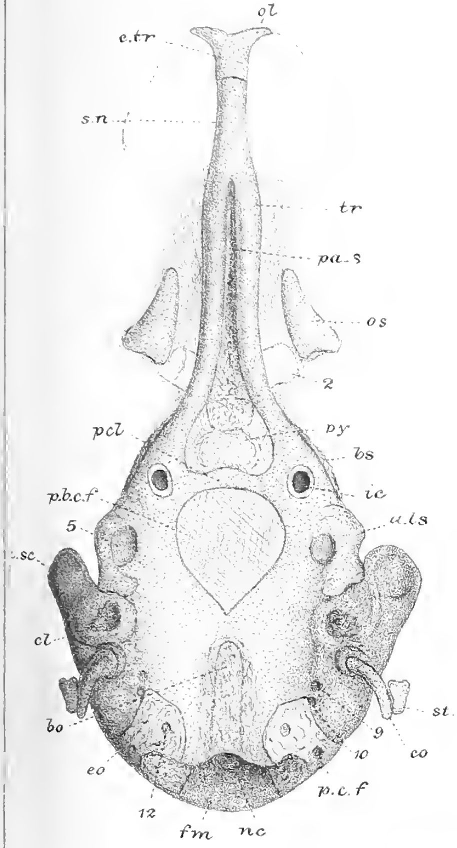


K. P. del. ad nat.
West Newm. lith.

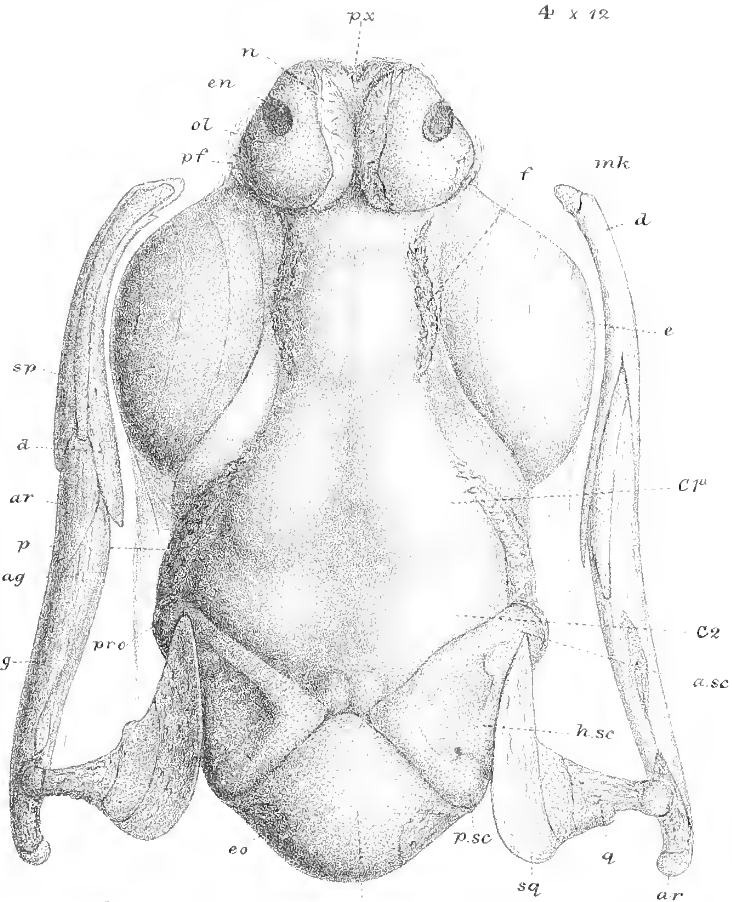
W. West & Co. imp.

Common Snake.

5 x 12



4 x 12



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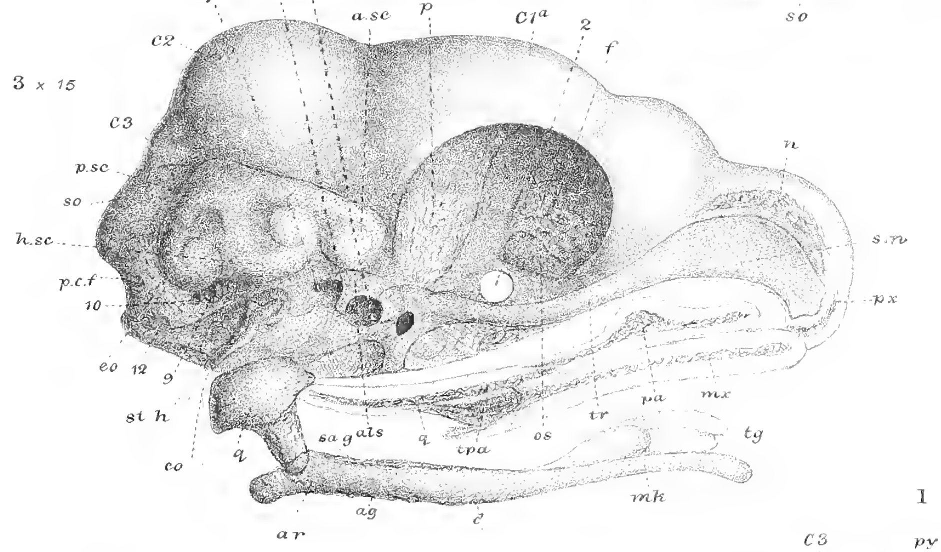

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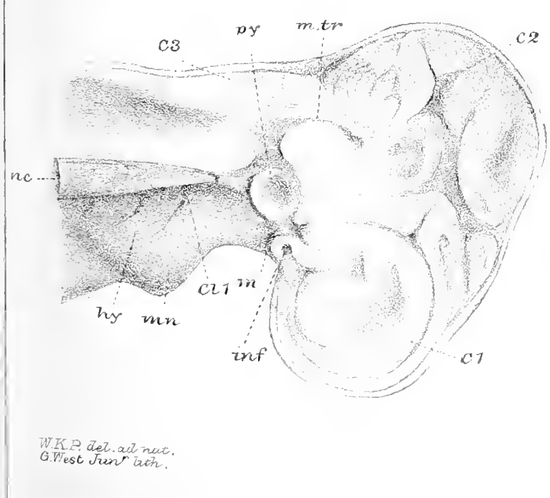
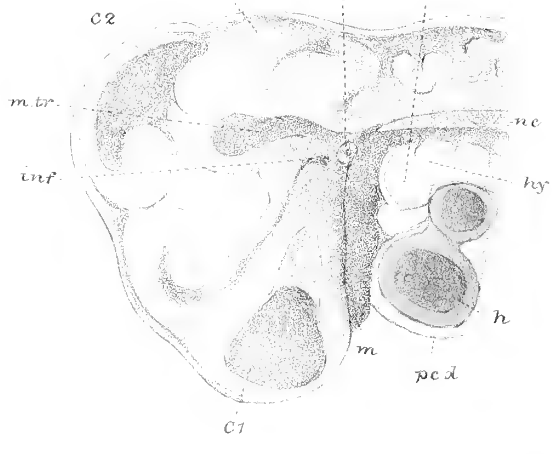


1 x 17

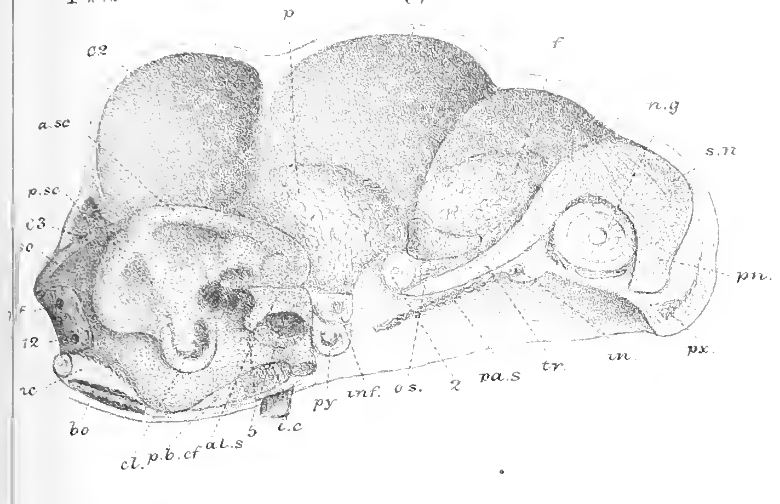


W.K.P. del. ad nat.
G.West Jun^{r} lith.

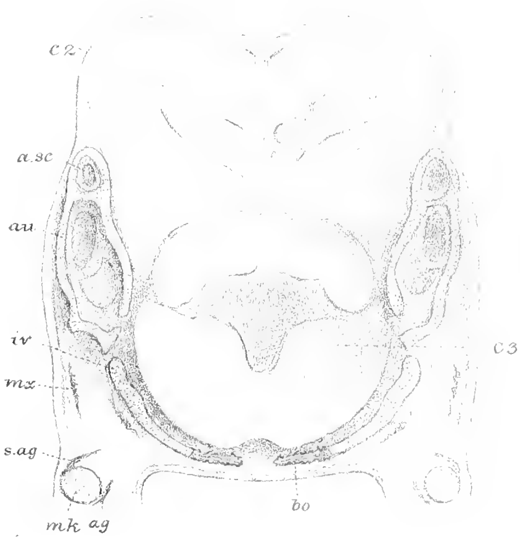
W.West & Co. imp.

Common Snake.

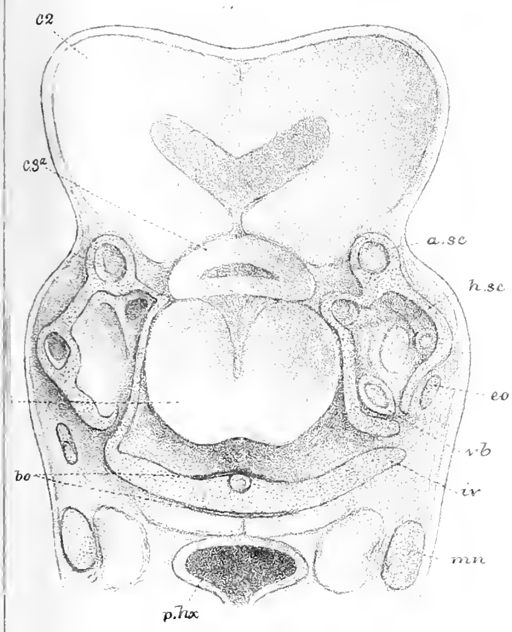
1 × 12



6 × 16



7 × 16



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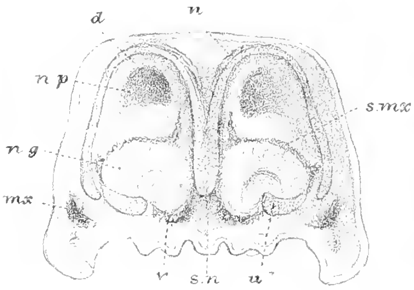
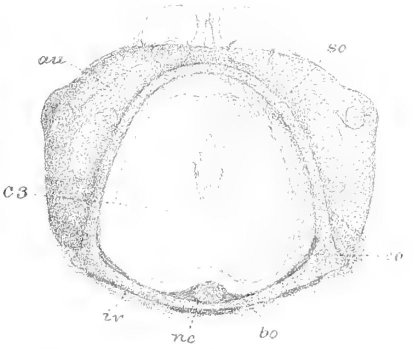
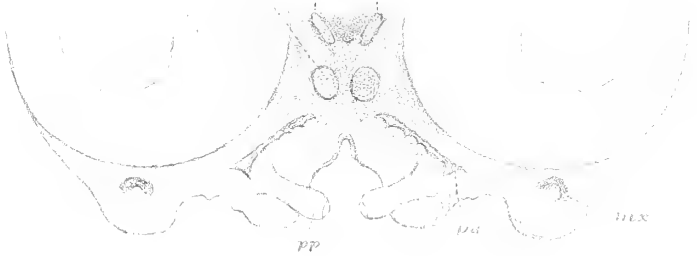


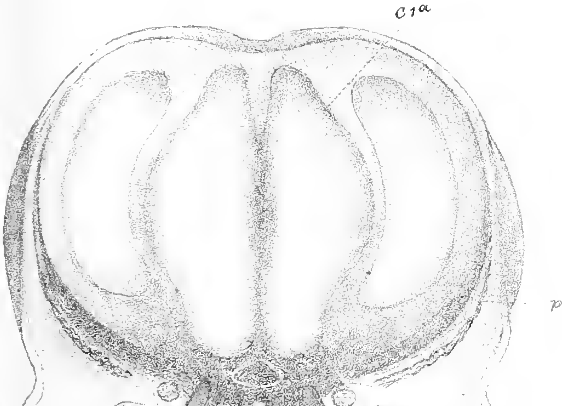
8 × 16



4 × 16



5 × 16



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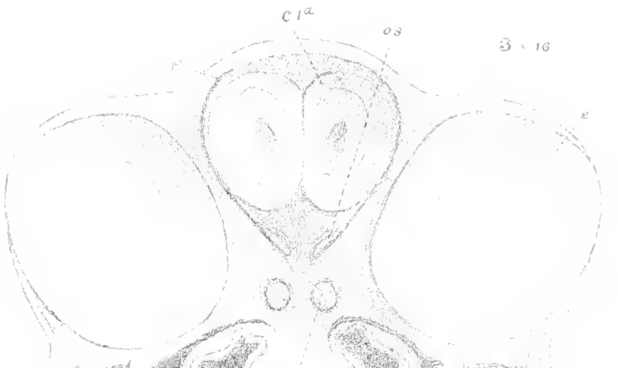
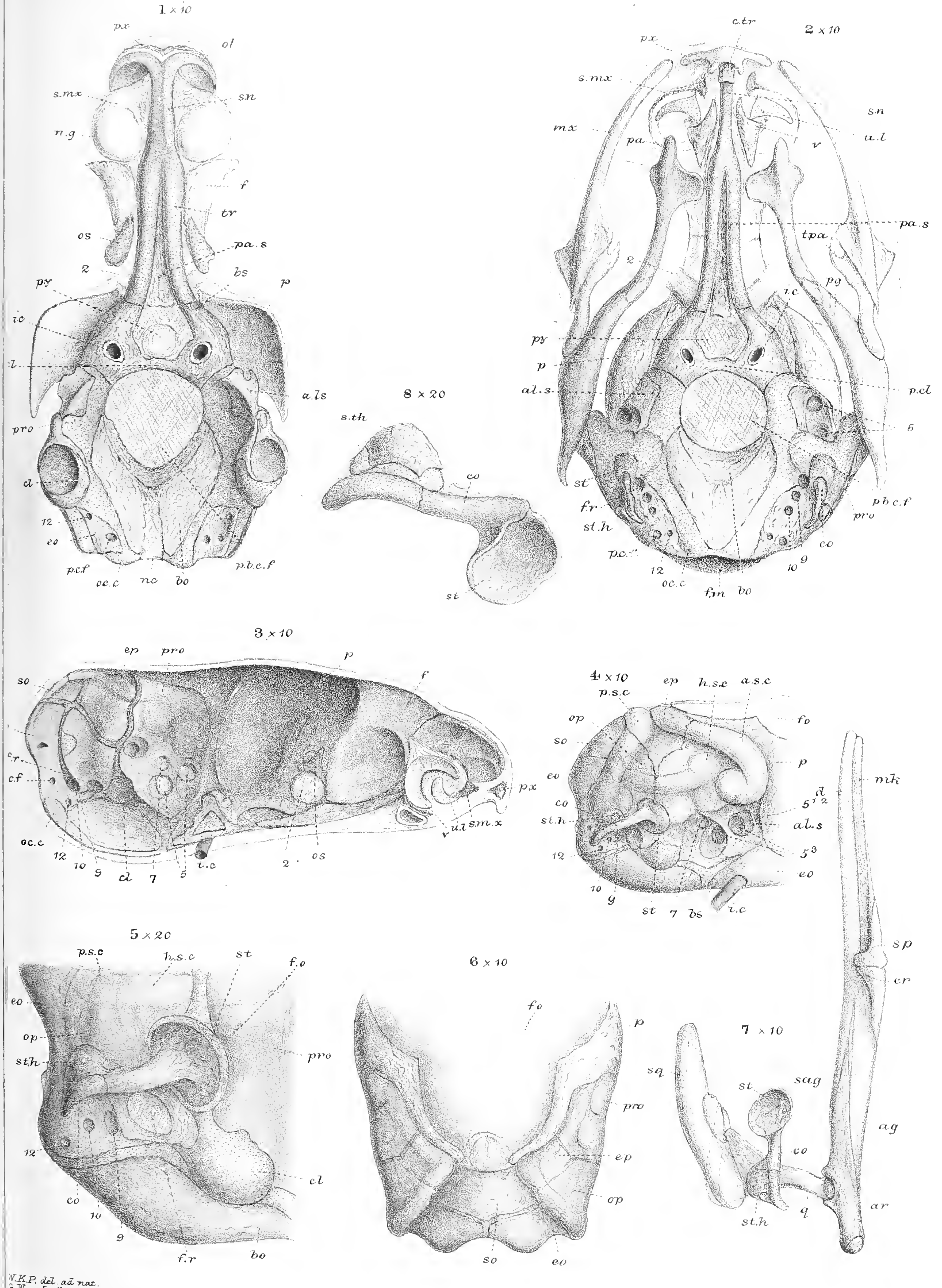


W.K.P. del. ad nat.
G. West Jun^r. lith.

W West & Co imp.

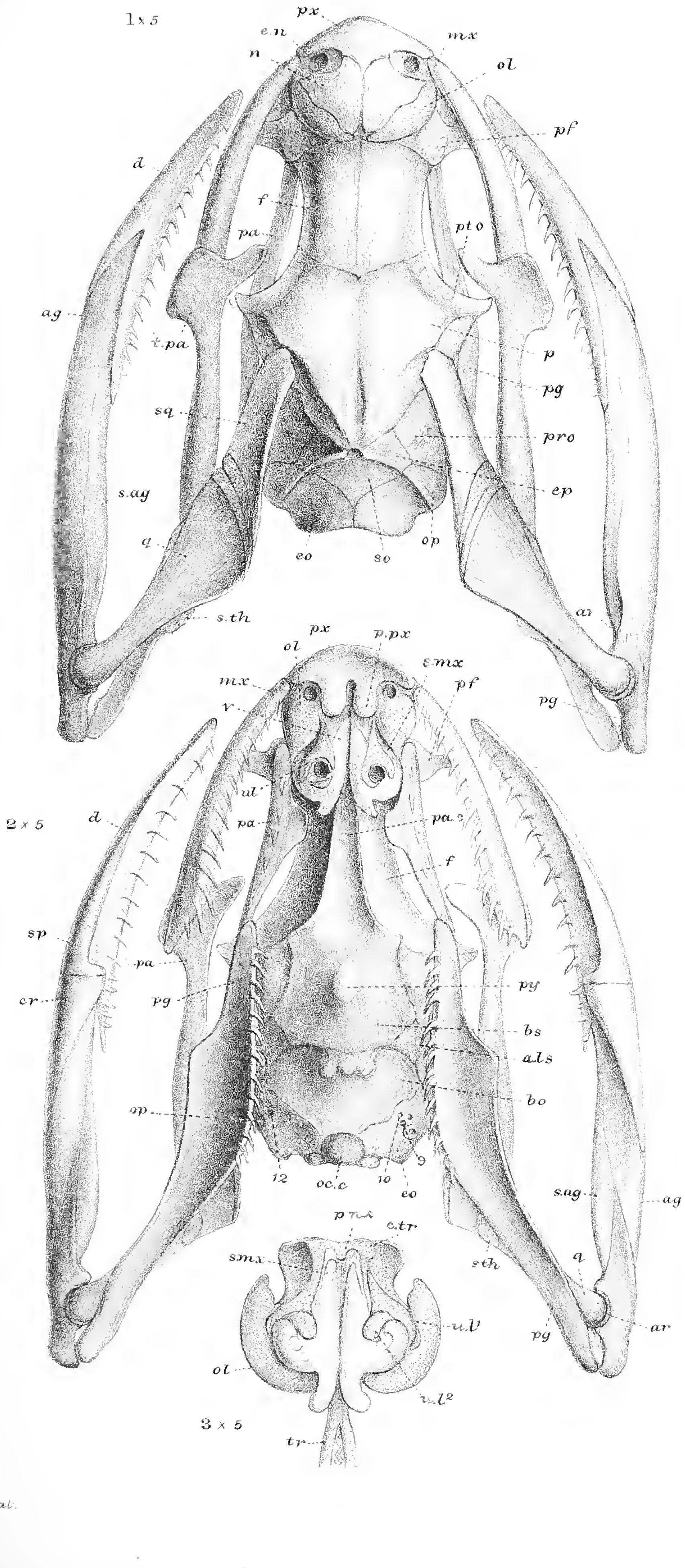
Common Snake



W.K.P. del. ad nat.
G West Jun.r lith.

W.West & Co. imp.

Common Snake.



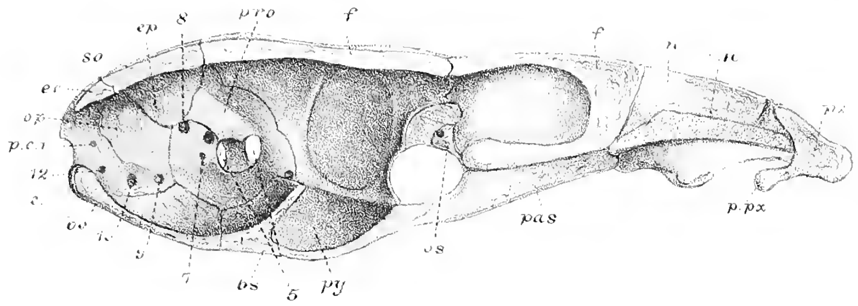
W. K. P. del ad nat.
G. West Junr. lith.

W West & Co imp.

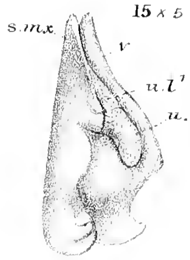
Common Snake.

16 × 5

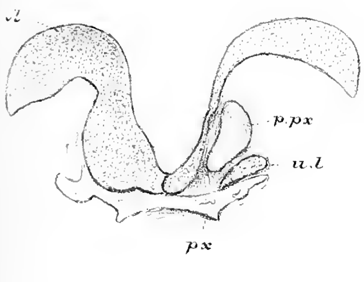
1 × 5



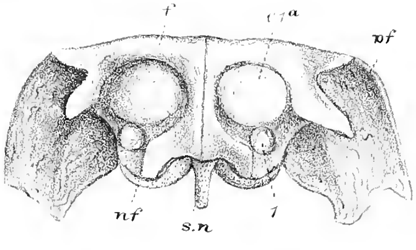
15 × 5



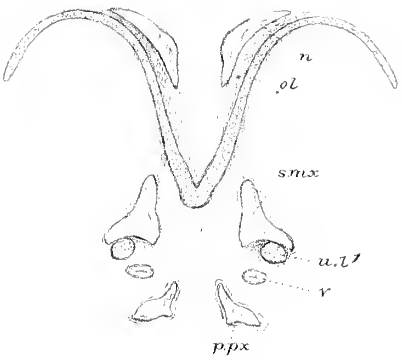
5 × 12



4 × 5



6 × 12



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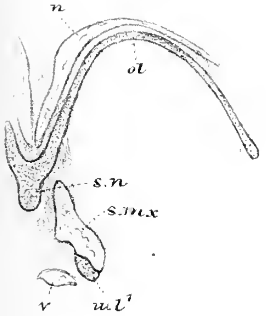
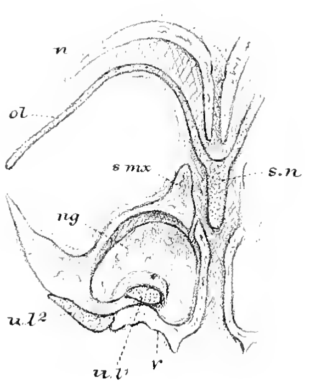
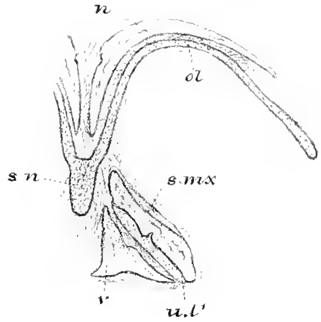


9 × 12



8 × 12



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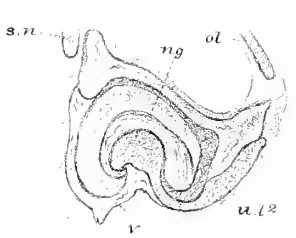
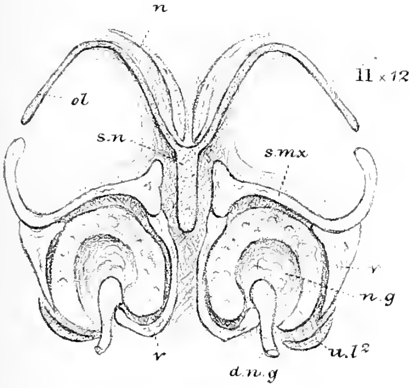
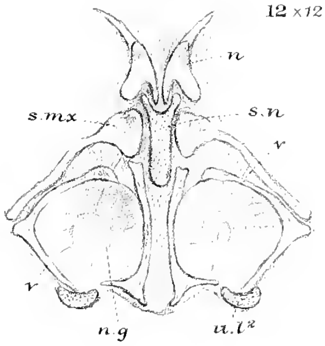


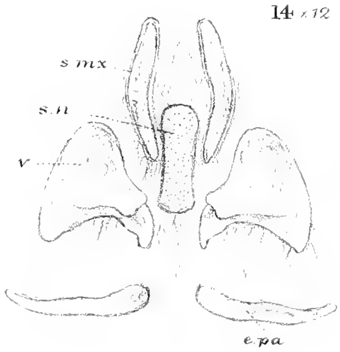
11 × 12



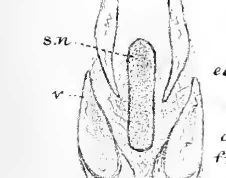
12 × 12



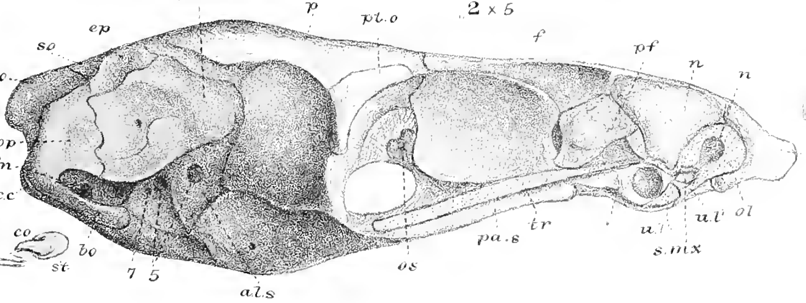
14 × 12



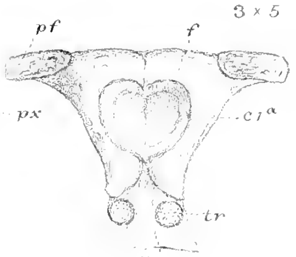
13 × 12



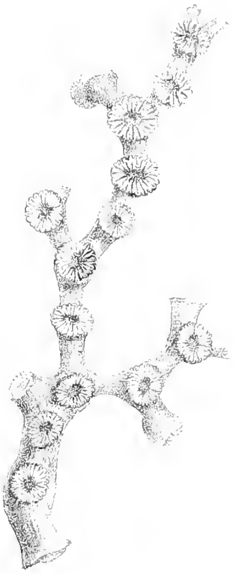
2 × 5

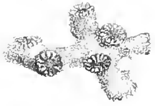


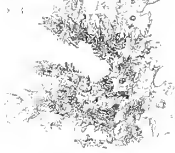
3 × 5



Common Snake.







JErxl n. ad nat. lith.

[illegible]

1, 2 SPORADOPORA. 4 ASTYLUS. 6 ALLOPORA
3 SPINIPORA. 5 STYLASTER 7 ERRINA.

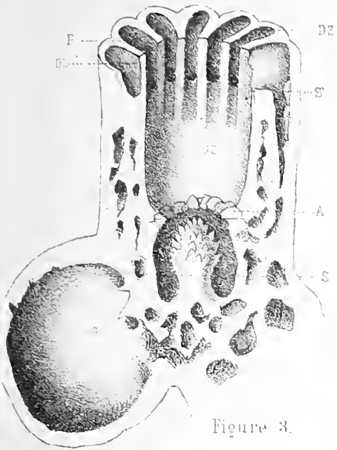


Figure 3.

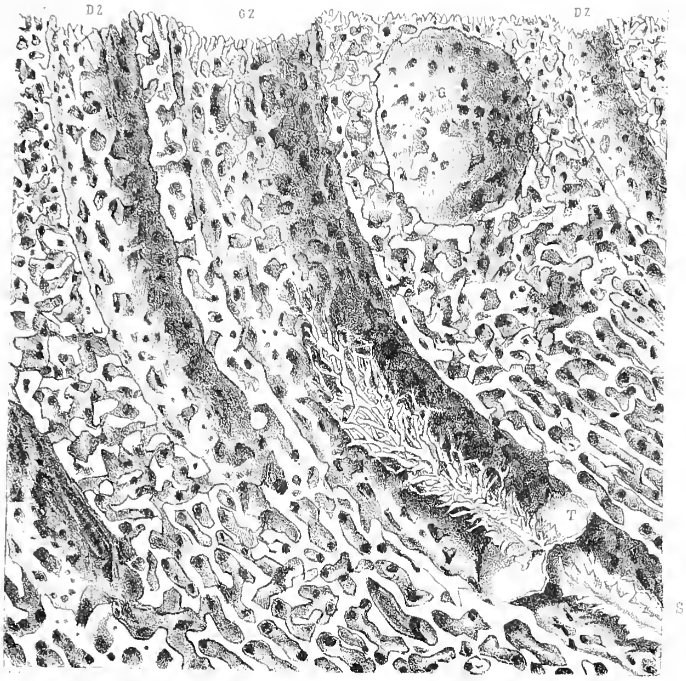


Figure 1 × 40

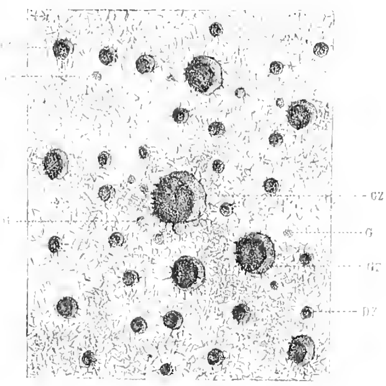


Figure 2 × 10

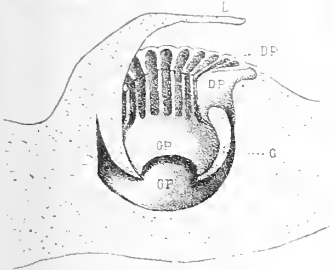


Figure 7.

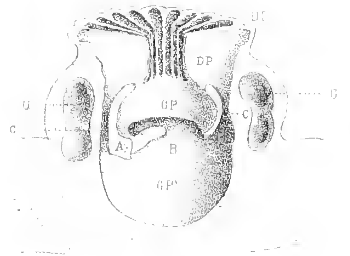


Figure 8.

Figure 4 × 7

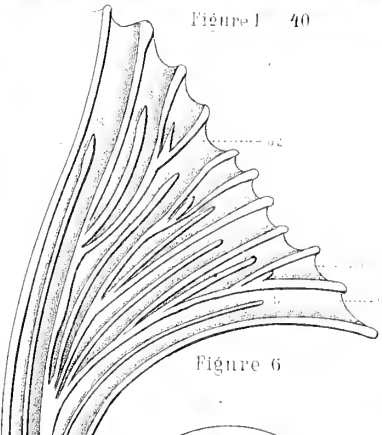


Figure 6

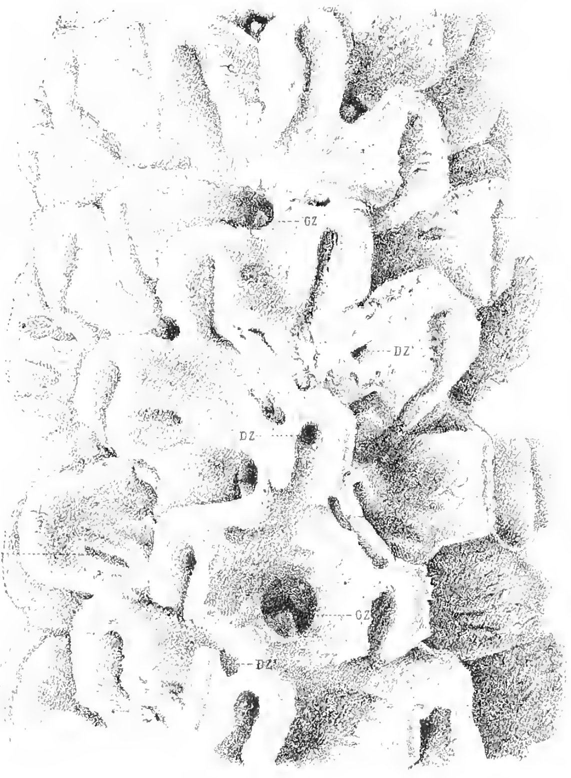


Figure 5 × 16.

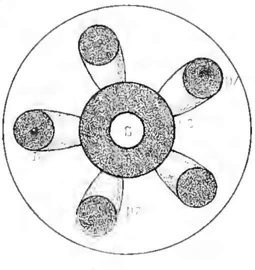


Figure 11.

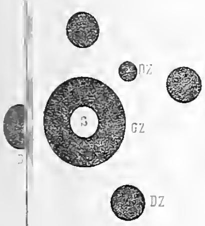


Figure 9.

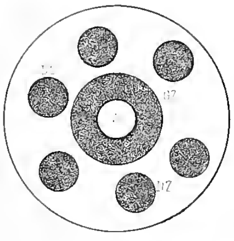


Figure 10

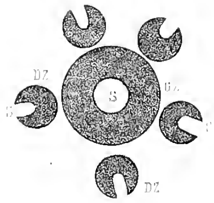


Figure 12.

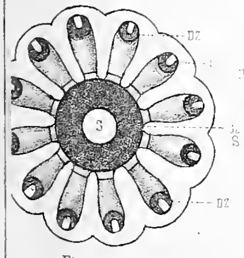


Figure 13.

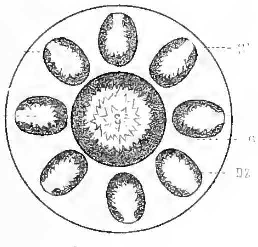


Figure 14

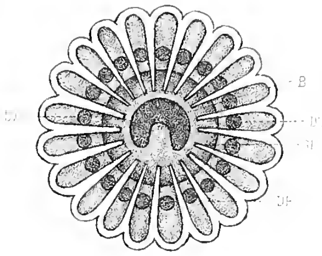


Figure 15

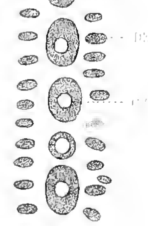
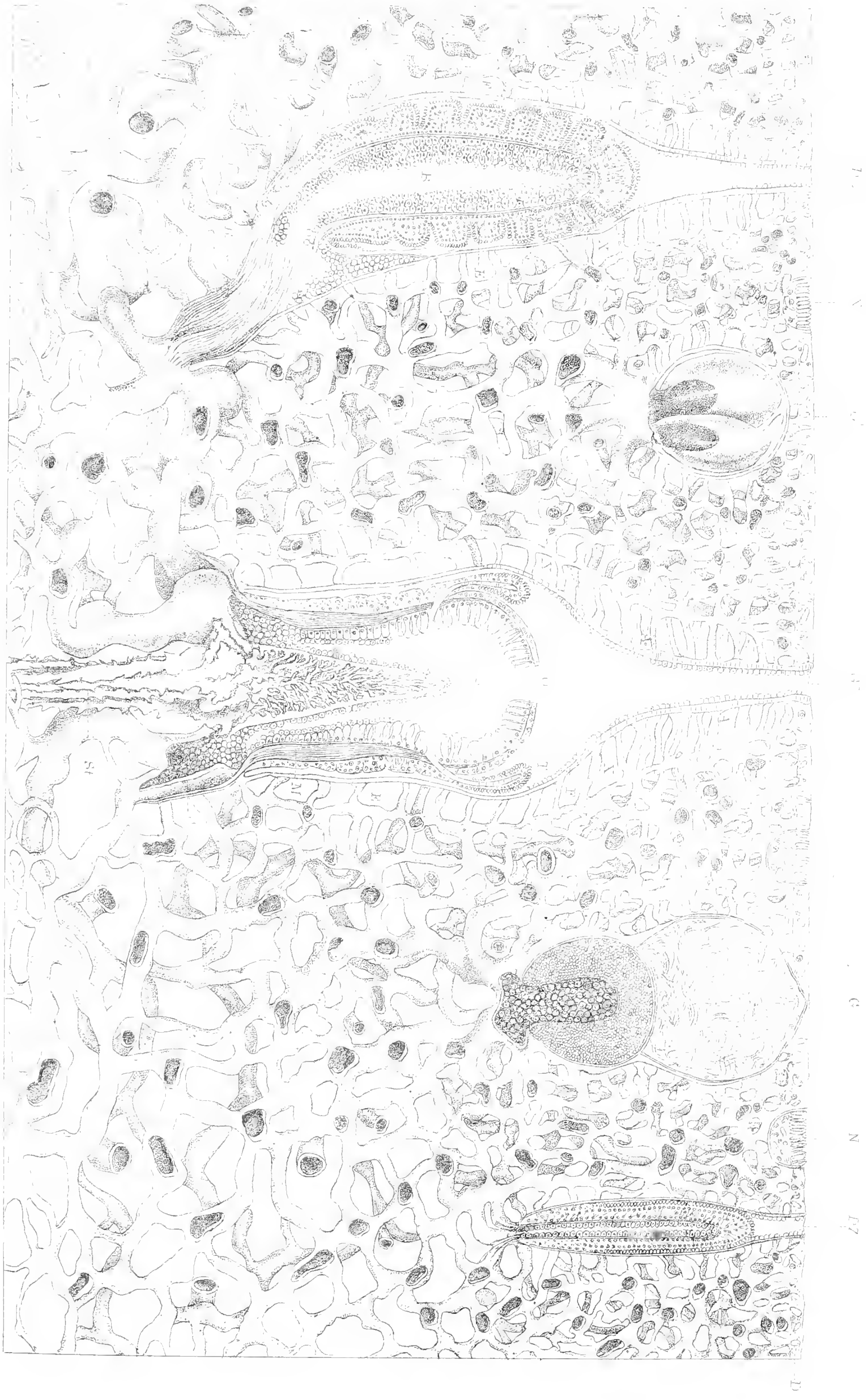


Figure 16

osel & Stewart del

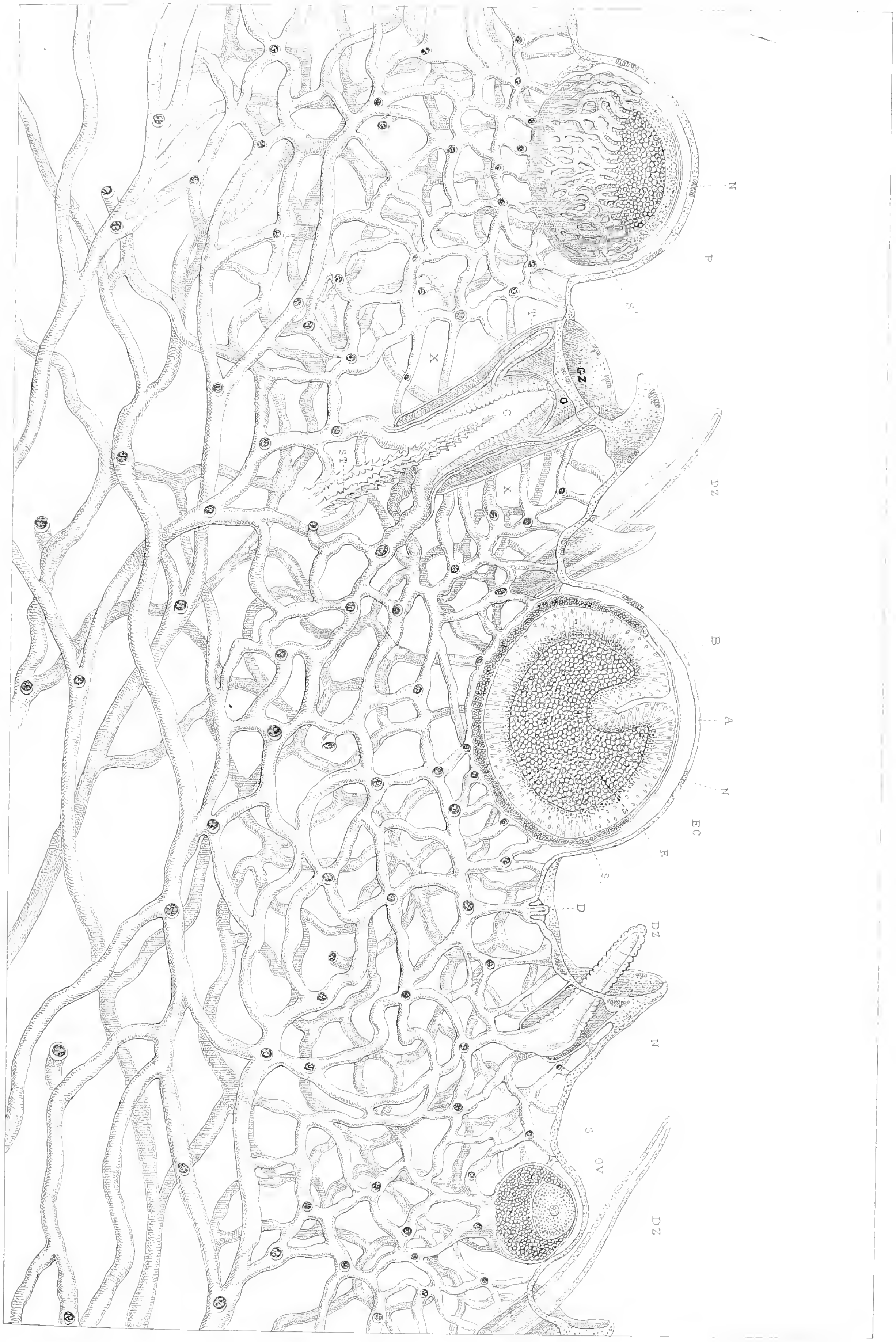
1, 2), SPORADOPORA. 3, 13, 14, STYLASTER. 7, CRYPTOHELIA. 8, 15, ASTYLUS. 4, SPINIPORA.
5, LABIOPORA. 6, 16, DISTICHOPORA 10, 11, 12, ALLOPORA.



Sporadopora dichotoma, x60

seley J.Erxleben, lith

SPORADOPORA.

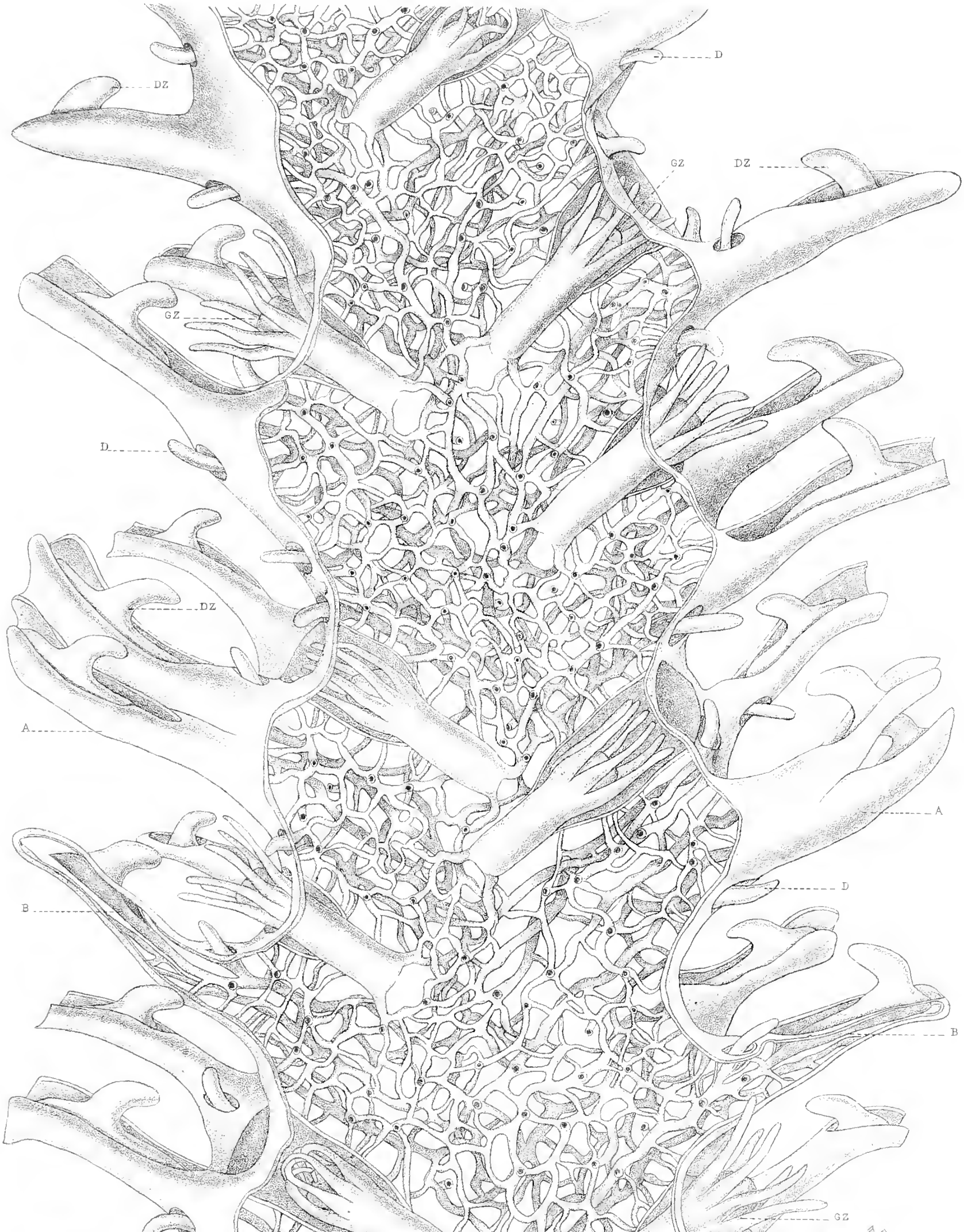


Errina labiata × 40.

Mos r, delt

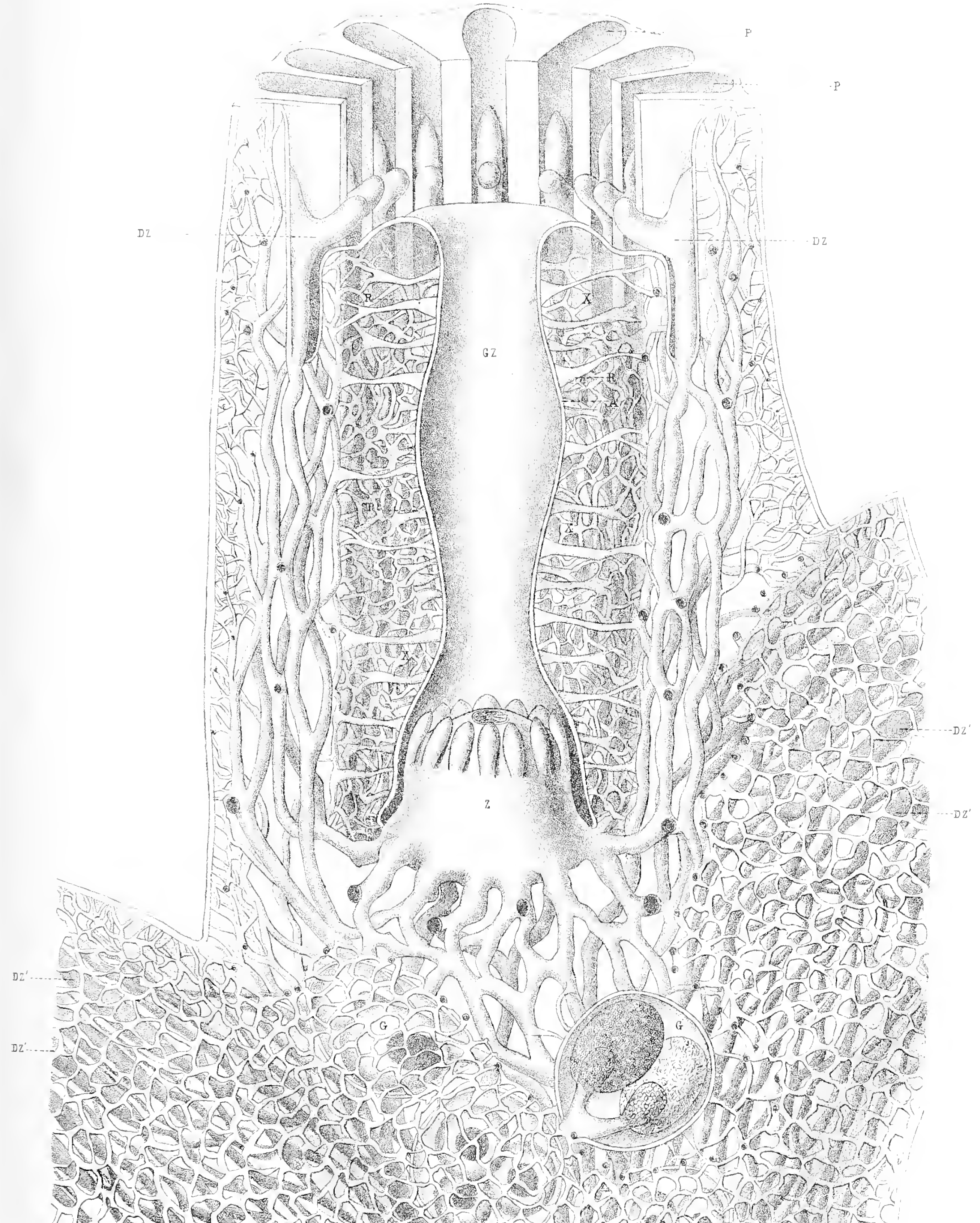
F Huth, Lithr Edinr

ERRINA.

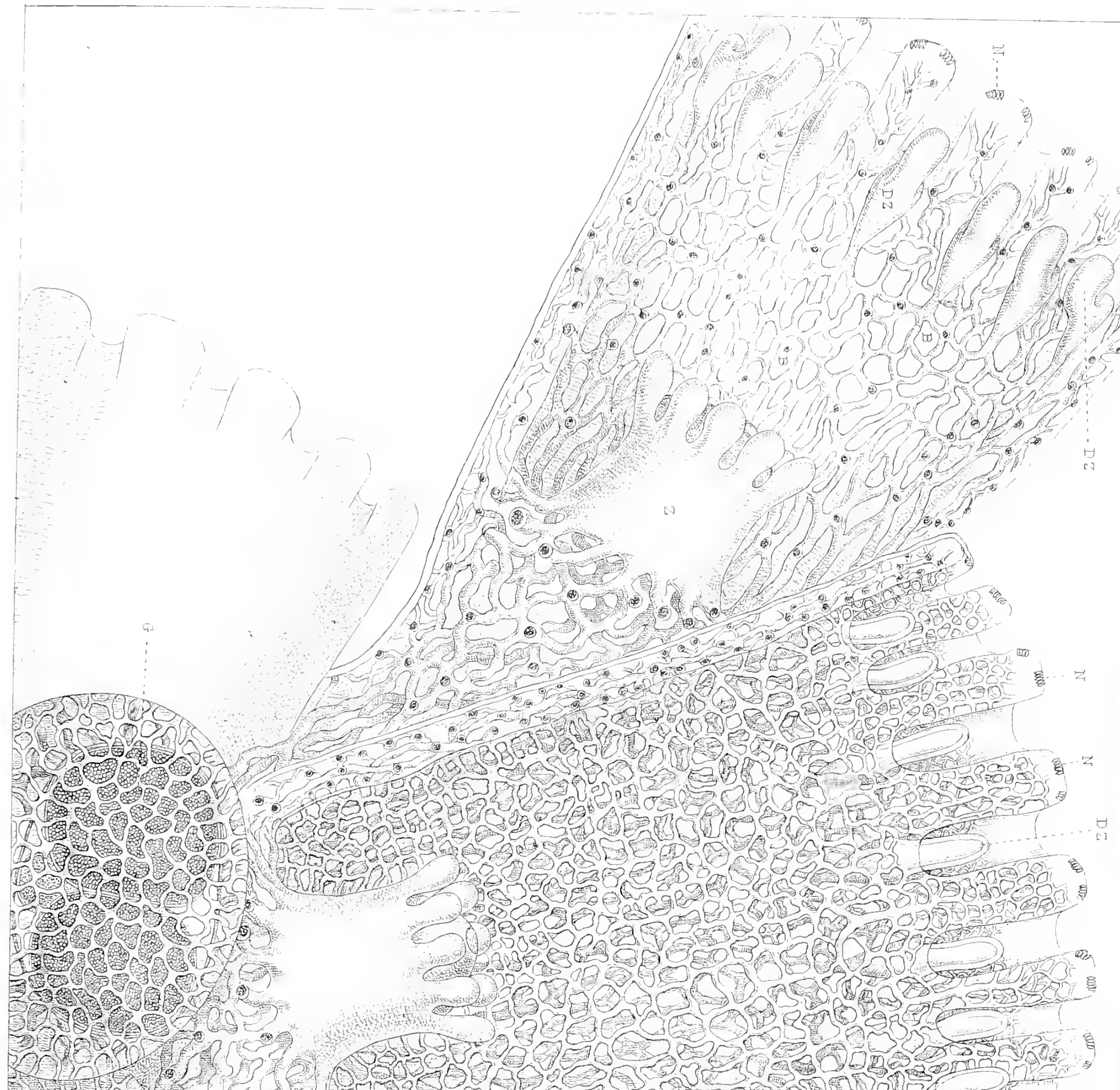


seley d.

SPINIPORA ECHINATA × 15.



ALLOPORA PROFUNDA × 60



Stylaster densicaulis × 60.

H. Moseley, delt.

F. Huth, Lithr Edinr

STYLASTER.

Figure 2. Pliobothrus symmetricus × 50.

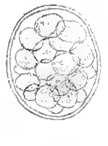


Figure 3. × 50.

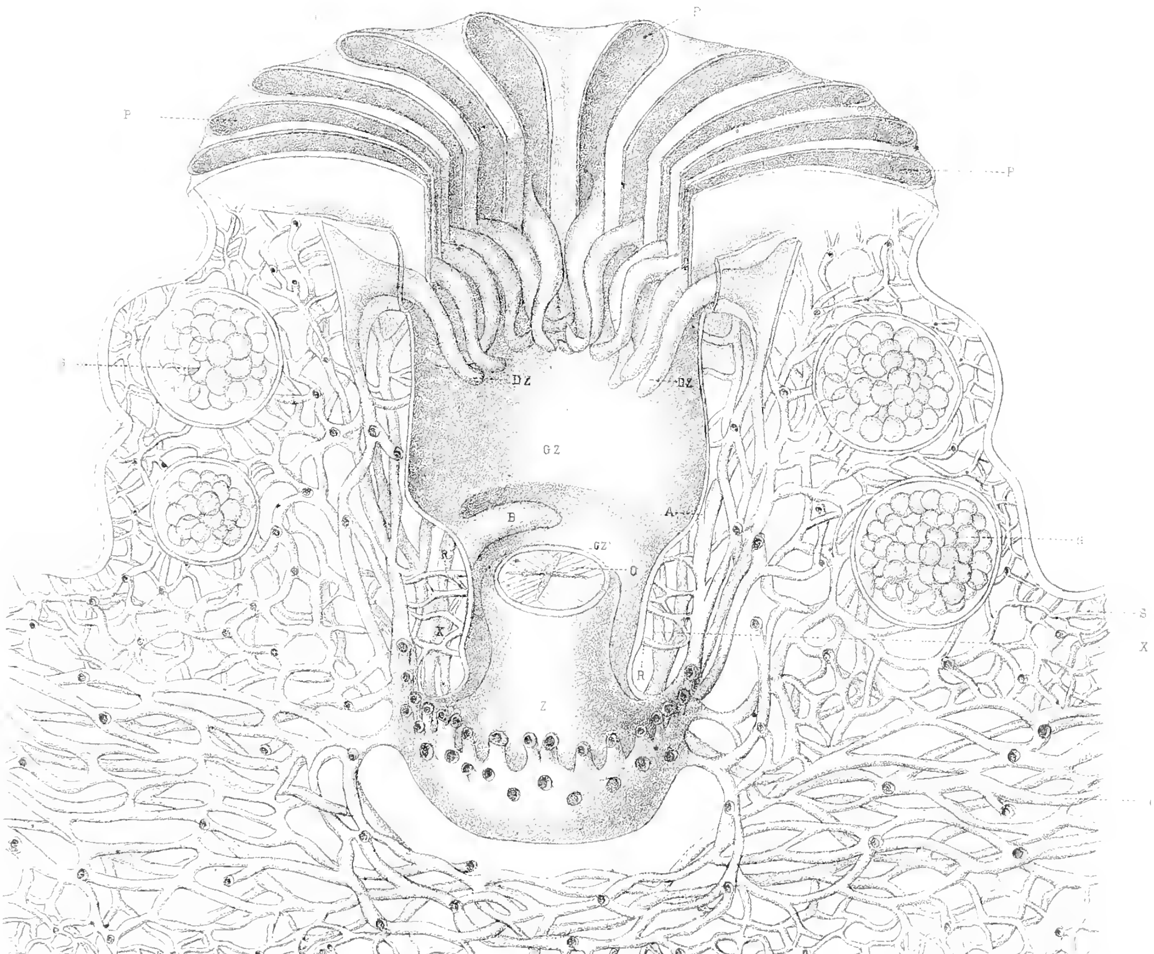
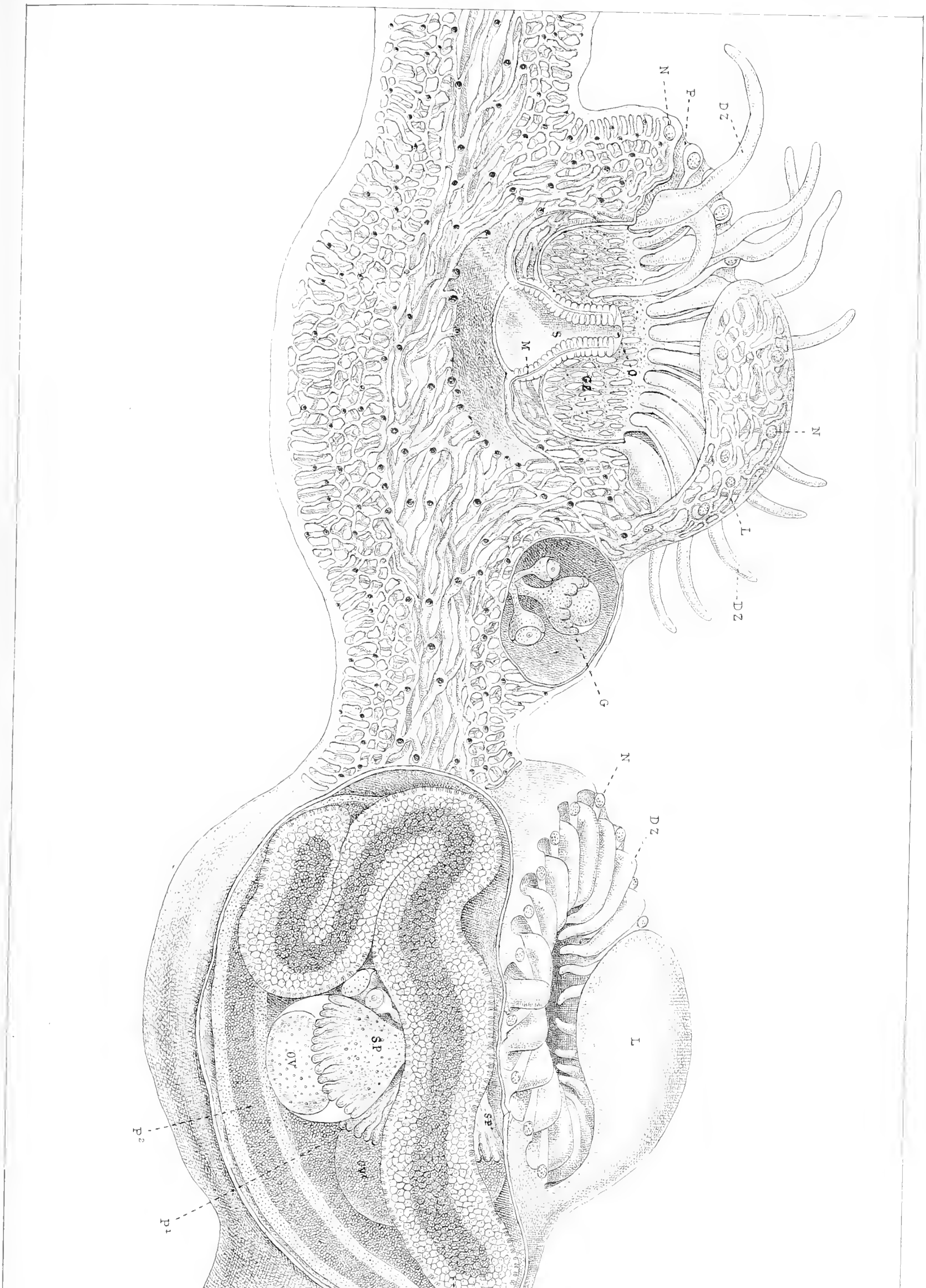


Figure 1. Astylus subviridis × 40.

1 ASTYLUS. 2 3 PLIOBOTHRUS.



Cryptohelia affinis × 30.

Mose[illegible], del^t

F. Huth, Lith^r Edin^r

CRYPTOHELIA.

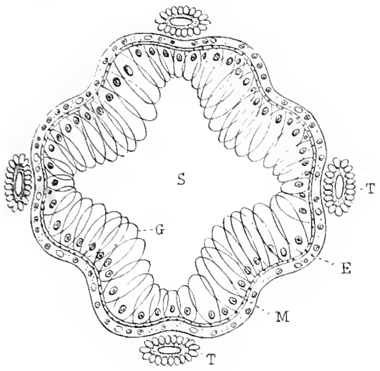


Fig 1. × 85.

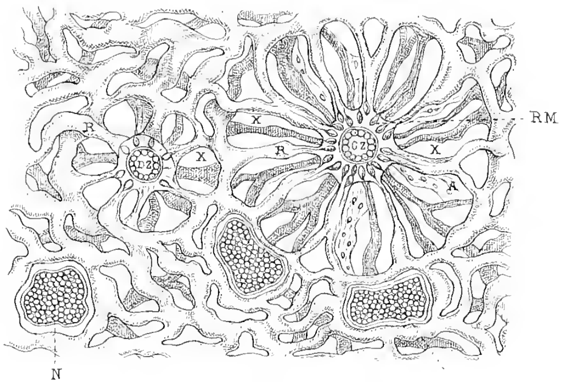


Fig. 3. × 70.

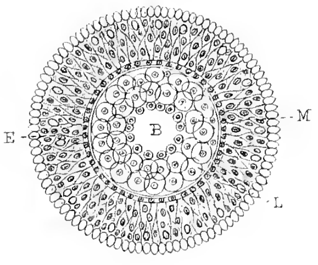
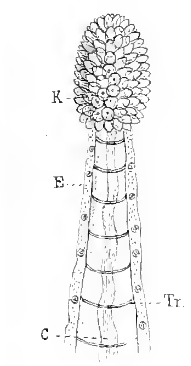
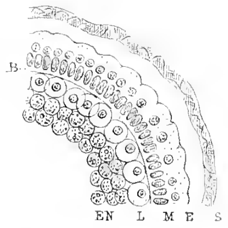


Fig. 2. × 85



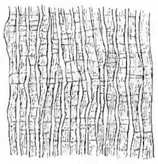
× 100.

Fig. 4.



× 100.

Fig. 5.

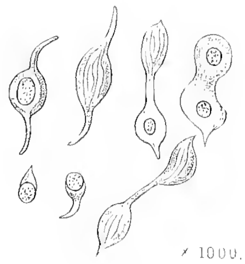


× 100.

Fig. 6.

× 100.

Fig. 7.



× 1000.

Fig. 8.

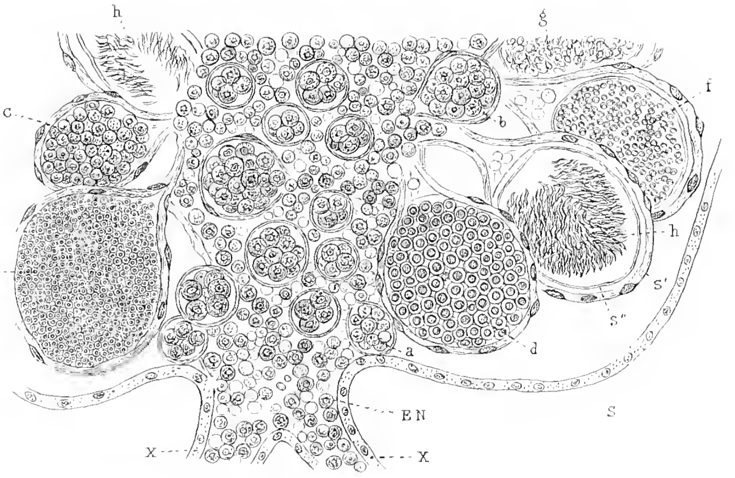
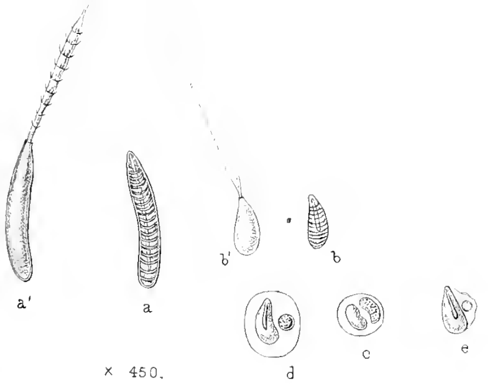


Fig. 10. × 300.



× 450.

Fig. 9.

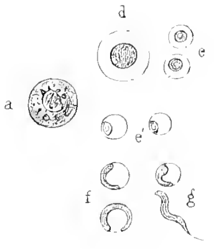
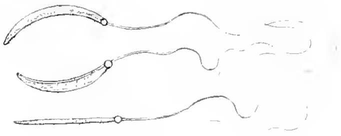


Fig 11.



× 900.

Fig 12.

Moseley, del.t

F. Huth. Lith.r Edin.r

1-9, 12. SPORADOPORA. 10. 11 ASTYLUS.

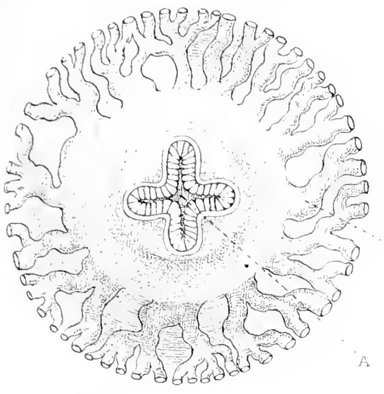


Fig. 1. × 50.

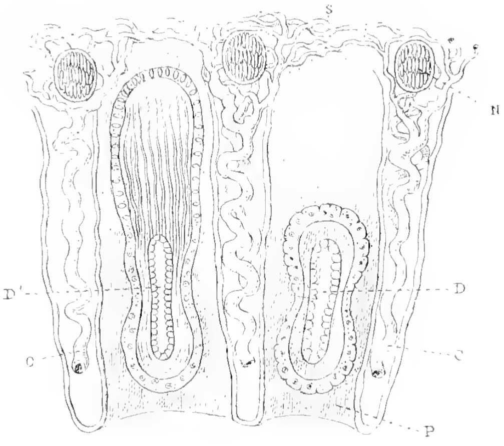
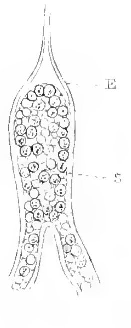


Fig. 2. × 100.



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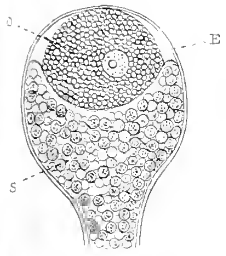


Fig. 4 × 150.

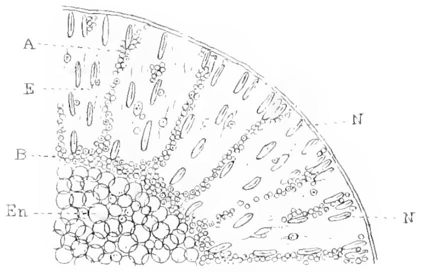


Fig. 7. × 125.

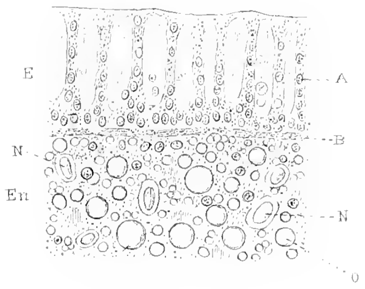


Fig. 9. × 150.

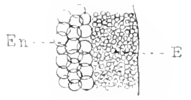


Fig. 5 × 125.

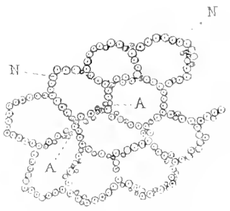


Fig. 8 × 120.

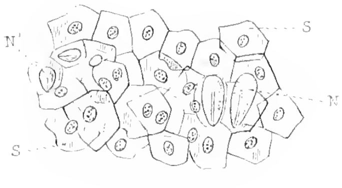


Fig. 10. × 200.

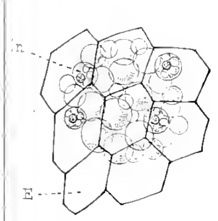


Fig. 6. × 200.

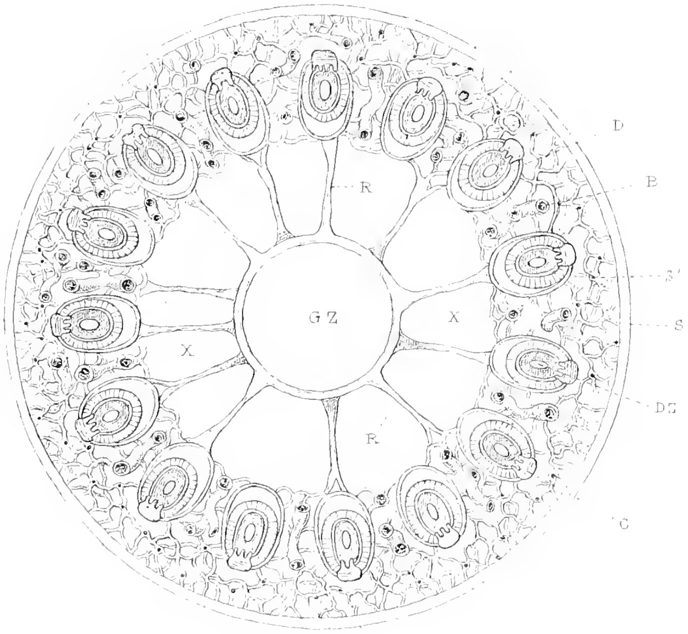


Fig. 12. × 50.

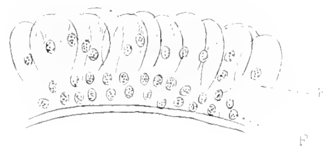


Fig. 11. × 200.

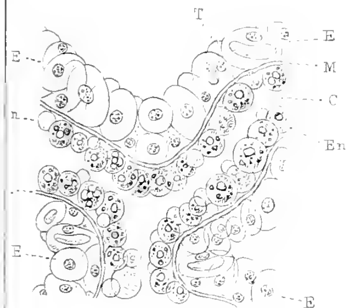


Fig. 13. × 320.

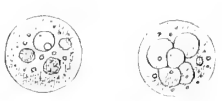


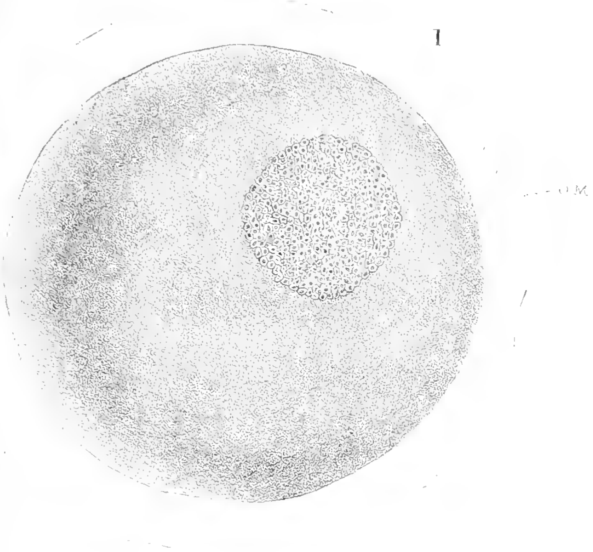
Fig. 14. × 900.

H. oseley, del^{t}

F. Huth, Lith^{r} Edin^{r}

ASTYLUS. CRYPTOHELIA ALLOPORA ERRINA.

1



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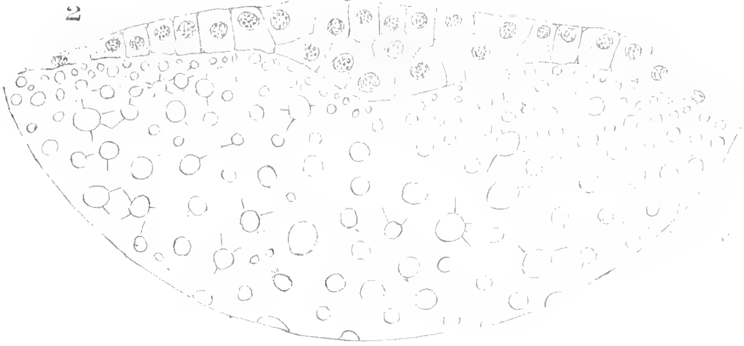

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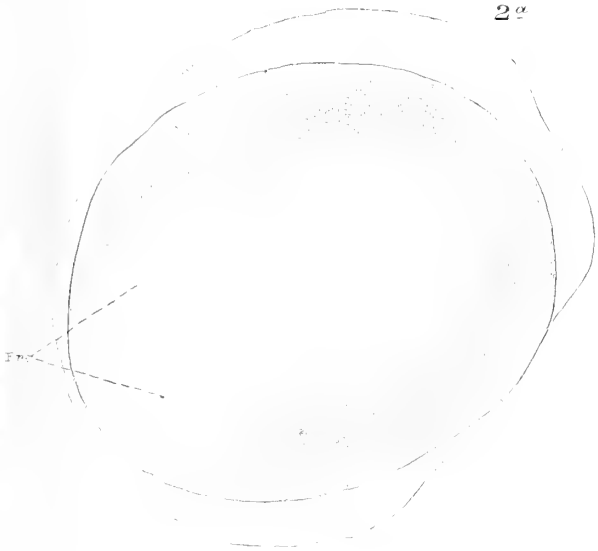

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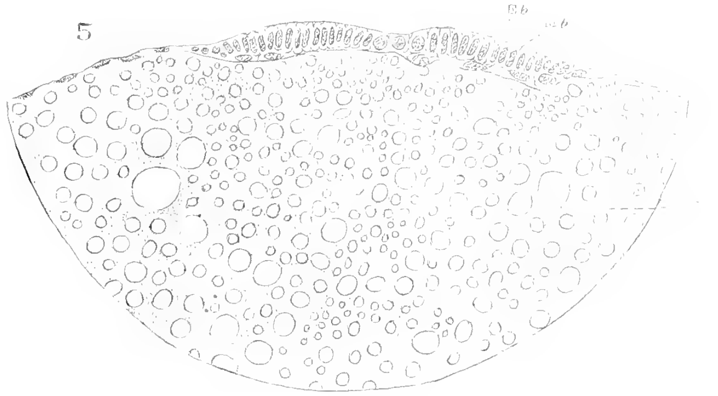

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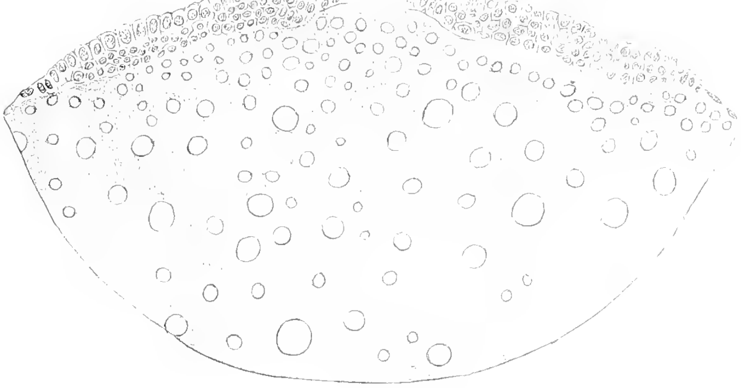

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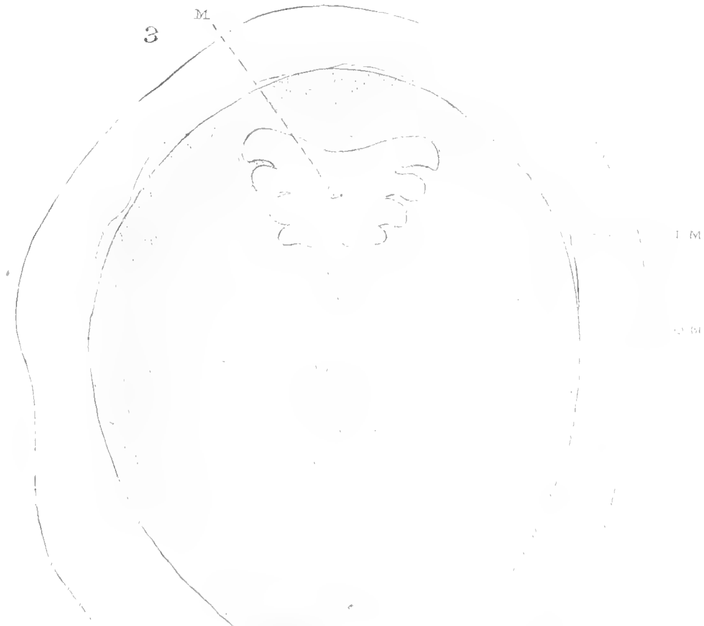

Wes. Newman & Co.

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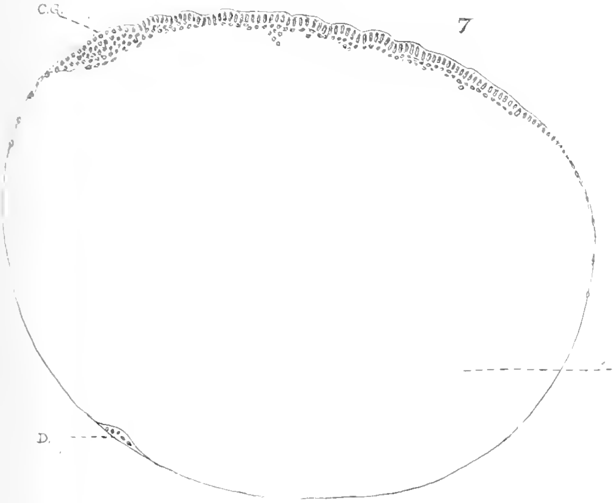

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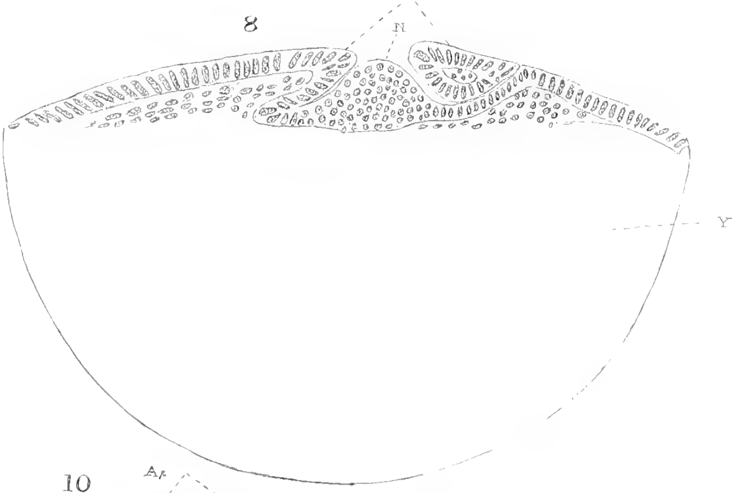

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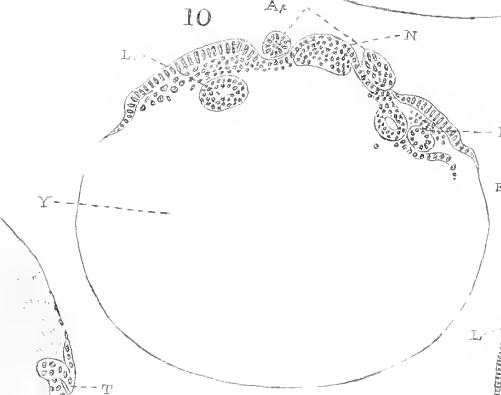

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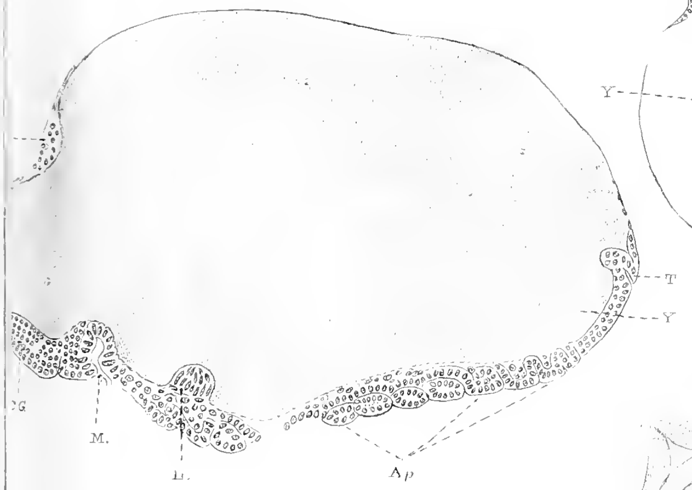

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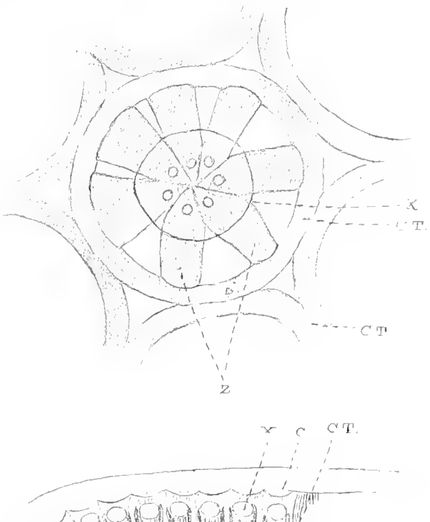

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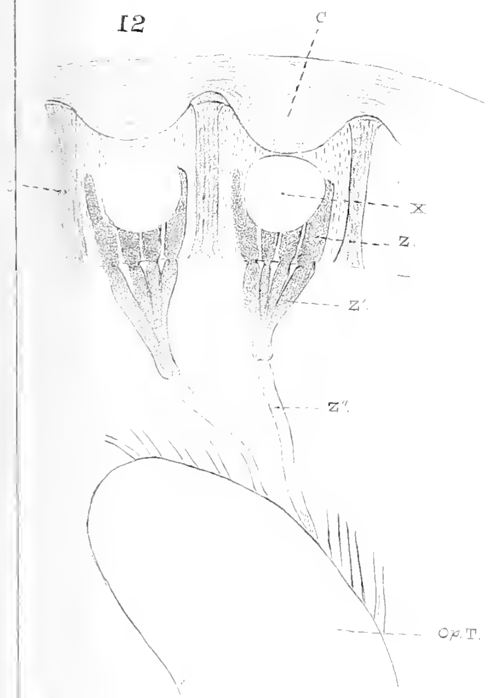

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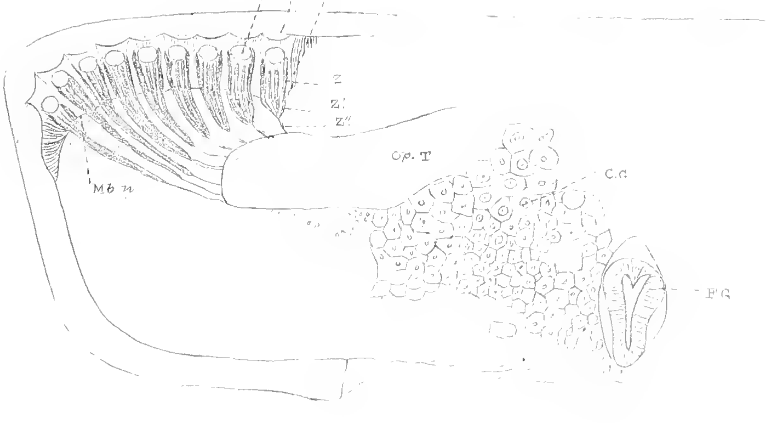

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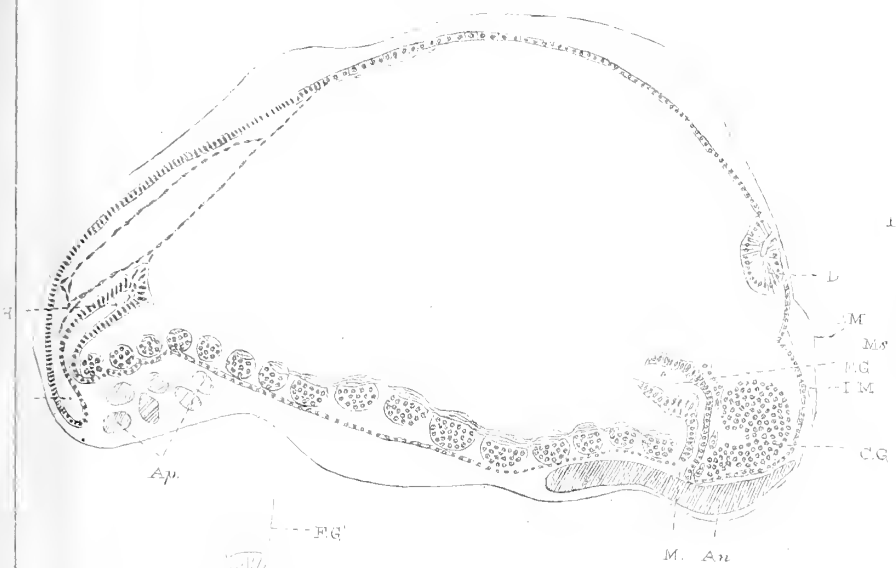

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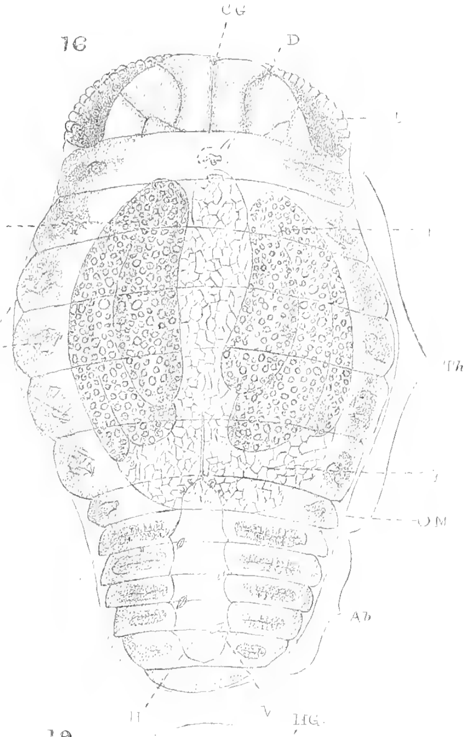

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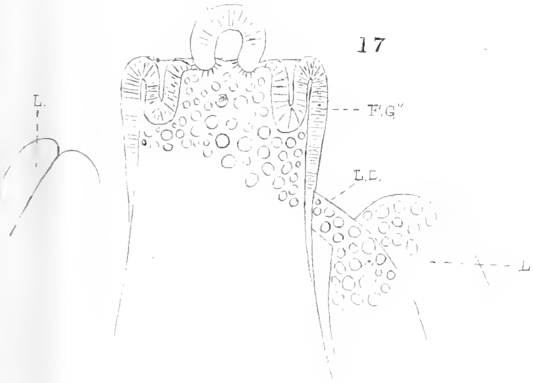

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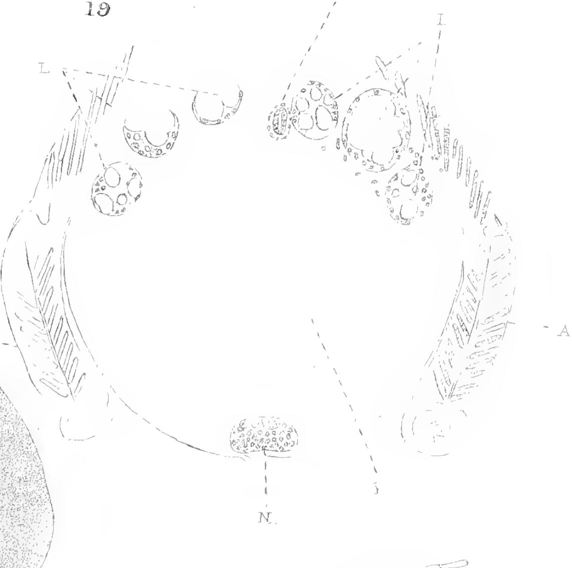

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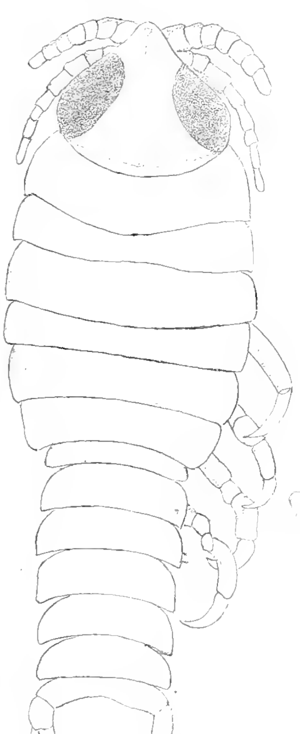

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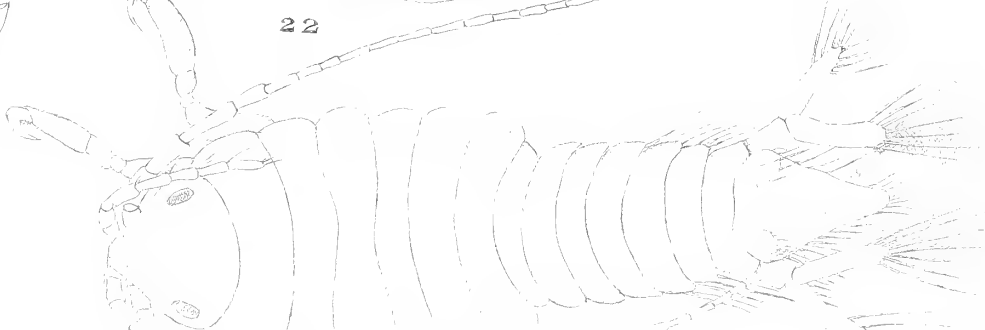

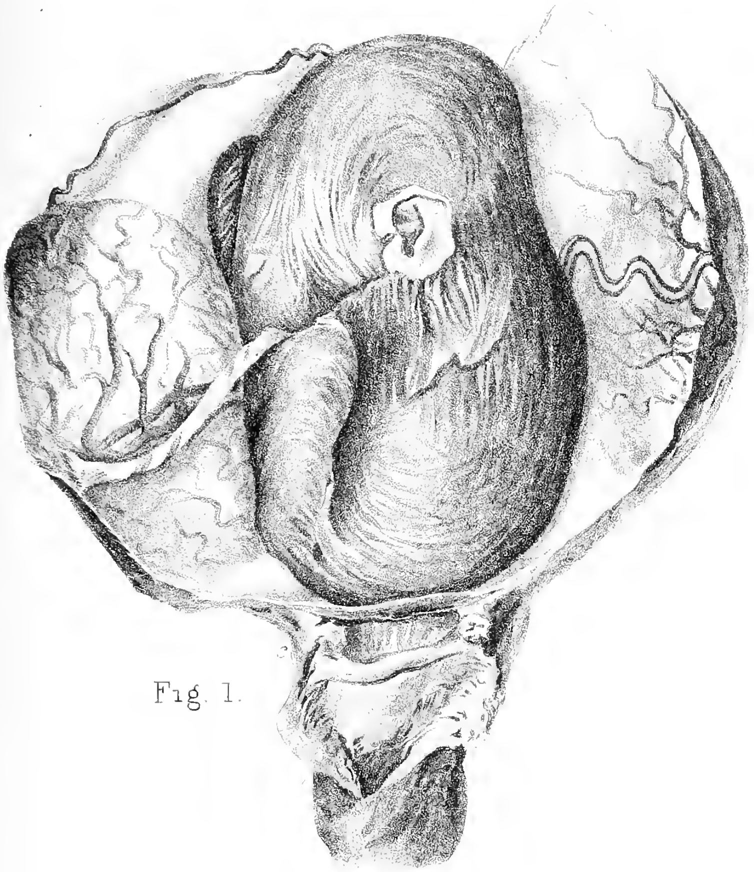

Fig. 1.

Fig. 4.

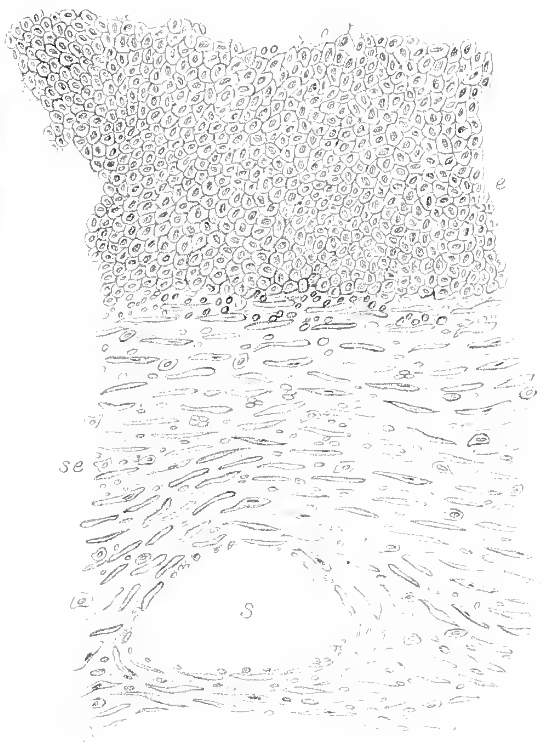

Fig. 3.

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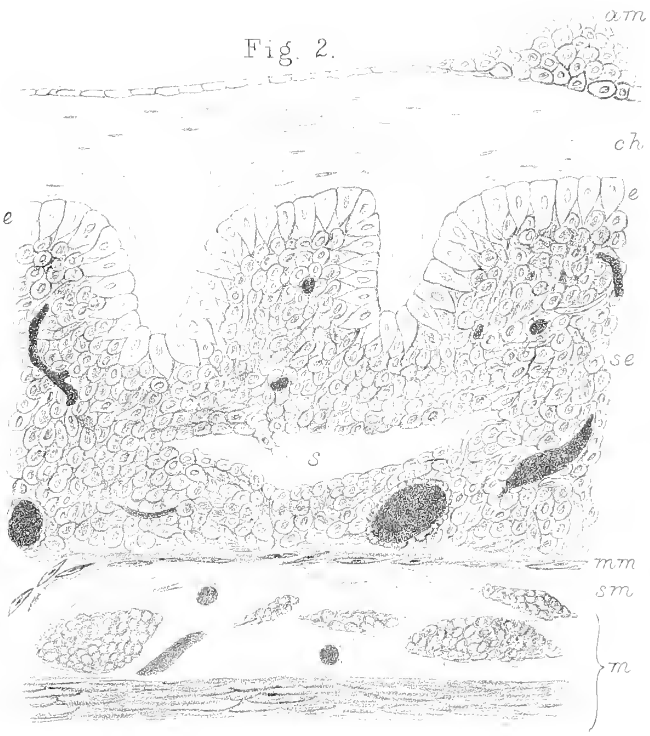
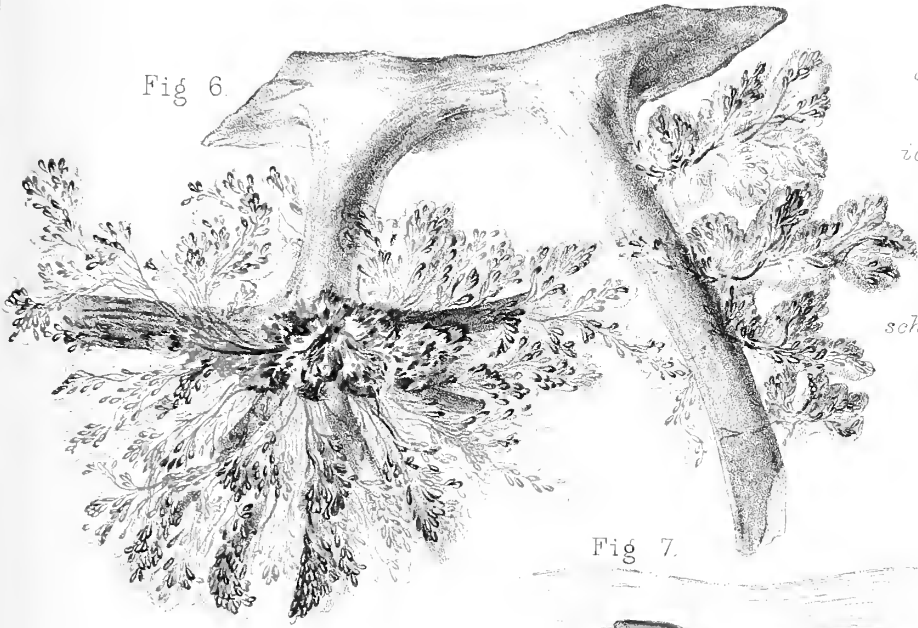


M^c Farlane & Erskine, Lith^rs Edin^r



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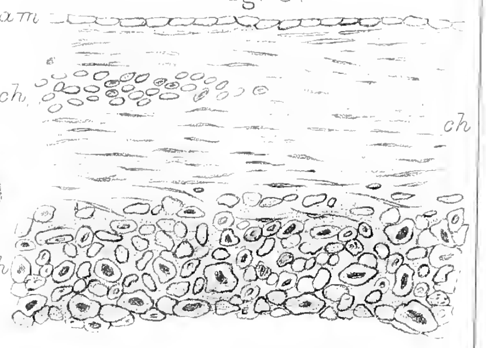

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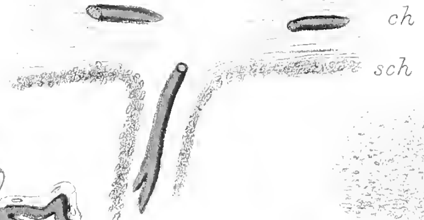

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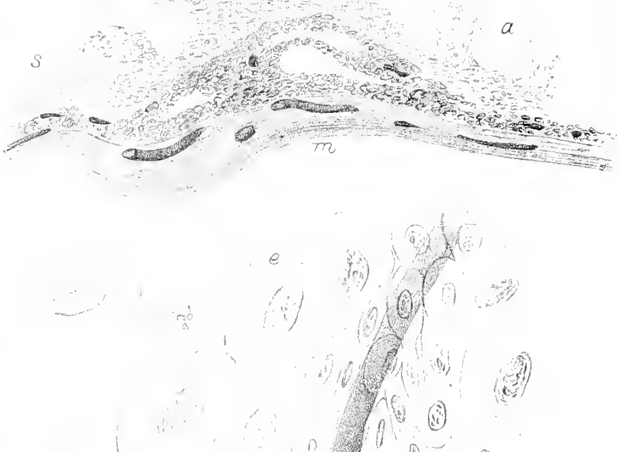
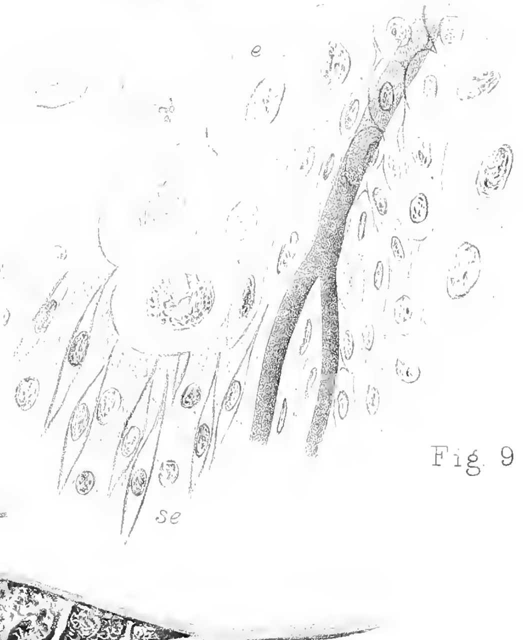


Fig. 10.

Fig. 11.



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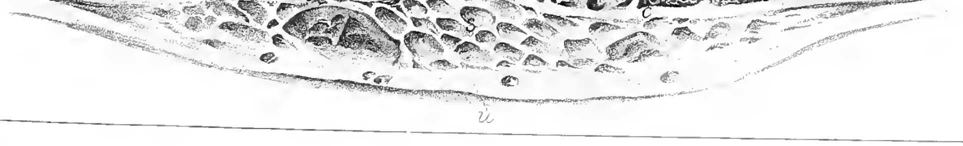

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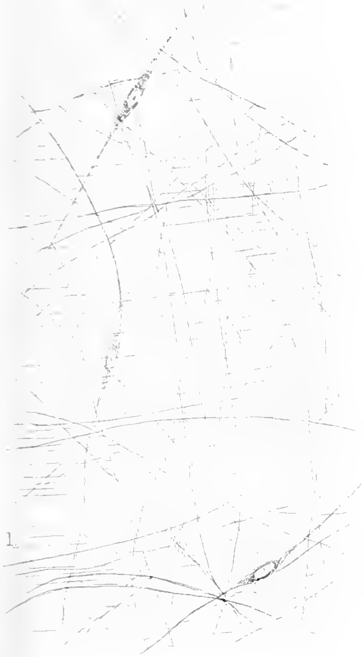

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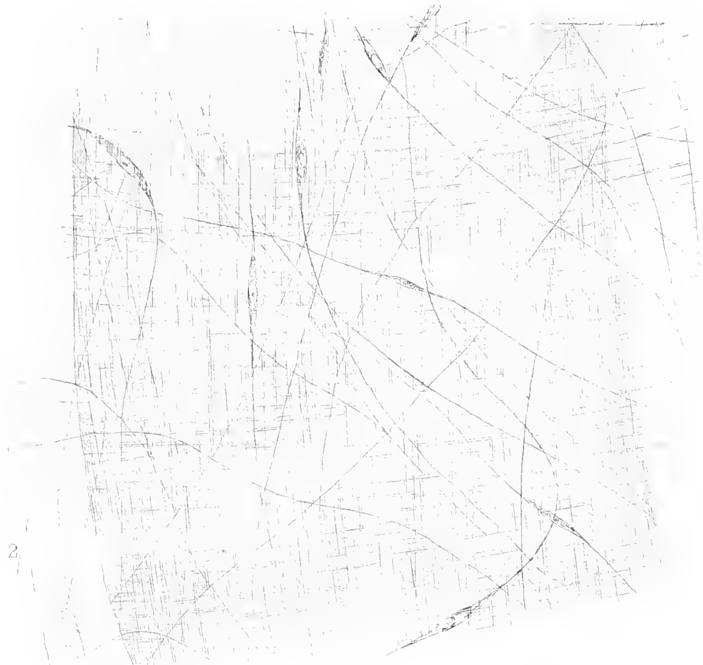

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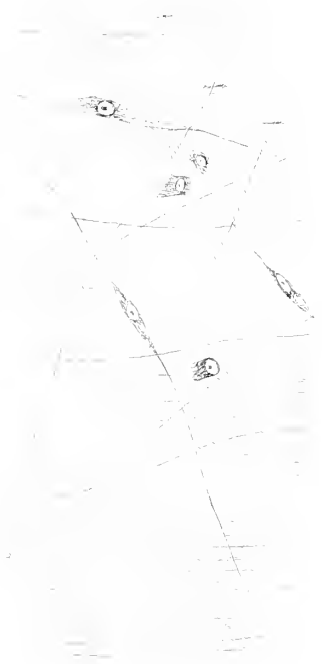

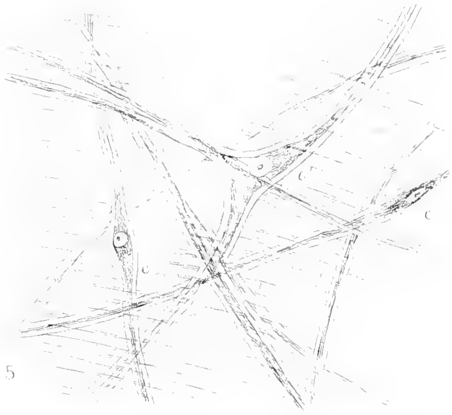

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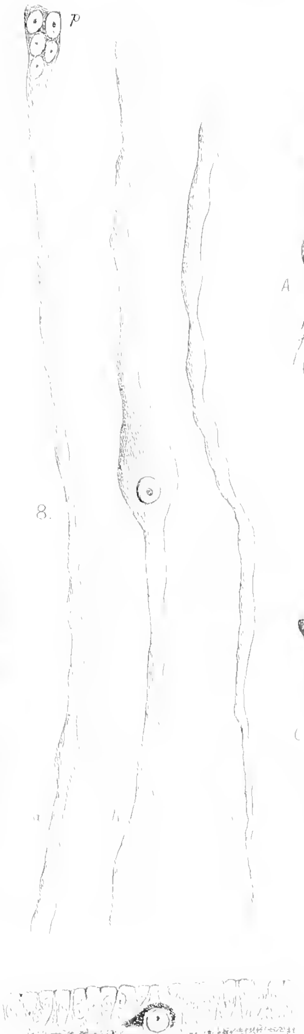

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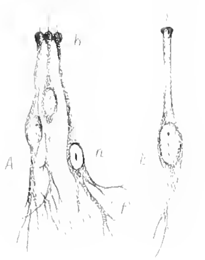

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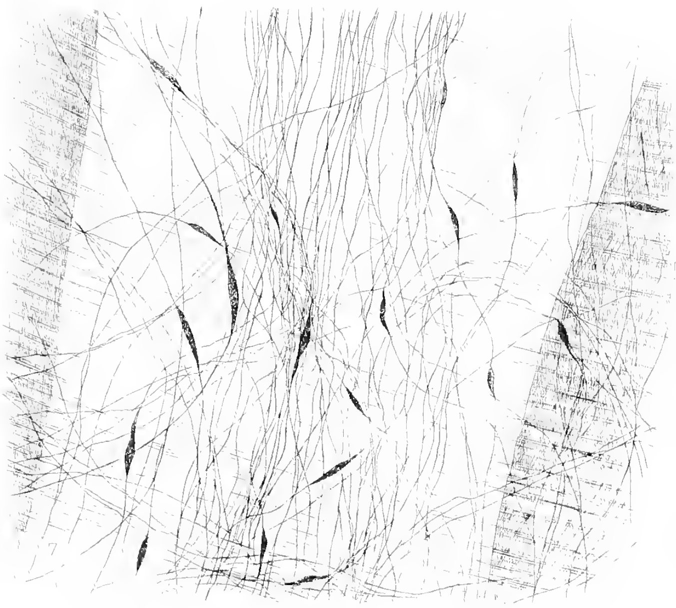

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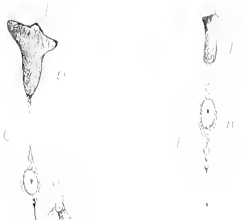
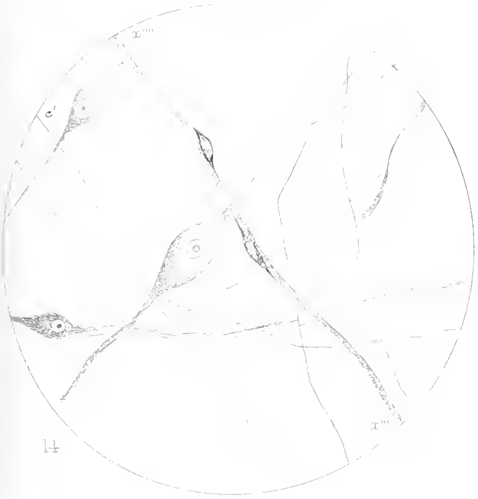


West Newman lith.

11

12

13



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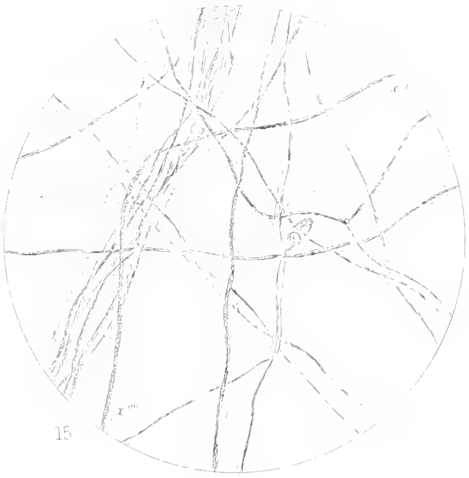

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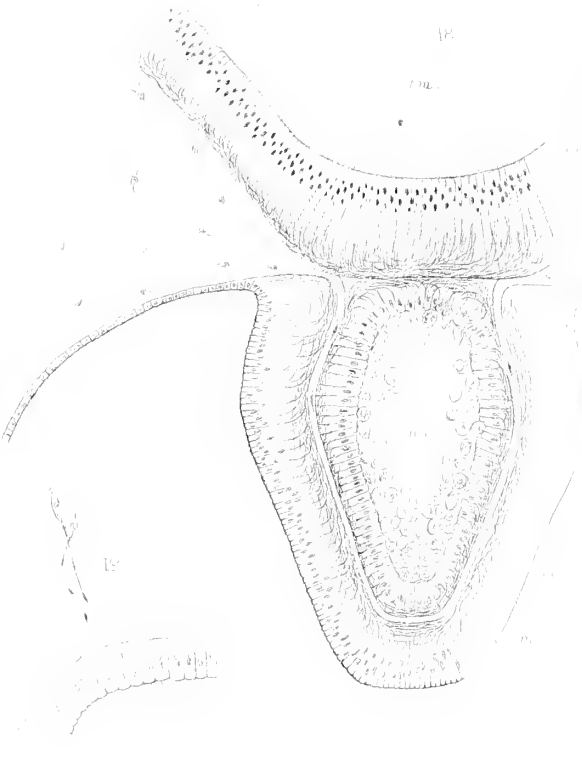
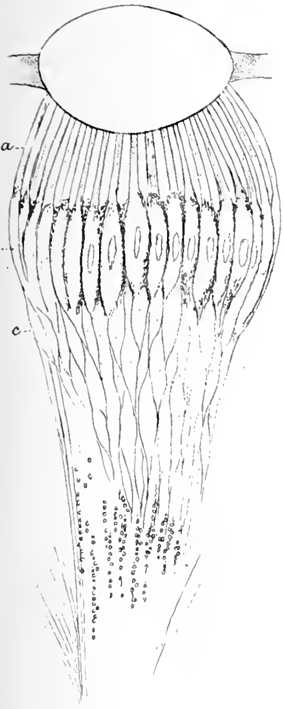


18

19

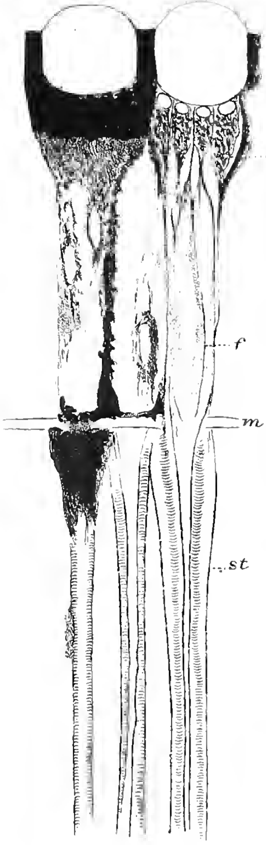
West, Newman lith.

1



1a

2



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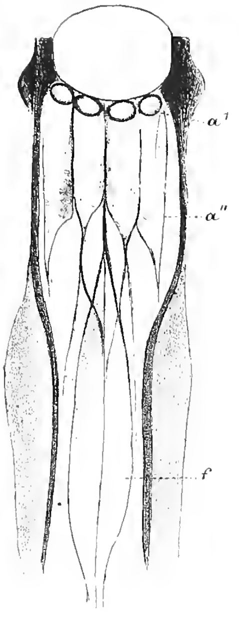

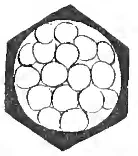

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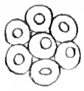
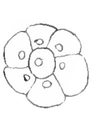
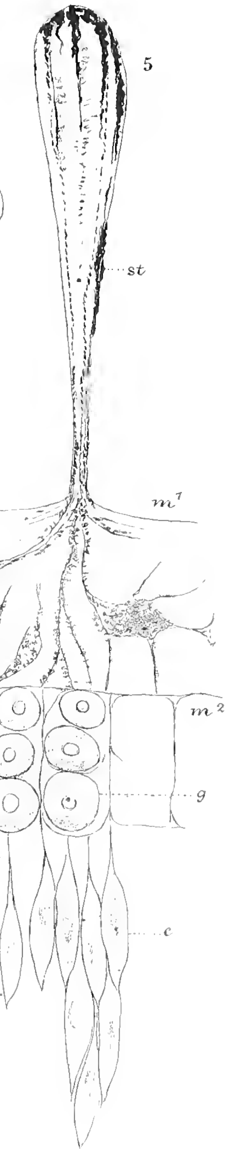




4a



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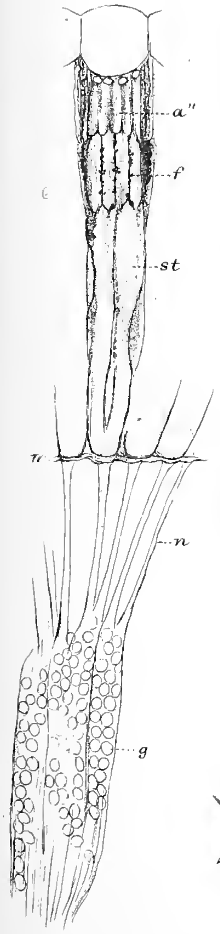

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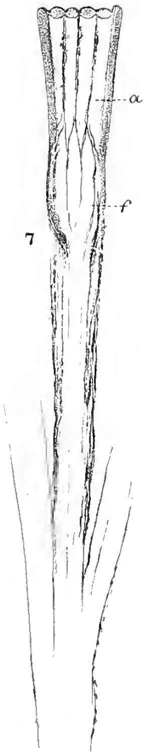

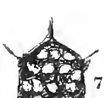

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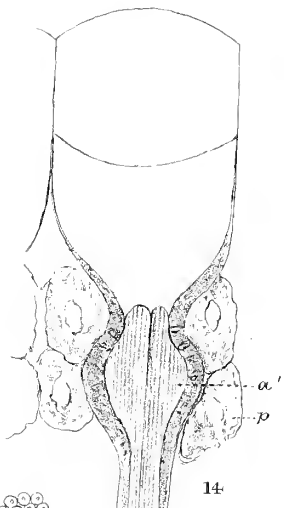

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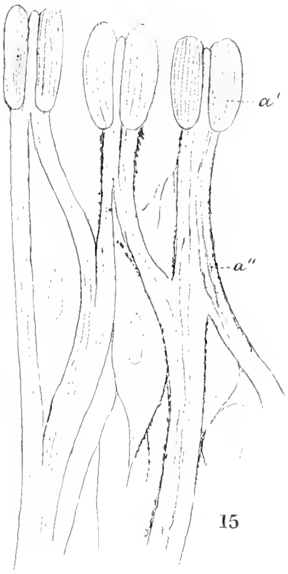

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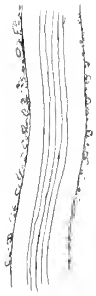

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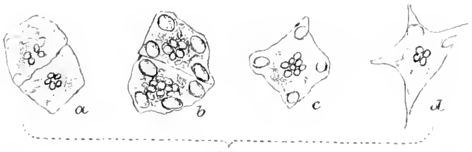

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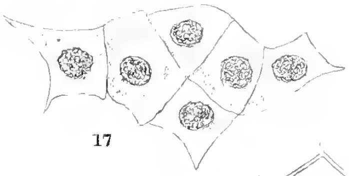

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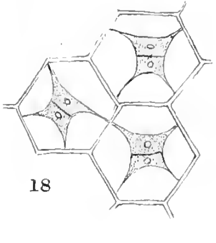

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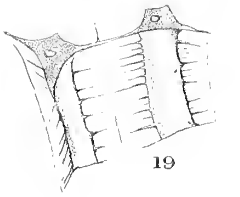

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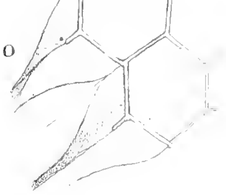


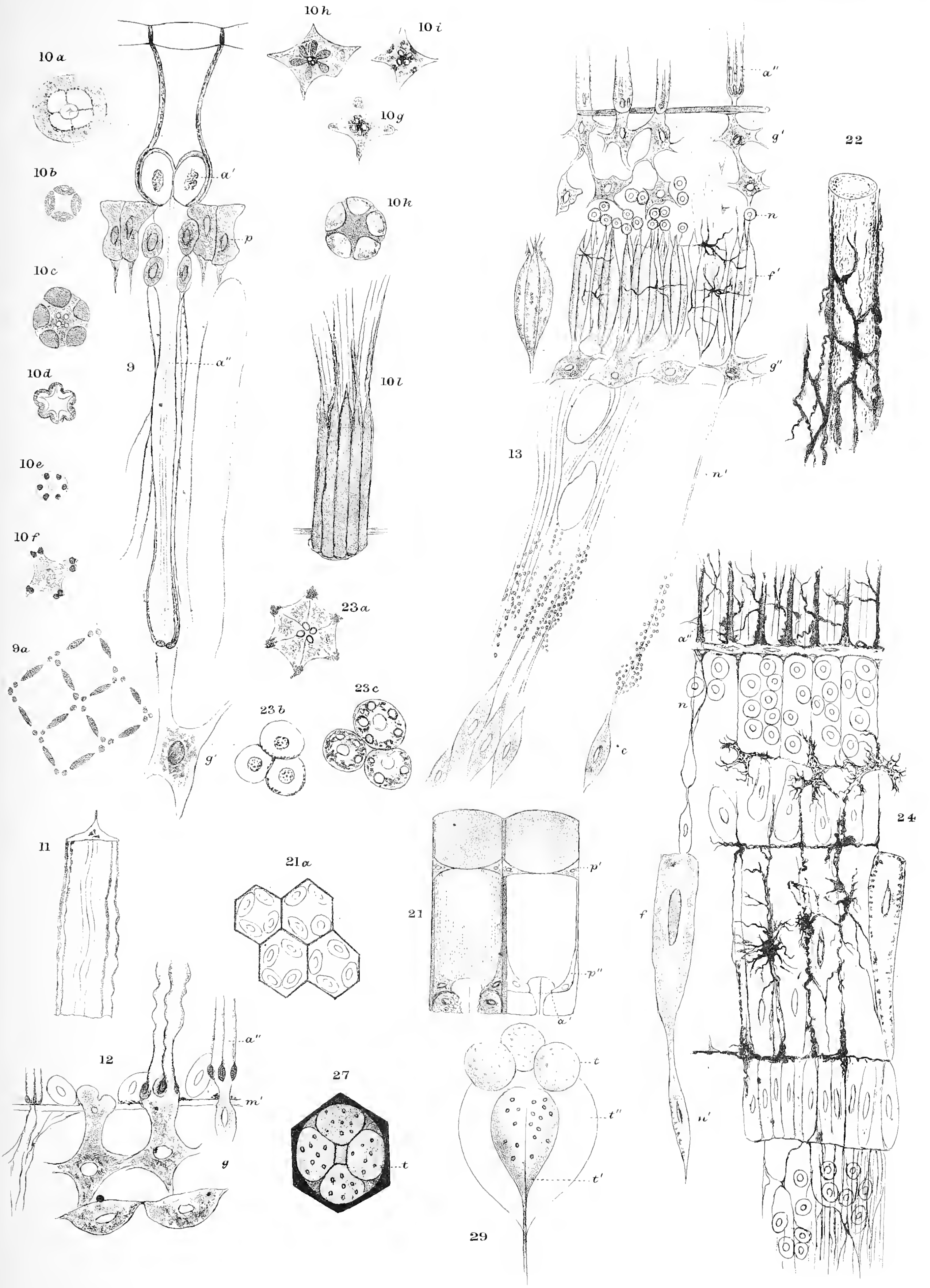
20

W.H. Wesley lith.

West, Newman & Co imp.

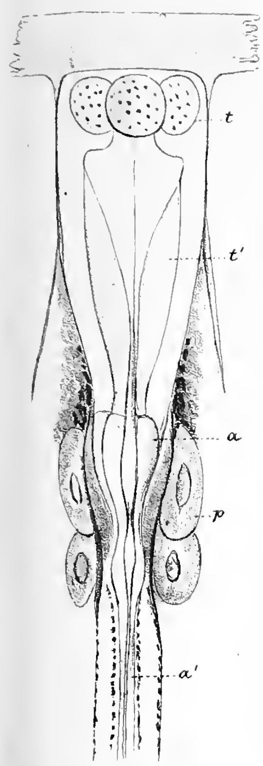
Lowne.

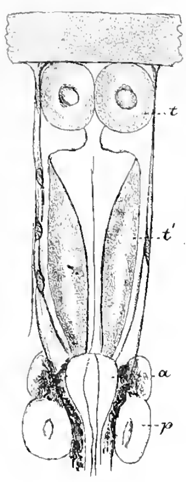
Phil. Trans. 1878. Plate 53.

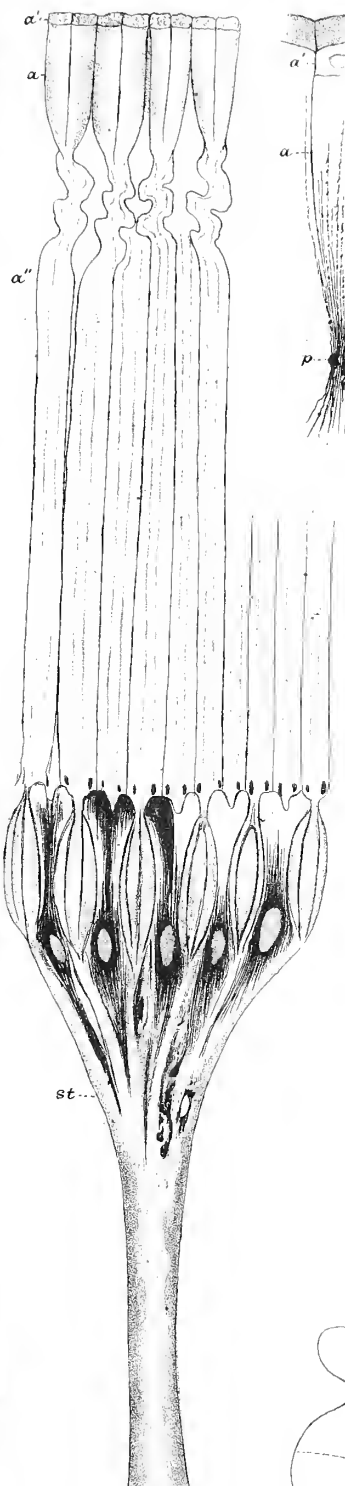


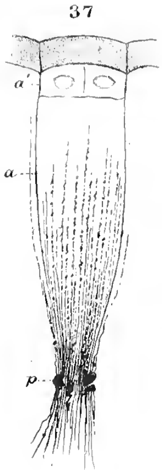
W.H. Wesley lith.

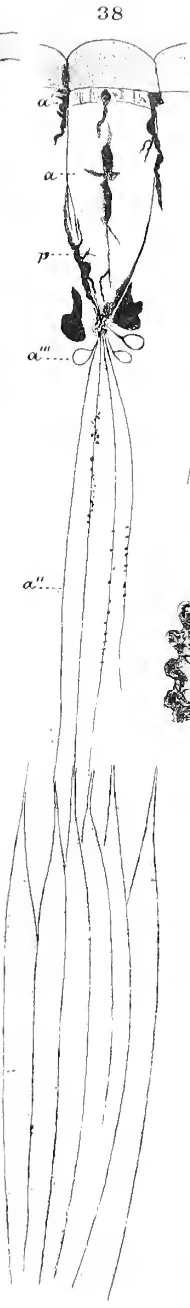
West, Newman & Co imp.

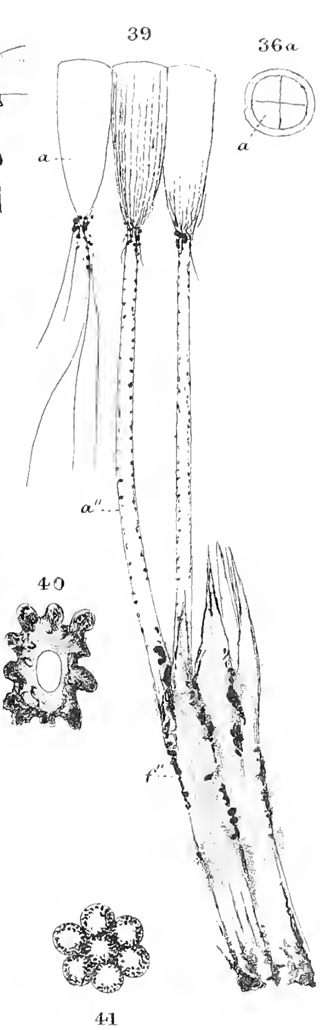


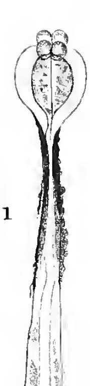


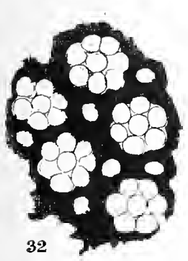


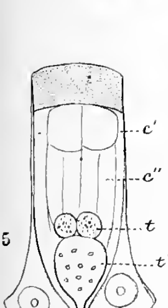


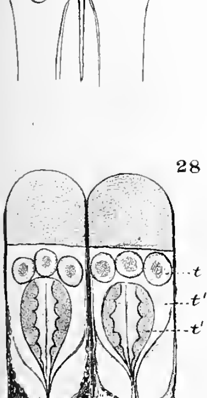


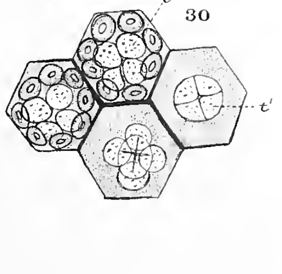


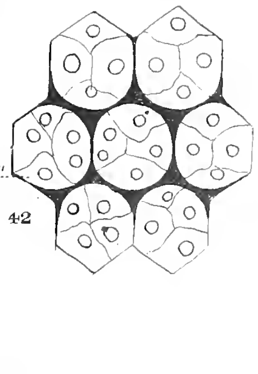


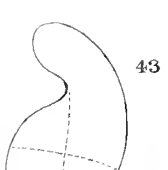


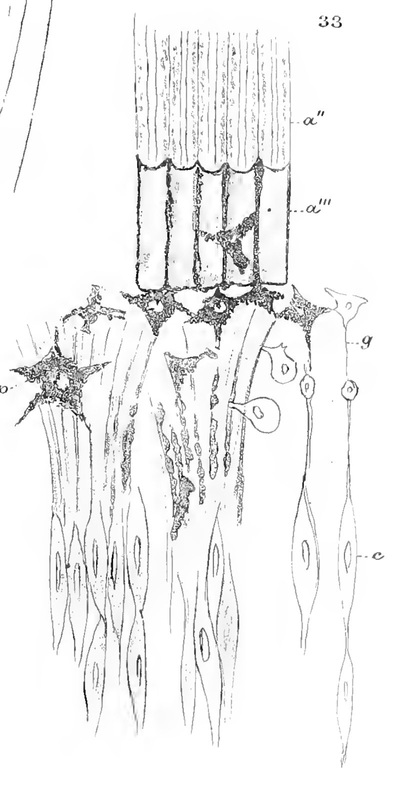












W. H. Wesley lith.

West, Newman & Co imp.

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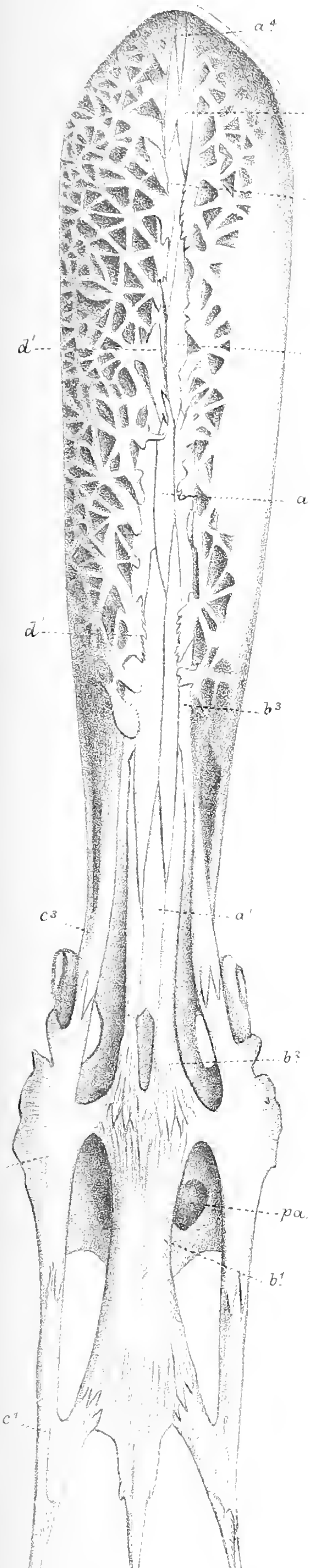

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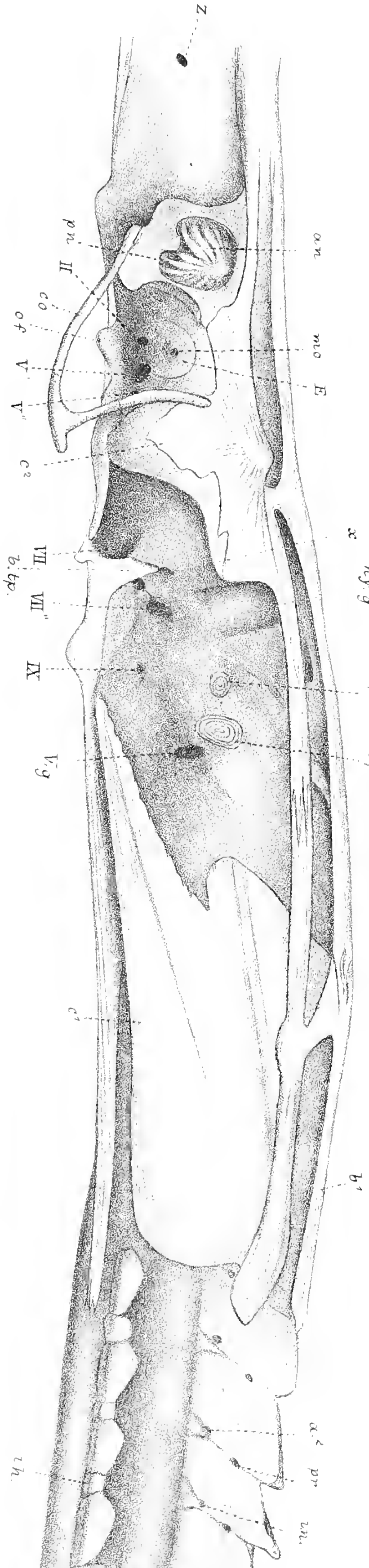

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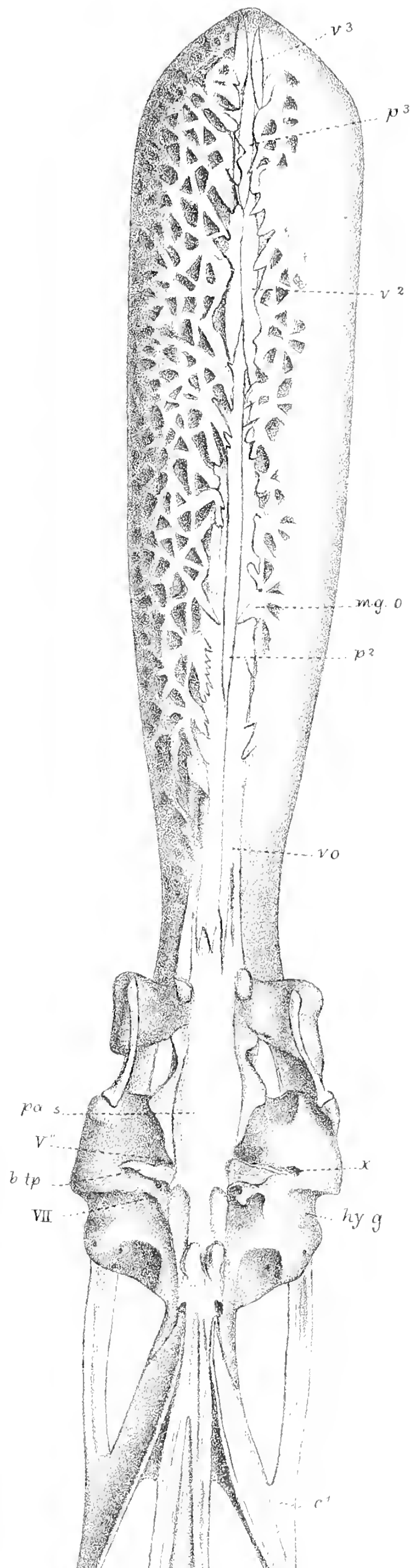
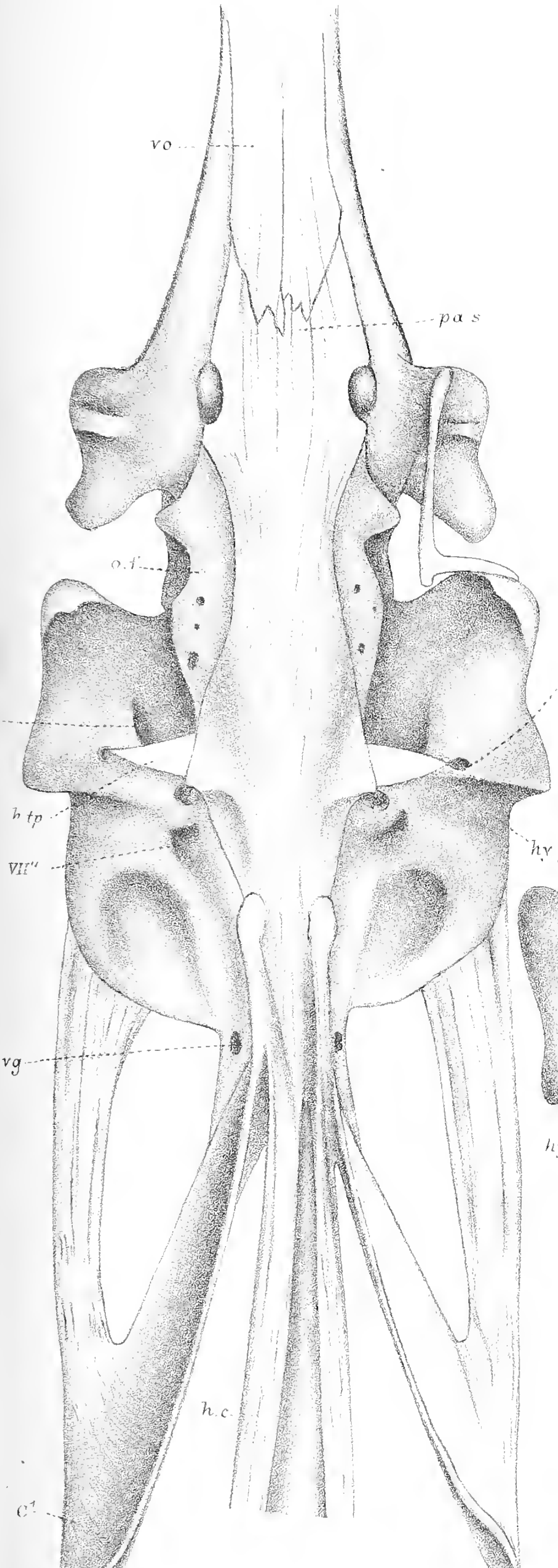


A.C. Haddon del. ad nat.
G. West Jun^r. lith.

West Newman & C^o imp.

4.



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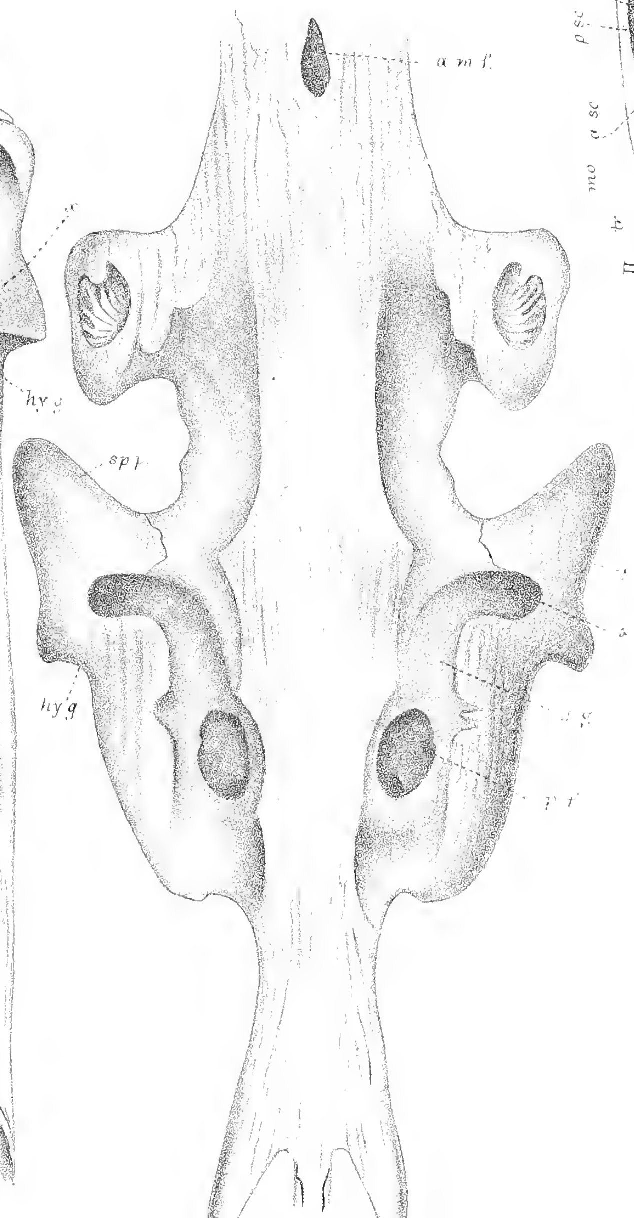

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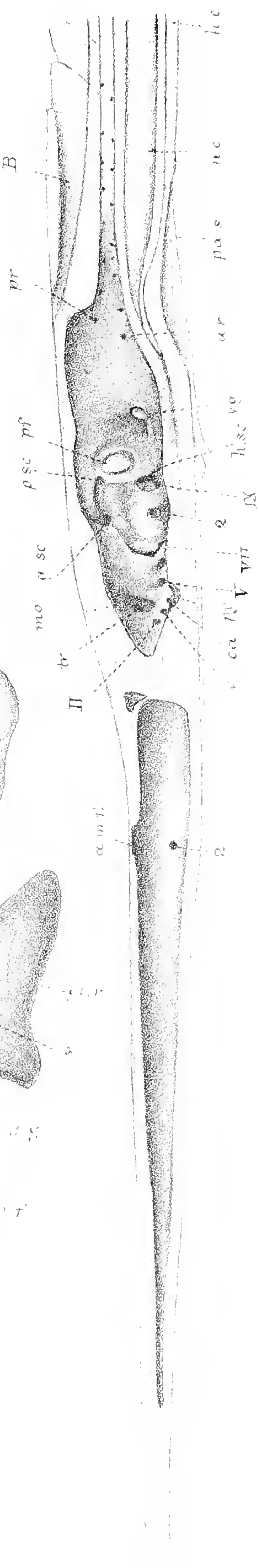
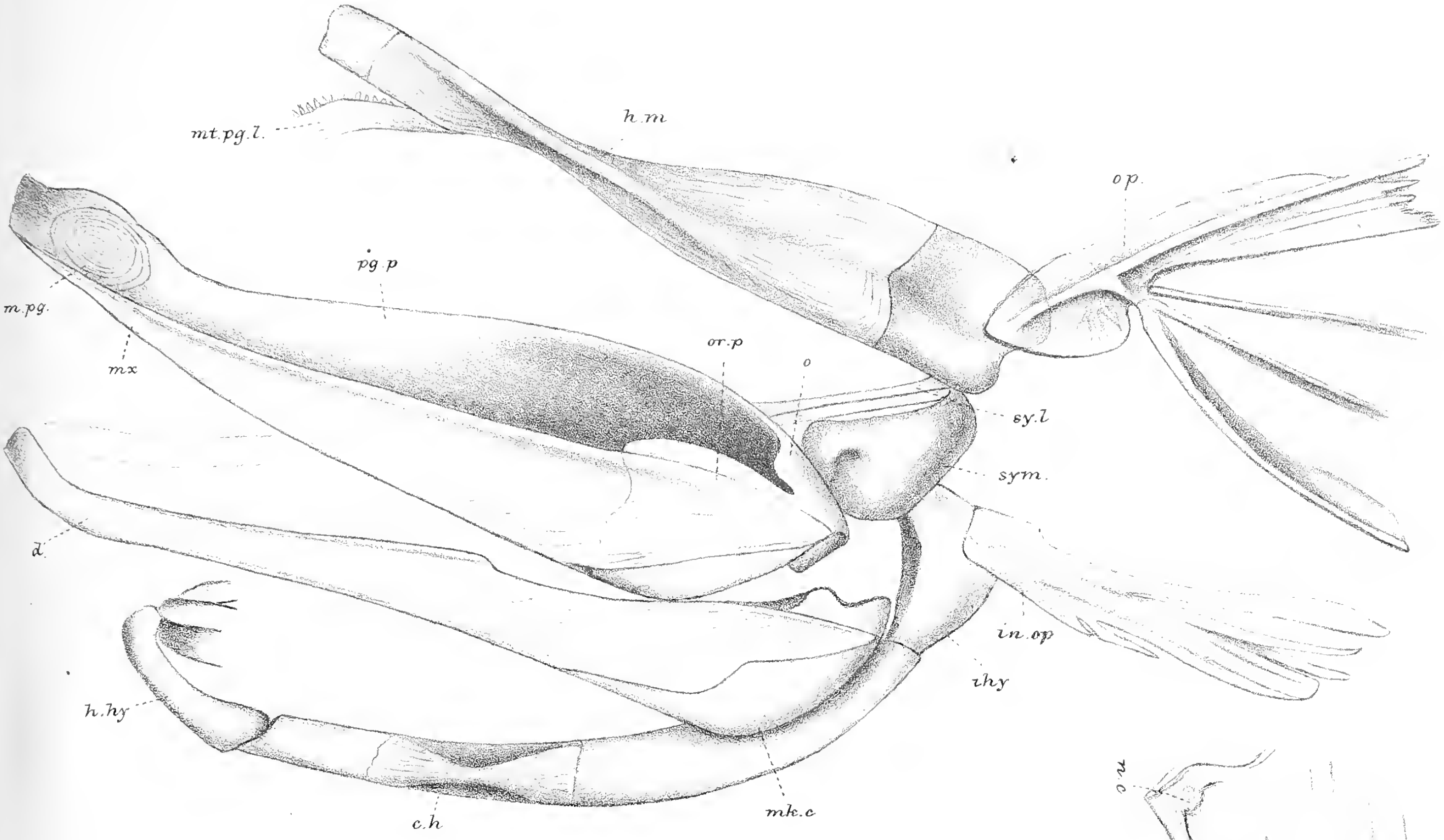
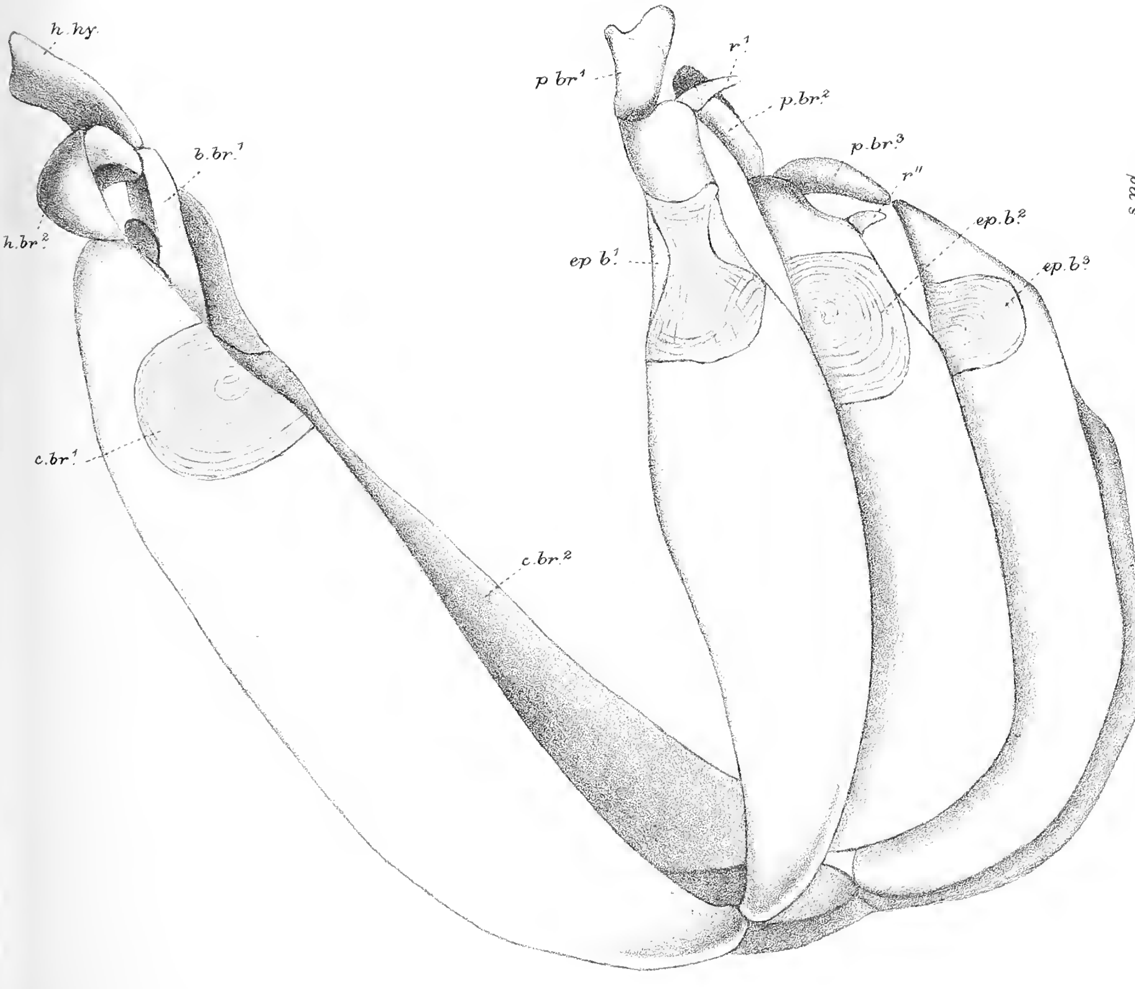
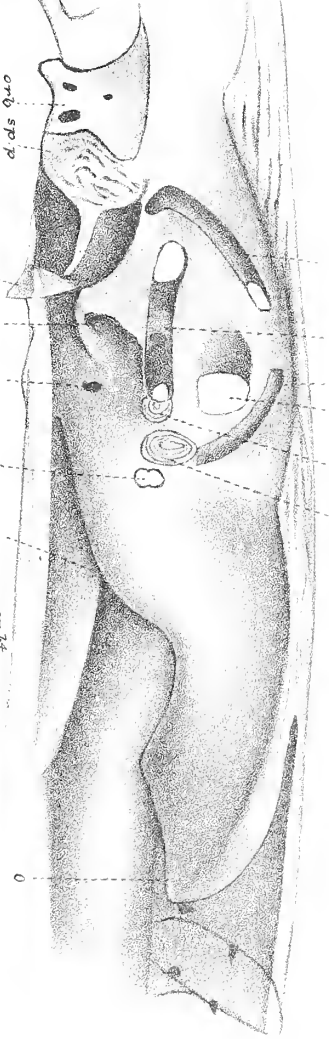


A.C. Haddon del. ad nat.
G West Junr. lith.

West Newman & Co. imp.



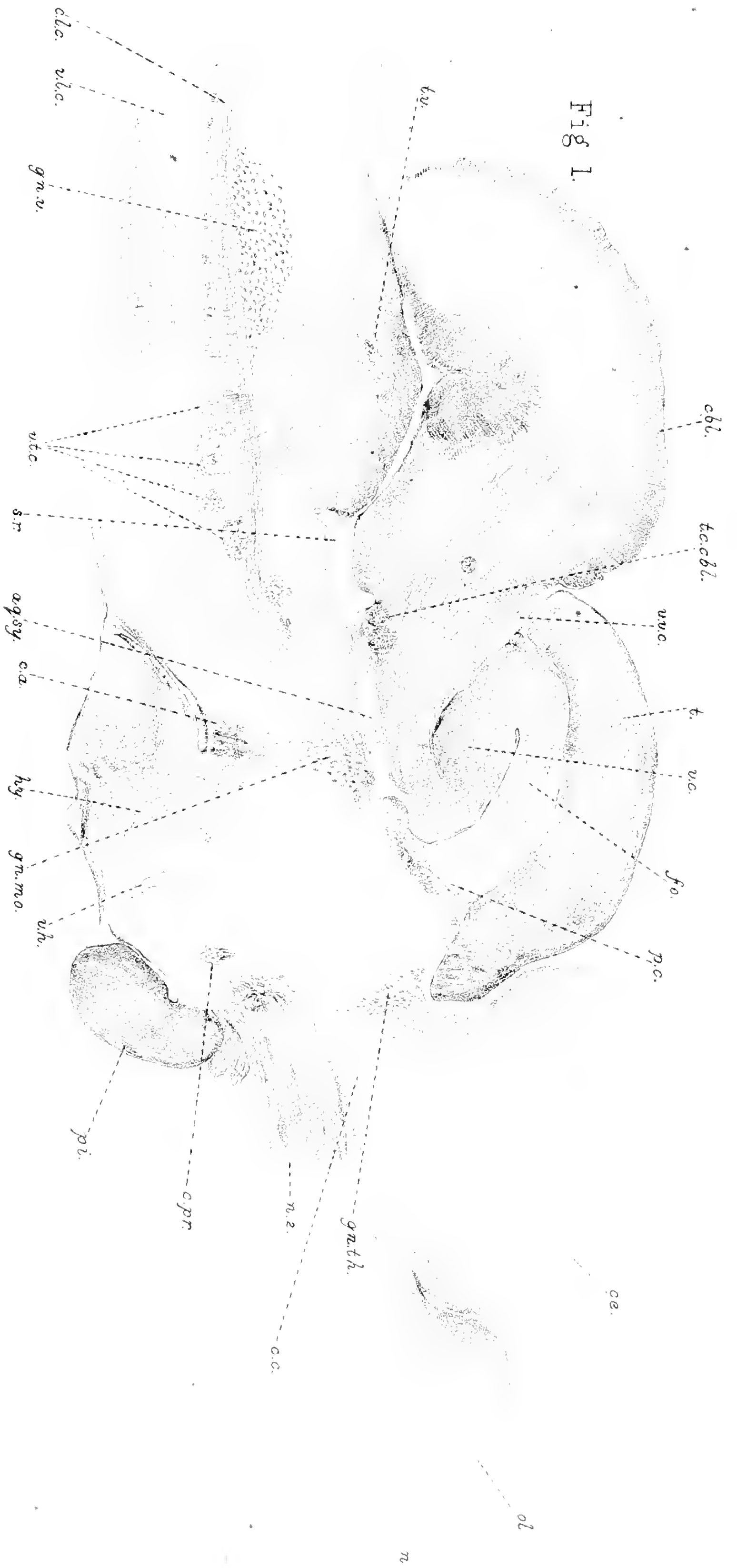




A.C. Haddon del. ad nat.
G. West Junr lith.

West, Newman & Co imp.

Fig 1.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gn.v.
v.t.c.
s.r
tc.cbl.
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t.
vc.
hy
fo.
gn.mo.
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pi.
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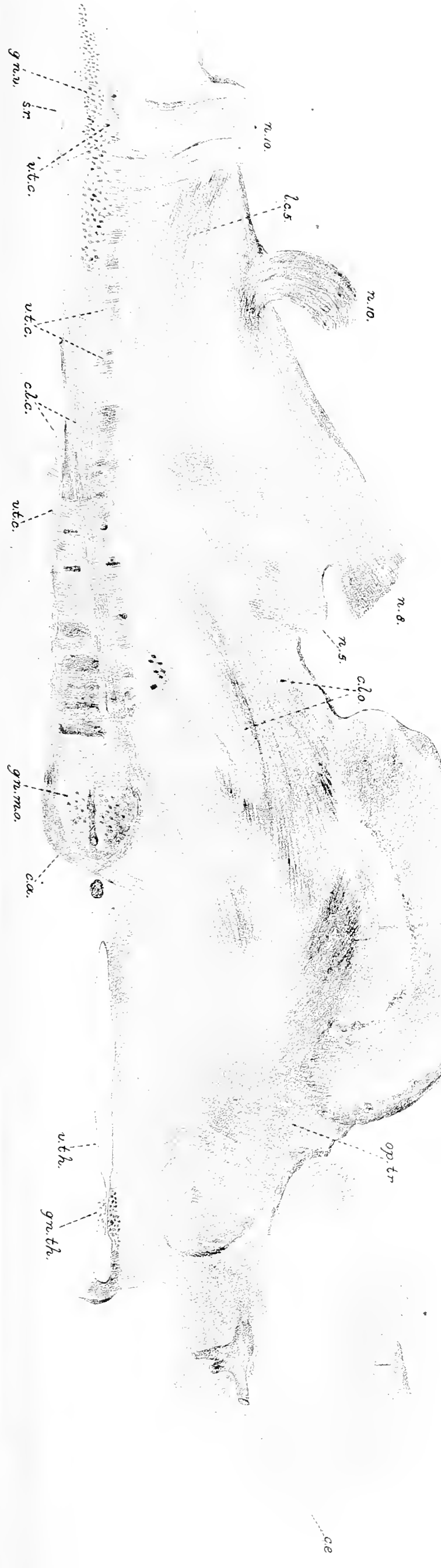
gn.th.
ce.
c.c.
ol
n.1.

Fig 2.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n.10
l.c.5
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n.8.
n.5.
c.l.o.
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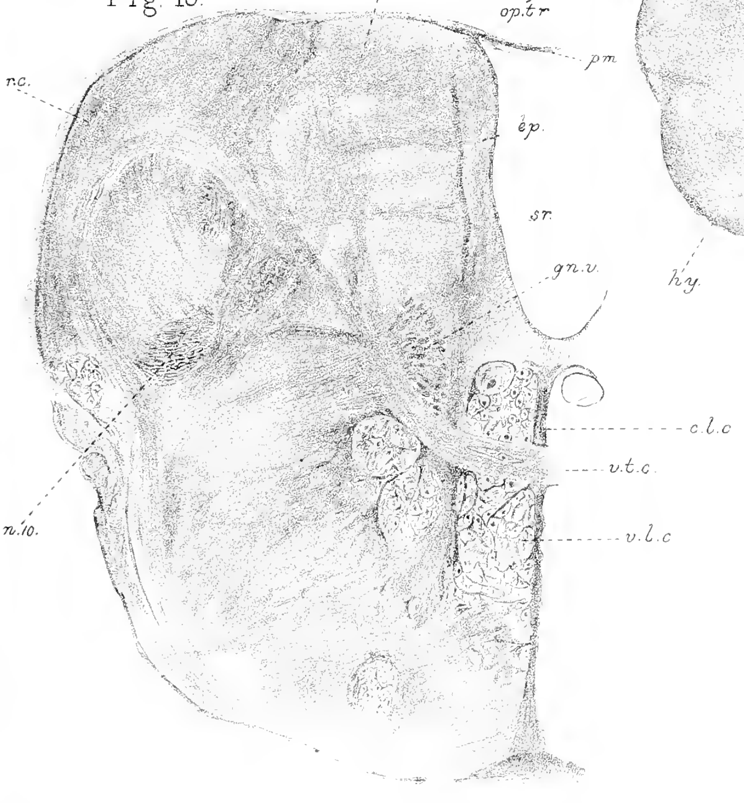
l.op.
op.tr
v.th.
gn.th.
ce

Fig. 3.

Fig. 10.



A. Sanders, del^t.

M^c Farlane & Erskine, Lith^{rs} Edin^r

Fig. 4.

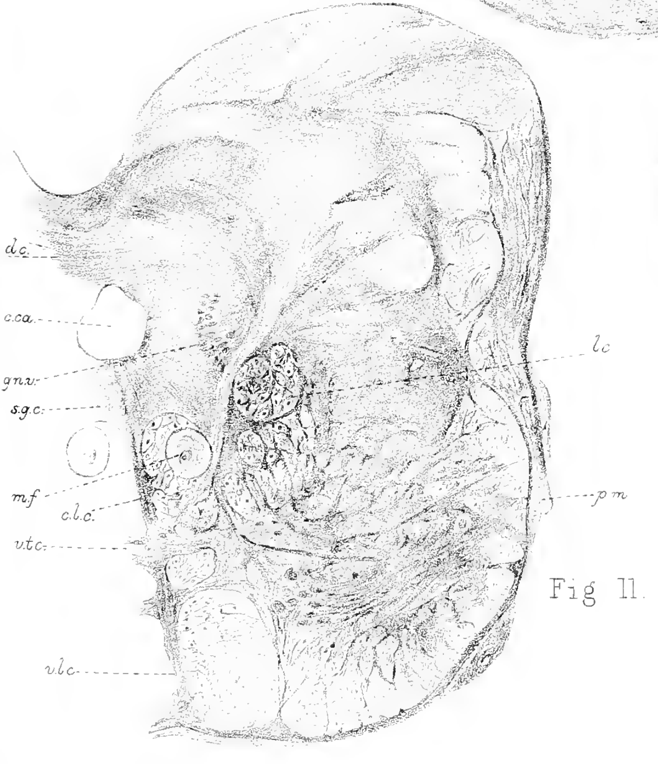


Fig 11.

A. Sanders, del.t

M.cFarlane & Erskine Lith.rs Edin.r

Fig. 5.

Fig. 23.

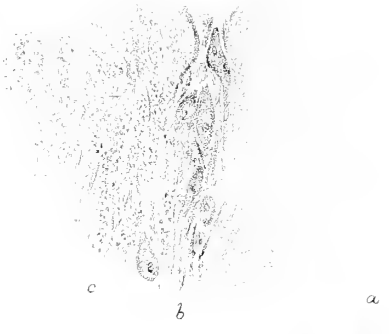


Fig. 14.

A. Sanders delᵗ. McFarlane & Erskine, Lithʳˢ Edinʳ.

Fig. 6.

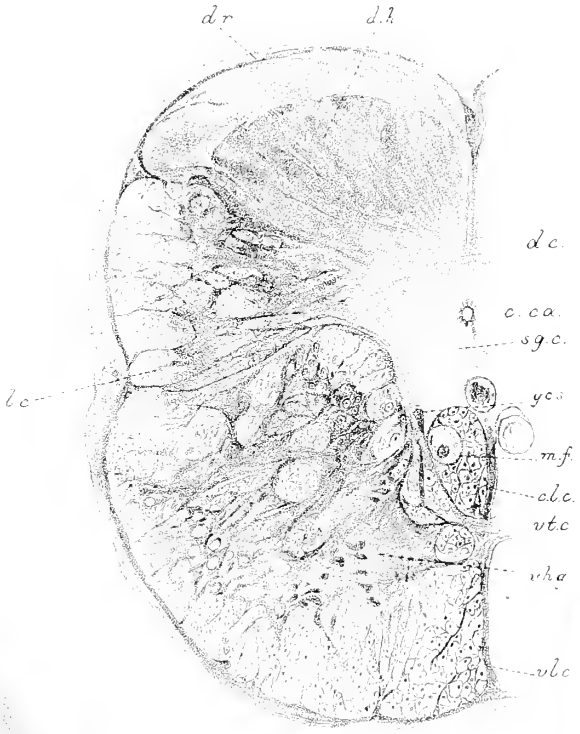


Fig. 12.

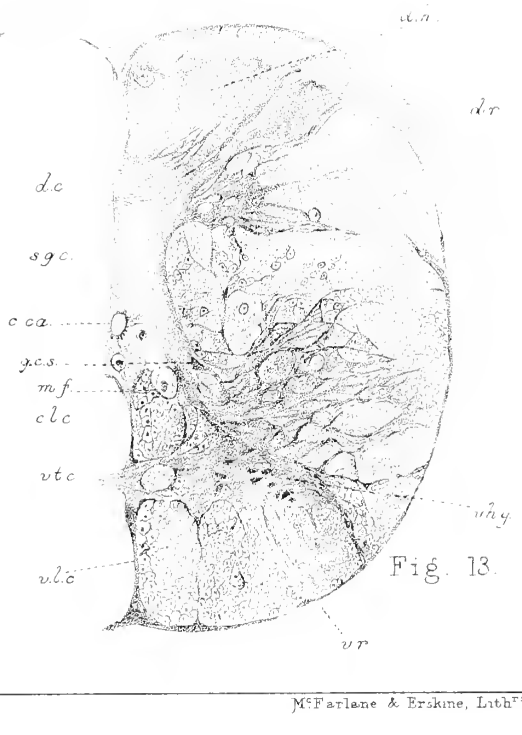


Fig. 13.

A. Sanders, del^t.

M^c Farlane & Erskine, Lith^rs Edin^r

Fig 15.

Fig 17.

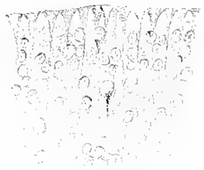


Fig. 16.

Fig 7

A. Sanders, delt
McFarlane & Erskine, Lithrs, Edinr

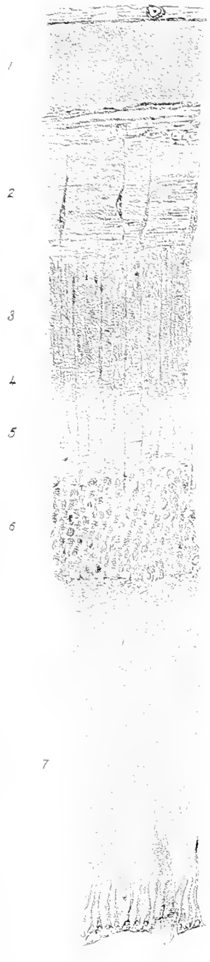


Fig. 18 × 170.

Fig. 18.

Fig. 19.

Fig. 8.

Fig. 20.

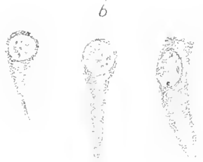


Fig. 20.

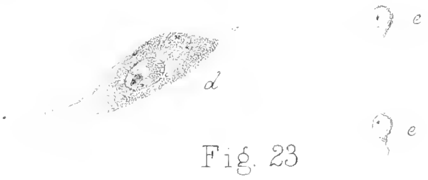


Fig. 23

A. Sanders, del.t
McFarlane & Erskine, Lith.rs Edin.r

Fig. 22 a.

Fig. 9.

Fig. 22 b.

Fig. 22 c.

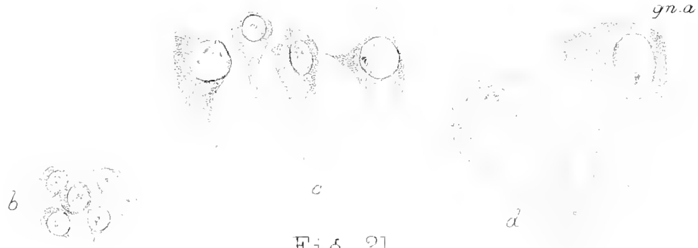


Fig. 21.

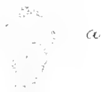
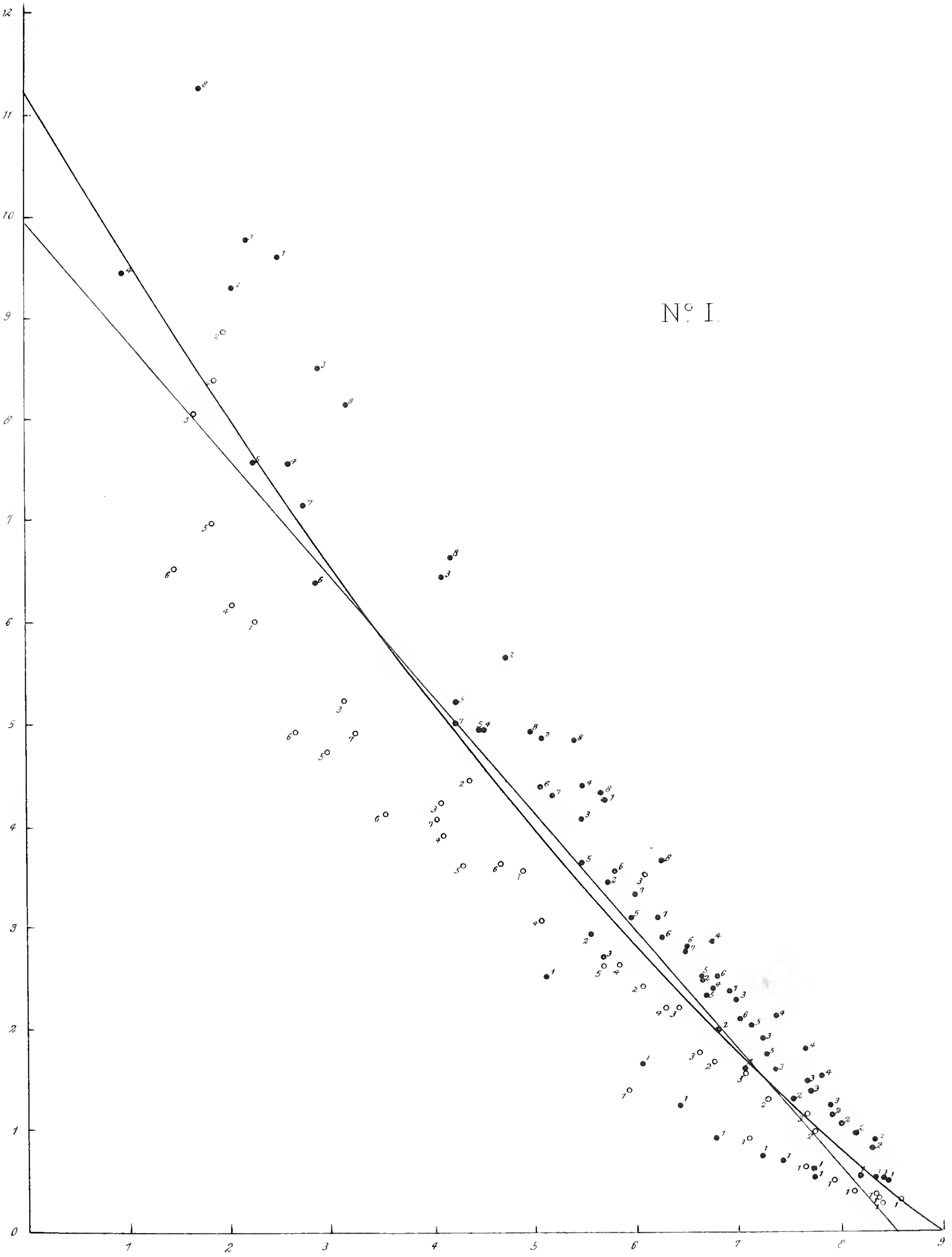


Fig. 21 a.

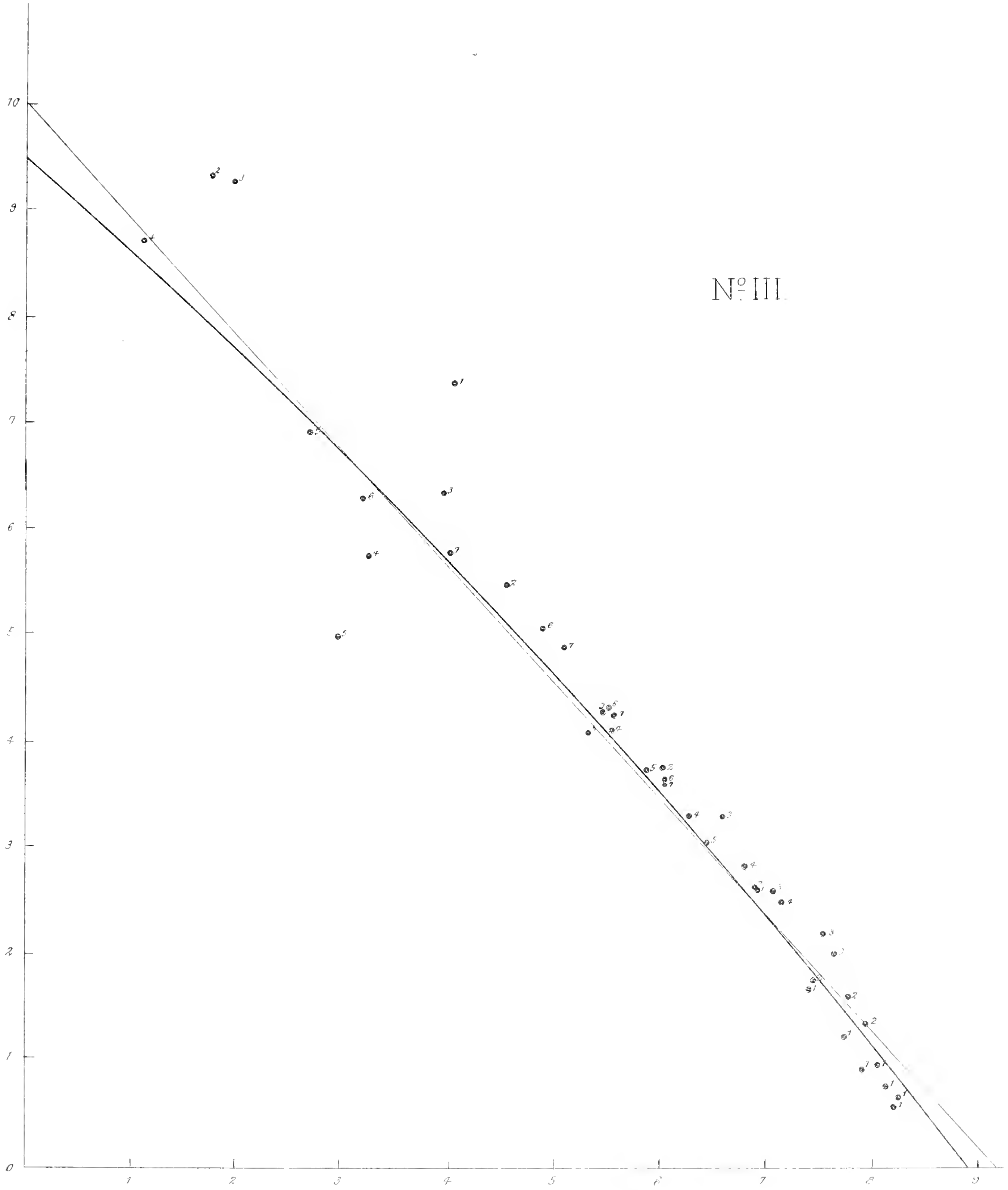
Fig. 24.

A. Sanders, delt. McFarlane & Erskine, Lithrs. Edinr.



No. I.

Harrison & Sons, lith. St Martin's Lane W.C.


N.º III.
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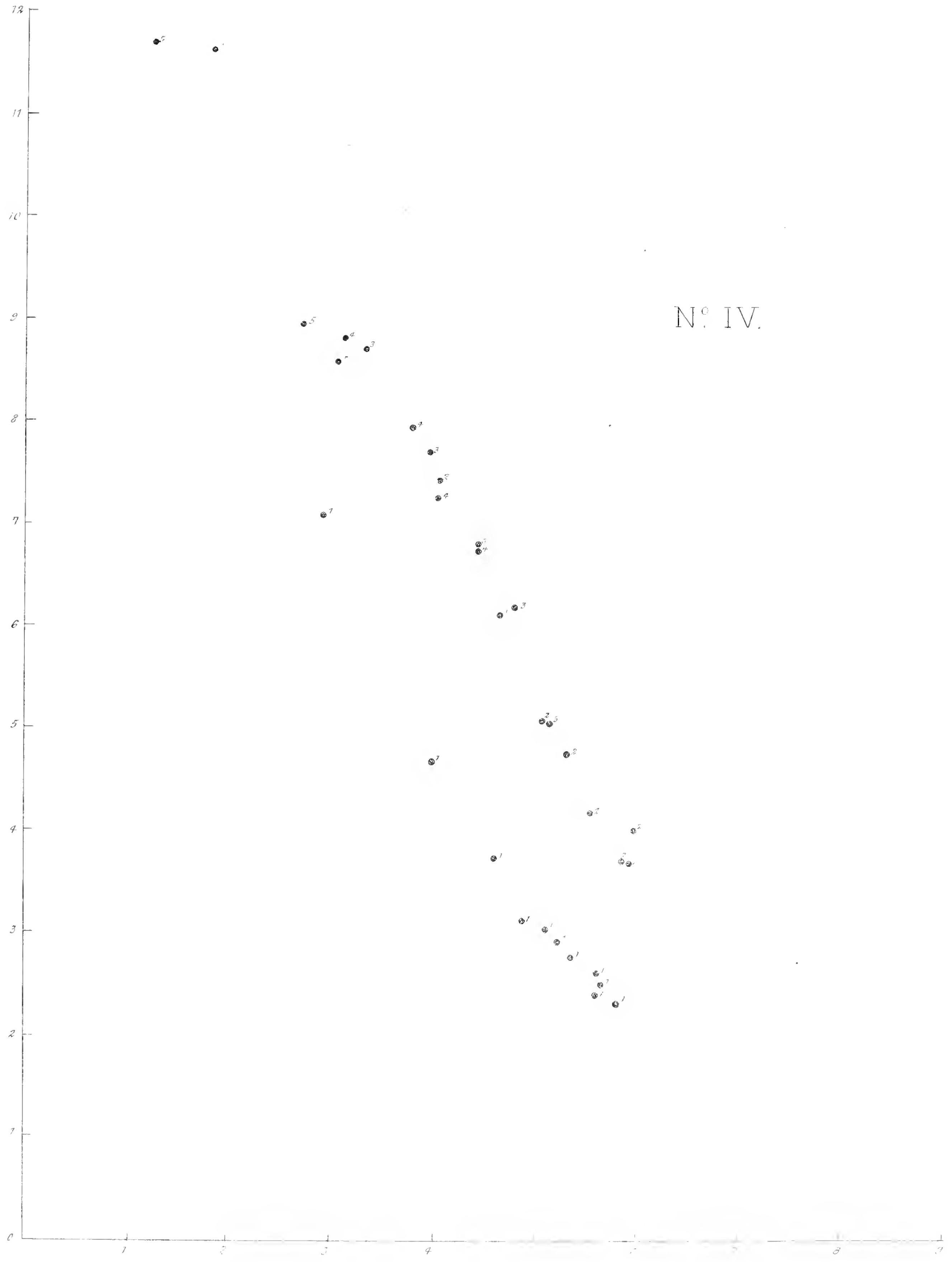


Nº IV.

[illegible]

Fig. 3.

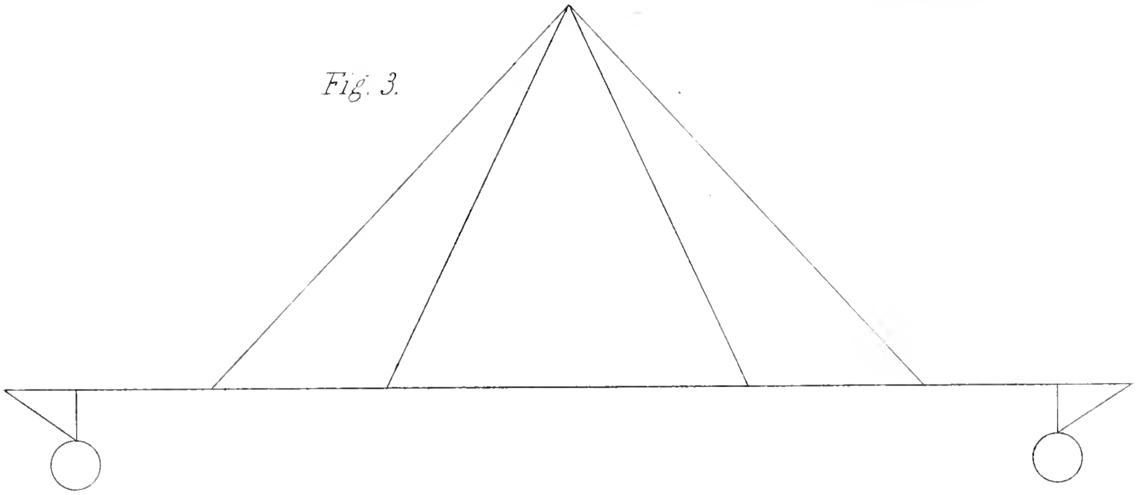
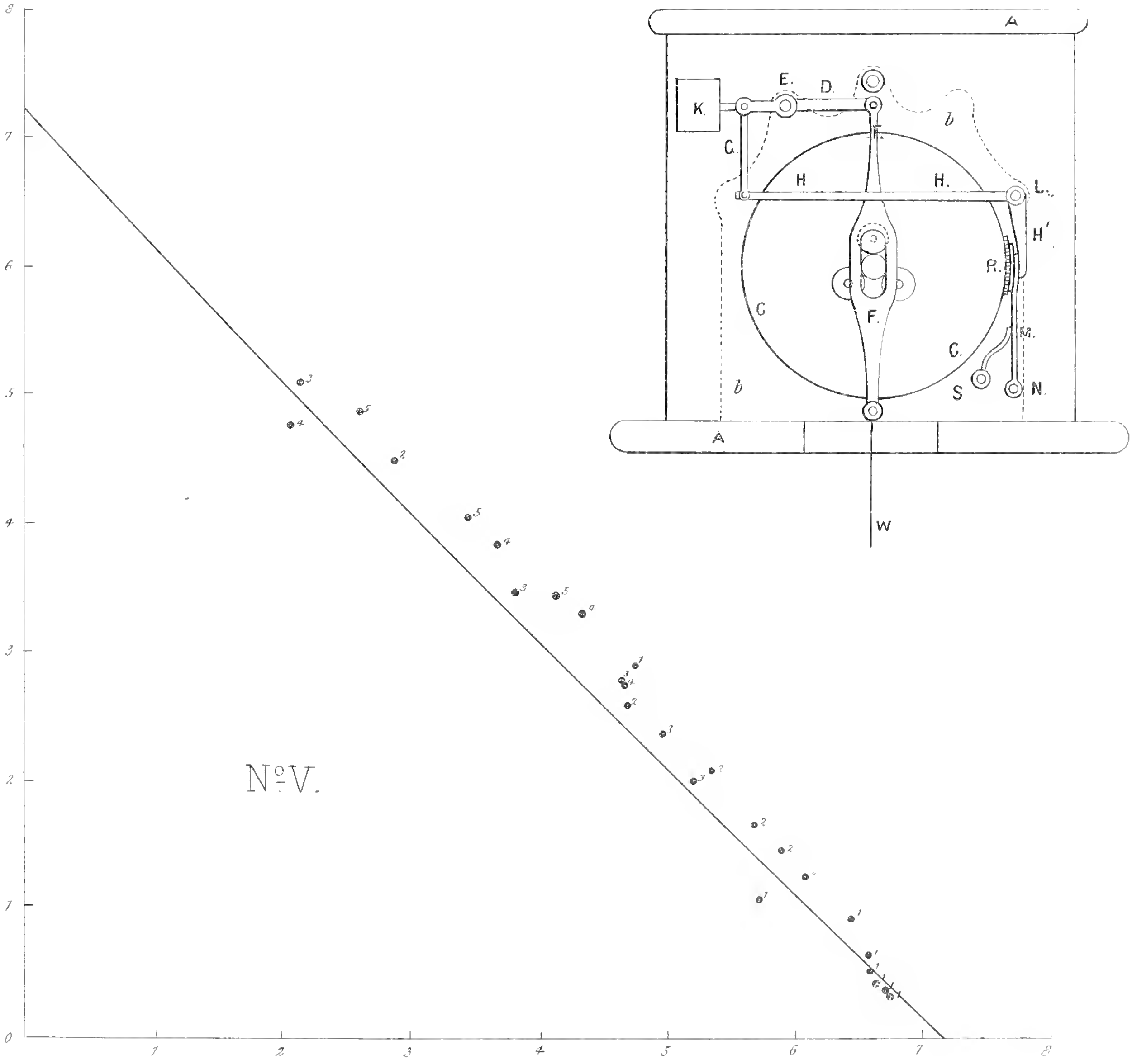


Fig. 4



Harrison & Sons, Lith. S^t Martin's Lane W.C.